$\eta_{\parallel}$ and $\eta_{\perp}$ are quite independent, and hence so are the expansions for $\eta_{\perp}$ and $\eta_{\parallel}$. That these independent expressions give rise to expansions that satisfy the SSR is greatly reassuring.

In order to see the effect of these new terms in the ϵ expansion for γ_1 and γ_{11}, we show in the accompanying Table the sums to order ϵ and order ϵ^2 of γ_1 and γ_{11} as well as the best series estimates. In every case the $O(\epsilon^2)$ term has effected a substantial improvement over the sum to $O(\epsilon)$, and in three dimensions all sums to $O(\epsilon^2)$ are within 3% of the series estimates. The agreement obtained by using $\gamma_{11}=\nu-1$ and summing to $O(\epsilon^2)$ is significantly worse.

We conclude that surface scaling is well supported by our calculations, but that the relation $\gamma_{11}=\nu-1$ due to Bray and Moore is incorrect.

After submission of this Letter, we became aware of the recent calculations of Diehl and Dietrich,[15] who confirm our result for $\eta_{\parallel}$ by an alternative calculation. They have also derived the scaling laws for surface exponents.

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Infinite Susceptibility Phase in Random Uniaxial Anisotropy Magnets

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The leading terms in the magnetic equation of state are calculated for models with random fields and random uniaxial anisotropies for dimensionalities $d<4$. In the random anisotropy case we find a new low-temperature phase, in which the magnetization vanishes but the zero-field susceptibility is infinite, because of algebraically decaying correlations. No phase transition is found for the random field case.

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It has recently been realized theoretically that when fluctuations are taken into account, then various types of randomness destroy long-range magnetic order in Heisenberg-like systems ($m>1$ spin components) with realistic dimensionalities $d<4$. Of particular interest are systems with (a) *random magnetic fields*,[1] where the randomness enters via

$$\sum_x [\vec{h}(x)\cdot\vec{S}(x)], \quad [\vec{h}(x)]_{av}=0, \quad [|\vec{h}(x)|^2]_{av}=\Delta,$$

and (b) *random uniaxial anisotropy*,[2] where the randomness arises via $-D\sum_x[\hat{n}(x)\cdot\vec{S}(x)]^2$, where $\hat{n}(x)$ is a unit vector with random direction. Sys-

tems with random off-diagonal exchange interactions (e.g., random isotropic dipolar interactions) should exhibit the same behavior as in case (b).[3]

Although various arguments exist for showing that these models have no long-range order for $d<4$, not much is known about their actual magnetic properties. In this paper we present the first calculation of the equation of state for these models.

In the case of random uniaxial anisotropy we find that *although there is no long-range order (the magnetization M goes to zero with the field H) at any finite temperature T, the magnetic susceptibility ($\chi \sim M/H$) diverges at a finite temperature T_c, and remains infinite for all $T<T_c$.*[4] The shape of the isotherms has the form $H \sim M^{\delta}$ at T_c and $H \sim M^{\delta_1}$ for all $T<T_c$. Neglecting conventional critical fluctuations we find

$$\delta=(10-d)/(6-d) \text{ and } \delta_1=(8-d)/(4-d).$$

We have also calculated the two-spin longitudinal- and transverse-correlation functions for $T<T_c$. In the limit $H \to 0$ (i.e., $M \to 0$) both approach the limit $1/q^2$, giving rise to *a power law decay* of the form $1/r^{d-2}$.

There is no such transition in the random-field case, for which χ remains finite for all $T>0$.

The random anisotropy model was originally formulated to account for the magnetic behavior of amorphous rare-earth alloys.[5] Much work has been devoted to mean-field treatments of the model, which predict a ferromagnetic state, and various experiments have been interpreted as agreeing with these predictions.[6,7] Particular attention has been given to measurements of the magnetic equation of state of such alloys. These are usually presented in terms of "Arrott" plots[6,8-10] in which the ratio H/M is plotted (for fixed temperature T) versus M^2. Mean-field theory predicts that H/M should be linear in M^2 [Fig. 1(a)]. In usual ferromagnetic cases, isotherms in the paramagnetic phase ($T>T_c$) intercept the H/M axis at the inverse susceptibility, χ^{-1}. For $T<T_c$ they intercept the M^2 axis at the spontaneous magnetization[11] M_s^2.

In many of the existing experiments on amorphous rare-earth alloys,[6,8,9] Arrott plots were extrapolated from large values of H/M down to $H/M=0$, apparently yielding a finite M_s^2. However, the experimental curves show deviations from linearity, and it has not been clear how exactly to extrapolate them to low fields.

