ment, and to Dr. Robert Harvey for a critical reading of the manuscript. A useful comment from Dr. J. Pipkins concerning the collisional effect is acknowledged. Helpful comments on the evolution of the equilibrium distribution function of Eq. (5) from Professor T. O'Neil are most appreciated.

[1]J. C. Hosea and W. M. Hooke, Phys. Rev. Lett. 31, 150 (1973).

[2]H. Takahashi *et al.*, Phys. Rev. Lett. 39, 31 (1977).

[3]D. Swanson, Phys. Rev. Lett. 36, 316 (1976); J. Jacquinot *et al.*, Phys. Rev. Lett. 39, 88 (1977); F. Perkins, Nucl. Fusion 17, 1197 (1977); H. Takahashi, J. Phys. (Paris), Colloq. Suppl. 38, C6-171 (1977).

[4]H. Ikegami *et al.*, Phys. Rev. Lett. 19, 778 (1967).

[5]R. Dandl *et al.*, Oak Ridge National Laboratory Report No. TM-6703, 1979 (to be published).

[6]R. Dandl, private communication.

[7]T. Antonsen and W. Manheimer, Phys. Fluids 21, 2295 (1978).

[8]See, for example, R. Davidson, *Methods in Nonlinear Plasma Theory* (Academic, New York, 1972), p. 8.

[9]See, for example, D. C. Montgomery, *Theory of the Unmagnetized Plasma* (Gordon and Breach, New York, 1971), p. 374.

[10]T. Stix [Nucl. Fusion 15, 737 (1975)] found runaway particle energies from a quasilinear treatment of the cyclotron harmonic heating. The present nonlinear resonance, however, has the stable equilibrium points of the Bessel functions. This adiabatic trap of the phase trajectory was discussed for high n modes by R. Aamodt and S. E. Bodner [Phys. Fluids 12, 1471 (1969)] and A. Timofeev [Nucl. Fusion 14, 165 (1974)].

[11]This nonlinear enhancement resulting from a velocity-space instability differs from that due to the linear enhancement of the left-circularly polarized component of the fast wave found by H. Takahashi, Princeton Plasma Physics Laboratory Report No. PPPL-1545, 1979 (unpublished).

[12]T. O'Neil, Phys. Fluids 8, 2255 (1976).

[13]J. Hosea *et al.*, Princeton Plasma Physics Laboratory Report PPPL-1554, 1979 (unpublished).

Observation of Spherical Ion-Acoustic Solitons

Y. Nakamura, M. Ooyama, and T. Ogino[(a)]

Institute of Space and Aeronautical Science, University of Tokyo, Komaba, Meguro-ku, Tokyo, Japan

(Received 21 May 1980)

Spherically converging positive and negative ion-acoustic pulses are investigated experimentally. Their behavior agrees with computer simulations based on the fluid model of plasma. Large positive pulses are identified as solitons.

PACS numbers: 52.35.Mw

Solitons have been of considerable interest in many fields in recent years.[1] In particular, one-dimensional ion-acoustic solitons have been extensively studied experimentally. Dependence of velocity and width on their amplitude has been verified.[2] The number and amplitude have been measured as a function of initial spatial density perturbation.[3] Development of initial perturbations has been observed and compared with numerical simulation.[4] A slight phase shift upon the head-on collision of solitons has been confirmed.[5] Characteristics of expanding and converging cylindrical solitons have been measured.[6] Scattering of solitons at the line of symmetry has been observed recently.[7]

The theoretical and experimental understanding of spherical solitons is less well developed than the planar and cylindrical cases. Maxon and Viecelli[8] have derived the dimensionally modified Kortweg–de Vries (KdV) equation appropriate to small-amplitude, spherically symmetric waves. Numerical solutions of the equation show that a small residue develops and moves inward behind the ingoing soliton.[8] Ko and Kuehl[9] have examined the modified KdV equation analytically and numerically in detail and have shown that the solitons continually lose energy and that the energy is transferred to the trailing structure (residue). Shelf formation by plane ion-acoustic solitons in a density gradient is theoretically predicted.[10] Since the modified KdV equation is only valid for the disturbance which is located at a sufficiently large radius, Ogino and Takeda[11] studied the ingoing spherical solitons by the computer simulation with use of a fluid model. All computer simulations described above use a soliton as an initial condition. However, in experiments, a sinusoidal pulse is used, which eventually develops into solitons. Spherical outgoing ion-acoustic solitons are launched from small hemispherical probes and the basic properties such as the velocity and the width as a function of amplitude have been verified by Hershkowitz *et al.*[12] Ze *et al.*[13] have observed the inelastic collisions of two spherical outgoing ion-acoustic solitons having different centers of symmetry. In previous experiments mentioned above, the existence of two- and three-dimensional solitons instead of solitary waves has not been confirmed since collision property is not observed. In this Letter, we report an experiment on the collision of spherical ion-acoustic pulses which confirms that the solitary pulses are solitons.

The experiment was performed in a cubic multidipole device of side 65 cm. A plasma was created by a discharge between tungsten filaments and walls. A spherical grid was made by combination of two semispherical screens with the diameter of 32.5 cm. The screens consisted of a mesh of 20 lines/in. made of 0.1-mm molybdenum wires. The grid was put on a stand made of 2-mm wires. Both the grid and the stand were floating. A plasma in the sphere was produced by primary electrons which passed through the grid. Lack of uniformity of the spherical grid is less than 1%. Density perturbations were excited with two small filaments in the grid. When a negative half-sine wave of amplitude V_{expt} is applied to the filaments, they emit electrons and the plasma potential in the grid decreases.[14] Because of this potential difference, ions are injected through the grid which form a compressional pulse. This method of excitation is basically equivalent to the usual one in double-plasma devices. The actual decrease of plasma potential in the grid was about $\frac{1}{2}V_{\text{expt}}$. Signals were detected by a small Langmuir probe which was biased positively to collect electron saturation current and was therefore sensitive to the perturbed electron density. The base vacuum pressure was 5×10^{-7} Torr. Data were taken in an argon plasma at a neutral pressure of $(1-4)\times 10^{-4}$ Torr. A xenon plasma was also used and basically similar phenomena were observed. The discharge voltage was 60–80 V and the discharge current was 100–200 mA. The plasma density was 10^8–10^9 cm^{-3} and the electron temperature T_e was 1.5–2.5 eV.

Computer simulations have been performed by solving the following dimensionless equations when the fluid model for ions and the Boltzmann distribution for electrons are assumed:

$$\partial\delta n/\partial t = -r^{-2}(\partial/\partial r)r^2 u(n+\delta n), \tag{1}$$

$$M[(\partial u/\partial t)+u(\partial u/\partial r)]+e\partial\varphi/\partial r = -3kT_i(\partial\delta n/\partial r)(n+\delta n)^{-1}, \tag{2}$$

$$(\partial^2\varphi/\partial r^2)+2r^{-1}\partial\varphi/\partial r = 4\pi e[n\exp(e\varphi/kT_e)-n-\delta n], \tag{3}$$

where standard notations are used. The plasma parameters used for the calculation are $T_e/T_i = 18$, $T_e = 2.3$ eV for argon plasma and the Debye length $\lambda_D = 6.5\times 10^{-2}$ cm. For boundary conditions, reflective or mirror boundaries ($u = dn/dr = 0$) are assumed[11] at $r=0$ and $r=19.6$ cm. For the initial condition, a Gaussian perturbation $\delta n_0 \exp[-k^2(r-r_0)^2/2]$ is considered, where $r_0 = 16.25$ cm. The wave number k is estimated by the frequency of the pulse and by the ion-acoustic speed. A sinusoidal pulse is also used as the initial condition to give essentially the same results.