In this Letter we relate the *theoretical* prediction concerning the *absence of long-range-order*

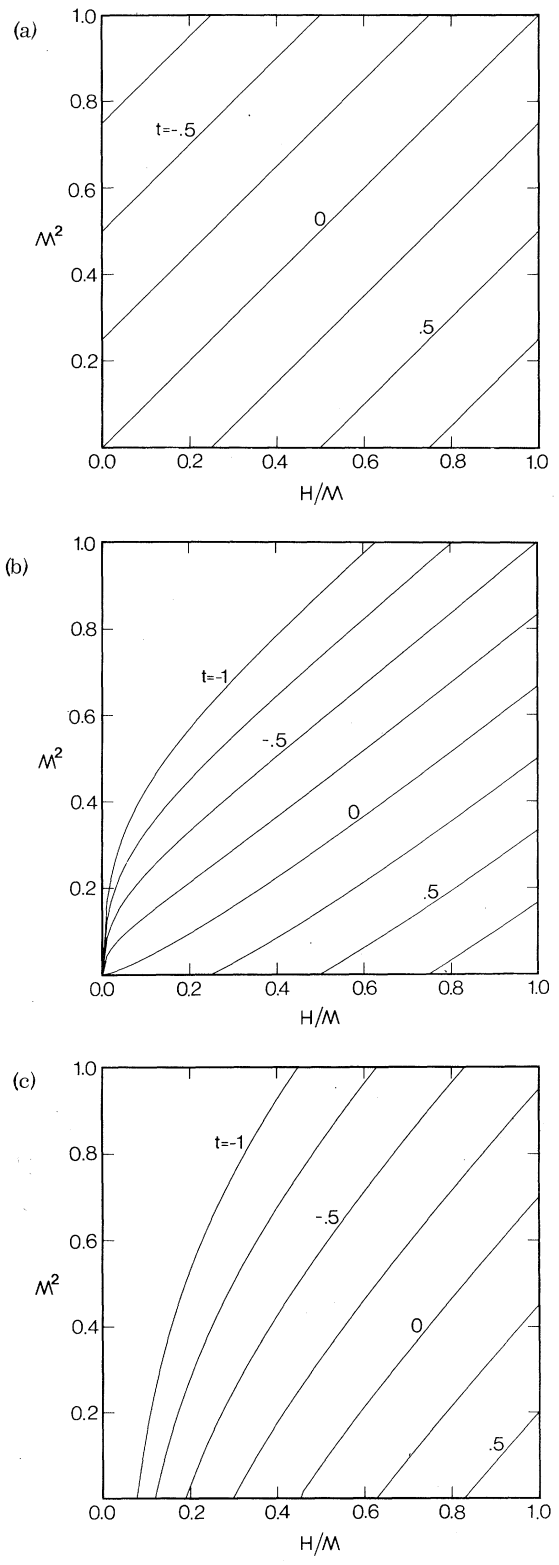

FIG. 1. Arrott plots. The numbers on the isotherms indicate the values of t. (a) Nonrandom case, (b) random anisotropy case, and (c) random field case.

with the *Arrott plot experiments*. Earlier theories dealt either with[1,12,13] $d>4$ or found an instability of the uniform nonrandom behavior for $d<4$.[1-3,14] Although that instability has been conjectured to yield a spin-glass ordering for large random uniaxial anisotropy,[15] no clear statements have been made as regards the detailed nature of the magnetic behavior of systems with random fields or random uniaxial anisotropies at $d<4$.[16]

We have calculated the equation of state of systems with random fields or random uniaxial anisotropies, to leading order in these random variables (Δ or D^2). Our results are presented in the form of Arrott plots in Figs. 1(b) and 1(c). As expected from the general arguments,[1,2] the isotherms *never intercept the M^2 axis*, confirming that for both models $M_s=0$. In the random anisotropy case [Fig. 1(b)], there exists a temperature T_c for which the isotherm first reaches the origin (as $H\sim M^{\delta}$). All isotherms for $T<T_c$ also approach the origin (as $H\sim M^{\delta 1}$). For relatively large values of H/M the isotherms look linear and might be extrapolated to a finite value of $M_s{}^2$. The effects of the random anisotropy, which give rise to the curvature of the isotherms towards the origin, begin to be felt around values of order $H/M\sim(D/J)^{4/\epsilon}$, where J is the exchange interaction and $\epsilon=4-d$. It is, therefore, important to take data for much smaller values of H/M than has generally been done[8,9] before extrapolating to $H/M\to 0$. In the random field case [Fig. 1(c)] one sees a similar curvature of the isotherms for small H/M. However, the lines never reach the origin. They always intercept the H/M axis at a finite value, yielding a finite susceptibility and, therefore, no transition.