An example of propagation of small ingoing pulse is shown in Fig. 1(a). Propagating inward, the positive pulse becomes larger because of spherical convergence and reflects at the center to be a negative pulse which propagates outward. The result agrees with the computer simulation shown in Fig. 1(b). Linearizing Eqs. (1)–(3), we easily obtain the following equation for the symmetric wave about the origin assuming that the wave is an ion-acoustic wave:

$$\partial^2\delta n/\partial t^2 = (T_e+3T_i)(\partial/Mr^2\partial r)(r^2\partial\delta n/\partial r). \tag{4}$$

The general solution of this equation has to remain finite at $r=0$, which results in the phase inversion.[15] The phase inversion is inherent to linear ingoing spherical waves. It is described in detail for spherical sound waves and light waves in textbooks.[16]

When the amplitude of the initial pulse was increased, the leading edge of the pulse steepened and a sharp peak formed as shown in Fig. 1(c). To judge the peak to be a soliton, its velocity was measured at $r\approx 3$ cm as a function of height. The velocity satisfies the relation $v = C_s(1+\alpha\delta n/n)$, where C_s is the ion-acoustic velocity. The constant $\alpha = 0.48$ which is nearly equal to the value of 0.37 obtained by the numerical simulation and which is also close to the value for outgoing solitons measured by Ze *et al.*[13]

The measured product of the height of the soliton and the square of its width D is constant, that is, $(\delta n/n)(D/\lambda_D)^2 \approx 14$ which agrees with the value (≈ 12) obtained from the numerical simulation.

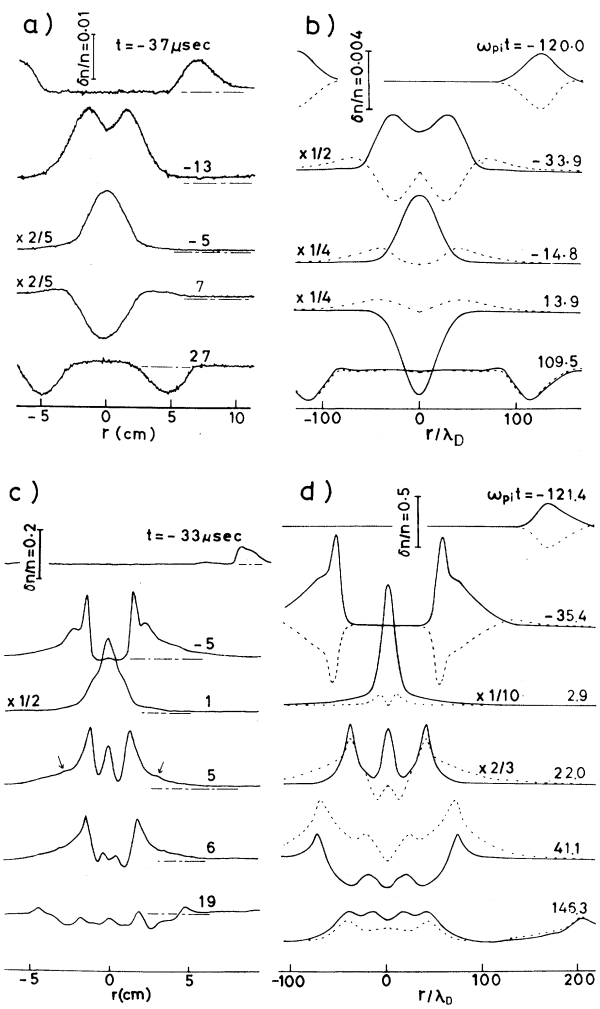

FIG. 1. Observed and calculated propagations of density perturbations for ingoing positive pulses. As numerical results are symmetric around the center, parts of left side are not shown. Dash-dotted curves give levels of n. Dotted curves are calculated velocity u/C_s of ions. (a) Experiment: $V_{\rm expt} = -0.3$ V and the width T of exciting pulses is 24 μsec. (b) Simulation: $\delta n_0/n = 1\times10^{-3}$ and $k\lambda_D = 6.4\times10^{-2}$. (c) Experiment: $V_{\rm expt} = -3.5$ V and $T = 12$ μsec. (d) Simulation: $\delta n_0/n = 0.15$ and $k\lambda_D = 6.4\times10^{-2}$.

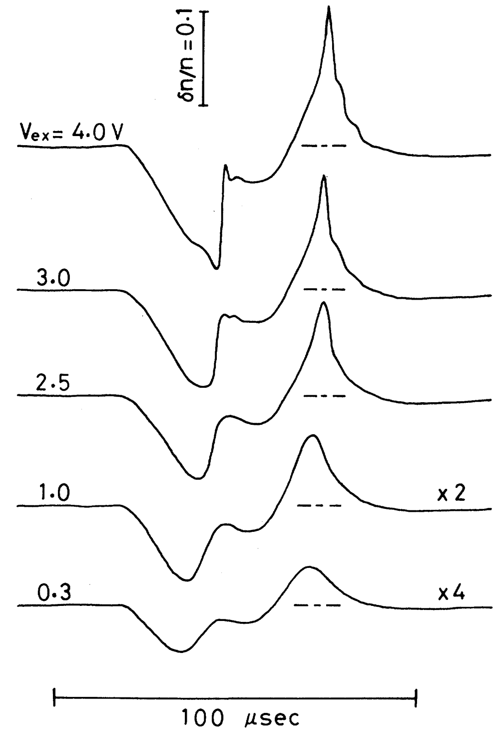

FIG. 2. Observed density perturbations at a fixed position ($r = 3$ cm) for several exciting voltages. Dash-dotted curves show level of n. $T = 26$ μsec.

The soliton leaves behind itself a density perturbation, e.g., $r = 5$ cm at $t = -5$ μsec in Fig. 1(c), which remains positive and is called the residue or shelf.[9] It is caused by the decrease in number of particles in the pulse. The number N of particles in the pulse is proportional to $r^2D\delta n$. For the linear pulse, the width D is constant and $\delta n \propto r^{-1}$, which is the solution of Eq. (4). As a result of this, $N \propto r$, which shows that the positive ingoing pulse emits some ions outward. A small outward drift which is clearly seen behind the pulse, e.g., $\omega_{pi}t = -33.9$ in Fig. 1(b), decreases the density in the pulse.

For the soliton, a simple calculation[12] which neglects a damping due to the residue gives that $\delta n \propto r^{-4/3}$ (the present simulation gives $\delta n \propto r^{-1.2}$). Therefore, $N \propto r^2 D\delta n \propto r^{4/3}(r^{1.4})$; here the relation that $D^2\delta n$ is constant is used. It implies that the residue of the soliton is larger than that of small positive pulse.

The observed behavior of the soliton agrees qualitatively with that obtained by computer simulation. Having converged to the center, the soliton reflects keeping the form of positive peak. At a radius of 3 cm, the speed of the peak is much larger than C_s and its amplitude decays rapidly since it is on a large outward flow as seen from the curve $\omega_{pi}t = 41.1$ in Fig. 1(d). Following the peak, a density depression propagates outward. It is the extension of the linear properties of phase inversion. In this density depression, small waves propagate outward as seen at 19 and 23 μsec in Fig. 2(a). The oscillation of density at the center excites the waves. As the soliton arrives at the center, the density goes up abruptly to $\delta n/n \lesssim 2$ and goes down with a temporal width which is equal to the period of the following oscillations. The outgoing waves are also seen in the computer simulation shown in Fig. 1(d). The phenomenon is similar to the one observed when a stone is thrown into the center of a pond. It is not the scattering of incident soliton, which is observed for the cylindrical soliton by Nishida *et al.*,[7] since almost all the energy of the converging soliton is carried out by the reflected peak and the density depression.

A phenomenon which is not predicted by the computer simulation is production of accelerated ions whose velocity is about $2C_s$ and which are indicated by arrows when $t = 5$ μsec in Fig. 1(c). The velocity agrees with the one measured by an energy analyzer. They originate at $r \approx 1$ cm when $t = 0$ and seem to be a precursor generated by the large electric field of the soliton.