More explicitly, we find that in both cases the equation of state may be written in the form

$$H/M=t+M^2+A(m-1)(H/M)^{-\epsilon/2}, \tag{1}$$

where $t=(T-T_c)/T_c$, T_c being the ordering temperature of the nonrandom problem, and where the units of M have been choosen to make the coefficient of M^2 equal to unity. The parameter A is given by

$$A=[a_A(D/J)^2/m(m+2)]M^2 \tag{2}$$

in the random anisotropy case and by

$$A=a_F(\Delta/J) \tag{3}$$

in the random field case. The d-dependent coefficients a_A and a_F are of order unity for $d=3$. The curves in Fig. 1 were drawn by using $\epsilon=1$ and $A(m-1)=0$, $0.5M^2$, and 0.3, respectively. Note that in the random anisotropy case $A\to 0$ as $m\to\infty$, and the effects predicted here disappear.

We briefly describe the derivation of Eq. (1), first for the random field case. We start with the usual Ginzburg-Landau-Wilson Hamiltonian with a random-field term,[1] and add a uniform field in the 1-direction, $H\sum_x S^1(x)$. We next follow the standard diagrammatic methods for calculating the equation of state[17]: We shift $S^1\to S^1+M$, and rewrite the Hamiltonian as $H=H_0+H_1$ where

$$H_0=-\tfrac{1}{2}\int(r_L+q^2)S^1(q)S^1(-q) - \tfrac{1}{2}\int(r_T+q^2)\sum_{\alpha=2}^{m}S^{\alpha}(q)S^{\alpha}(-q) \tag{4}$$

and H_1 contains the remainder. In Eq. (4), r_L and r_T are the *exact* (renormalized) longitudinal and transverse susceptibilities, which satisfy the relations $r_T=H/M$ and $r_L=(\partial H/\partial M)$. We now make a perturbation expansion in H_1 and finally average over the random fields. In addition to the usual terms[17] there arise terms involving the random fields. If we assume a Gaussian distribution of the field, only terms in which pairs of fields have been multiplied and averaged over contribute.[1] The leading term in the expression for H thus becomes $-4(\Delta/J)uM(m-1)\int_q(r_T+q^2)^{-2}$, where u is the coefficient of S^4 in the Ginzburg-Landau Hamiltonian and where Δ has been normalized by J to make the coefficient of $q^2\vec{S}(q)\cdot\vec{S}(-q)$ equal to unity. All the other terms linear in Δ can be resummed in a way similar to that used by Nelson,[18] and can be shown to be small for $H\to 0$. An explicit evaluation of the integral then yields Eqs. (1) and (3) (with the normalization $4uM^2\to M^2$).[19] Note that the "conventional" terms in the perturbation expansion for H/M do not diverge as $H\to 0$. They will at most shift the value of T_c and modify some of the power laws in Eq. (1). By themselves, these terms give rise to the usual nonclassical equation of state.[11] We believe they will not change the qualitative nature of our results.

In the random anisotropy case, the shift $S^1\to S^1+M$ generates a term $D\sum_x n^1(x)M[\hat{n}(x)\cdot\vec{S}(x)]$, which is equivalent to a local random field $\vec{h}(x)=Dn^1(x)\hat{n}(x)M$. Substituting in the random-field result then converts Eq. (3) into Eq. (2).

It should be emphasized that our calculations have been carried out only to lowest order in Δ (or D^2). In cases with very large Δ or D^2 (or very close to the origin in Fig. 1), it may be necessary to include higher-order terms.

It is interesting to note that the effect of adding a cubic anisotropy of the form $v\sum_x\sum_{\alpha=1}^{m}[S^\alpha(x)]^4$ will be to replace H/M in the last term of Eq. (1) by $(H/M+4vM^2)$. Finite (not too small) values of the spontaneous magnetization are then not excluded, and one probably has a first-order transition into a ferromagnetic phase.[3]

We hope that this paper will stimulate detailed experimental studies of Arrott plots in the limit of very low fields, in order to search for our predicted new phase and to measure the exponents δ and δ_1.

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