Considering the fact that the density pulse has the property of phase reversal at the center, we can expect that a large negative pulse, after converging to the center, will change into a soliton which propagates outward. It is also predicted by the computer simulation with use of a negative initial value. It is confirmed by the experimental measurement given in Fig. 2, which shows signals versus time at a constant position. When the amplitude is small, the form of the outgoing positive pulse is similar to that of the negative pulse. When the amplitude is increased, the resultant pulse becomes larger and its width narrower. The velocity and the width of the outgoing peak were measured as a function of its height and they satisfy the conditions for the soliton stated before.

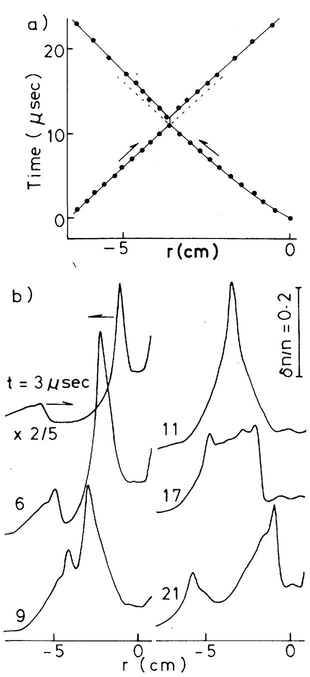

FIG. 3. (a) Trajectories of ingoing and outgoing soliton. (b) The collision of ingoing and outgoing soliton.

The observed sharp peaks are called solitons rather than solitary waves if they are unaffected by collisions.[9] To observe the collision, first a large negative pulse which became the outgoing soliton had been transmitted. Next, after 30 μsec, a large positive pulse which would eventually develop into the soliton was excited [Fig. 3(b)]. Both solitons collided at $r \approx 4$ cm and they passed through each other without losing their identity as seen in Fig. 3(b). Trajectories of both ingoing and outgoing solitary peaks are shown in Fig. 3(a). By this head-on collision, a phase delay which has been confirmed for planar solitons by Watanabe[5] occurs as clearly seen in Fig. 3(a). As a result of this, these peaks are identified as solitons.

In conclusion, ingoing ion-acoustic pulses are investigated with the spherical grid and with the novel way of excitation. The device is also suitable to perform an experiment on a converging ion beam. The behavior of linear waves agrees with theory. The large-amplitude pulses which agree with the computer simulation are identified as solitons by their collisional property. The residue which is peculiar to nonplanar solitons was first observed in the present experiment.

[(a)]Permanent address: Research Institute of Atmospherics, University of Nagoya, Toyokawa-shi, Aichi-ken, Japan.

[1]*Solitons in Action*, edited by K. E. Lonngren and A. C. Scott (Academic, New York, 1978).

[2]H. Ikezi, Phys. Fluids 16, 1688 (1973).

[3]N. Hershkowitz *et al.*, Phys. Rev. Lett. 29, 1586 (1972).

[4]E. Okutsu and Y. Nakamura, Plasma Phys. 21, 1053 (1979).

[5]S. Watanabe, J. Plasma Phys. 14, 353 (1975).

[6]N. Hershkowitz and T. Romesser, Phys. Rev. Lett. 32, 581 (1974); T. Chen and L. Schott, Plasma Phys. 19, 959 (1977).

[7]Y. Nishida *et al.*, Phys. Rev. Lett. 42, 379 (1979).

[8]S. Maxon and J. Viecelli, Phys. Rev. Lett. 32, 4 (1974).

[9]K. Ko and H. H. Kuehl, Phys. Fluids 22, 1343 (1979).

[10]D. J. Kaup and A. C. Newell, Proc. Roy. Soc. London, Ser. A 361, 413 (1978).
[11]T. Ogino and S. Takeda, J. Phys. Soc. Jpn. 41, 257 (1976).
[12]N. Hershkowitz *et al.*, Plasma Phys. 21, 583 (1979); F. Ze *et al.*, Phys. Fluids 22, 1554 (1979).
[13]F. Ze *et al.*, Phys. Rev. Lett. 42, 1747 (1979).
[14]Y. Nakamura and Y. Nomura, Phys. Lett. 65A, 415 (1978).
[15]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1966).
[16]J. W. S. Rayleigh, *The Theory of Sound* (Dover, New York, 1945); A. Sommerfeld, *Lectures on Theoretical Physics* (Academic, New York, 1964), Vol. IV.

Influence of a Magnetic Field on Flowing Superfluid ^{3}He

J. P. Eisenstein,[(a)] G. W. Swift, and R. E. Packard
University of California, Berkeley, California 94720
(Received 11 August 1980)

With use of a U-tube device, substantial effects of a magnetic field on flowing superfluid ^{3}He at saturated vapor pressure have been observed. Data are presented that show marked magnetic distortions of the otherwise isotropic B-phase order parameter. Above a critical temperature the magnetic field appears to destroy the B phase in favor of another type of state.

PACS numbers: 67.50.Fi

The superfluid B phase of liquid ^{3}He is believed to be a Balian-Werthamer state.[1] In equilibrium, and in the absence of external fields, the gap is isotropic over the Fermi surface. Unlike the A phase, there are no macroscopic anisotropy axes (neglecting the tiny dipole-dipole interaction) in the B phase. This leads to substantial distortions of the Balian-Werthamer (BW) order parameter when a modest (~1 kG) magnetic field or superflow (~1 cm sec^{-1}) is imposed.[2] The A phase can, in principle, remove these distortions by suitably orienting its internal degrees of freedom.

For the case of zero flow it is well known that a magnetic field destabilizes the B phase in favor of the A phase, resulting in the interposition of A phase between the normal Fermi liquid and the B phase at pressures below the polycritical point.[3] Below the A-B transition temperature, T_{AB}, the B-phase superfluid density, ρ_s, is depressed below its zero-field value, the depression lessening as the temperature is reduced. Fetter[2] has given a Ginzburg-Landau derivation of T_{AB} which, if weak coupling is assumed, is

$$1-T_{AB}/T_c = [7\zeta(3)/8\pi^2(1+\tfrac{1}{4}Z_0)^2](\gamma\hbar H/k_B T_c)^2, \qquad (1)$$

where $\zeta(3)$ is the Riemann zeta function,[4] γ the ^{3}He gyromagnetic ratio, k_B Boltzmann's constant, T_c the superfluid transition temperature, and Z_0 the first magnetic Landau parameter. Taking T_c =1.04 mK and $Z_0=-3.08$, both values appropriate to saturated vapor pressure,[5] then $1-T_{AB}/T_c$ $=0.045H^2$ for H in units of kilogauss.

The presence of an equilibrium superflow, v_s, further suppresses ρ_s in the B phase.[2] The A phase, believed to be an Anderson-Brinkman-Morel (ABM) state[1] can, in principle, orient itself to reduce the kinetic energy of flow. These combined effects may shift T_{AB} from the zero flow value but the magnitude of the effect will depend on which, if any, stable A-phase texture exists.

We have used a U-tube technique to study the influence of a magnetic field on superfluid flow at saturated vapor pressure. The device consists of two reservoirs connected by a narrow flow channel designed to lock the normal component. The channel is a circular pipe of 177 μm radius and 1.0 cm length. The U-tube reservoirs are the gaps of concentric cylinder capacitors. These are used to electrostatically drive and detect the flow. Our measurement technique consists of recording flow transients induced by applying fixed high-voltage steps to one capacitor reservoir. (This produces a pressure head quadratically dependent on the bias voltage. For our apparatus the typical 400 V bias gives a pressure head of approximately 9 dyn cm^{-2}.) The flow transients are digitized and an on-line computer calculates the initial slope. A solenoid is wound around the flow channel and produces a field of up to 2 kG parallel to it. The solenoid has trimming coils