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Thought Experiment to Determine the Special Relativistic Temperature Transformation

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The special relativistic transformation of temperature is not known with certainty and some authors believe it to be arbitrary within a certain range of possibilities. A procedure is proposed here which can in principle, and for the first time, lead to an experimental attack on this problem. Even though the experimental difficulties may be prohibitive, if the principle of the method is granted, then a definite transformation must exist.

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The temperature transformation of special relativity is still being discussed,[1-7] but no consensus has emerged, except that the law has the form

$$t(v)=[\gamma(v)]^a t(0), \quad \gamma(v)\equiv[1-(v/c)^2]^{-1/2}. \tag{1}$$

Here a is not known with certainty and $a=-1$ (Pauli-Planck), or $a=0$ (Landsberg) or $a=1$ (Ott). Some authors maintain that which transformation is adopted is a matter of convention. The many attempts to deduce the value of a from accepted theory appear to have failed,[1,8] nor is any experimental approach to temperature by studying two bodies which are in relative motion likely to succeed.[9]

The thought experiment proposed here proceeds in two steps. *Step 1:* Consider two uniform disks with constant angular velocities ω_1, ω_2 (in parallel planes which are close together) about centers A and B which are at rest in a common inertial frame I. The temperatures in I at a general radius r from the centers are, respectively,

$$T_1(r)=\gamma_1(r)T_1(0), \quad T_2(r)=\gamma_2(r)T_2(0), \tag{2}$$

where

$$\gamma_i(r)=[1-\omega_i{}^2r^2/c^2]^{-1/2}, \quad i=1,2; \quad \omega_i r<c. \tag{3}$$

Note that $T_1(0)$ and $T_2(0)$ need not be the same. We are interested in heat transfer between annuli *with the same linear velocity,* of radii r_1 and r_2 where

$$\omega_1 r_1=\omega_2 r_2, \tag{4}$$

in order to avoid the difficulties noted in Ref. 9.

The basis of Eq. (2) resides in general relativity,[10] but it is also easily accessible from the equivalence principle.[8,11] It has been used in the astrophysical context of rotating stars.[12] The idea of the use of high-speed rotors is also well-known in connection with the experimental confirmation of the time-dilation formula with the Mössbauer effect.[13] Note that the formula involved here relates to the frequency $\nu_1(r)$ absorbed at radius r to the frequency emitted in the rest frame of the emitter, $\nu_1(0)$, by $\nu_1(r)=\gamma_1(r)\times\nu_1(0)$, in close analogy with (2), when the emitter is at the origin. This experiment depends, however, on special relativity and the hypothesis: "If an ideal clock moves nonuniformly through an inertial frame, then the acceleration has no effect on the rate of the clock." The justification of this hypothesis depends again on general relativity. However, we are not here concerned with the details of this experiment or time-dilation effects. The relativistic rotating disk is in any case a much studied topic.[14]

We adopt the *hypothesis that if two bodies in close proximity are relatively at rest for a time, however short, then one can find experimentally if heat passes between them or not.* If this can be applied to the annuli of the two disks, then the possibility of thermal equilibrium for the appropriate portions of the two annuli occurs by (2) and (4) only if $T_1(0)=T_2(0)$. But the ω values can be different, the appropriate radii being given by (4). Would experiment confirm that $T_1(0)=T_2(0)$, and that for given $\omega_1 r_1 \equiv v$, equilibrium with the second annulus occurs at radii r_2 given by $\omega_2 r_2 = v$? To accelerate bodies, large enough to have a temperature, to high velocities presents formidable obstacles. However, we shall assume that it can be done and that (2) is thus confirmed. We have already presented reasons why Eq. (2) is a reasonable formula in any case.

Step 2: A special relativistic experiment is to shoot a small body B with constant velocity

$$v=\omega_1 r_1 \qquad (5)$$

in a plane parallel to, but close to, disk A with impact parameter r_1 in such a way that it is instantaneously at rest with respect to the annulus of radius r_1 as it passes just above it. Let the rest temperature of B be $t(0)$. One then has four independent parameters $T_1(0)$, $t(0)$, ω_1, and r_1. This experiment is of even greater difficulty than the last. However, adopting the above *hypothesis*, one may one day be able to find sets of values for which the annulus is in thermal equilibrium with B when the two are in close proximity. If relation (1) holds, the value of a can be determined as follows:

$$T_1(r_1)=t(v) \qquad (6)$$

implies $\gamma_1(r_1)T_1(0)=\gamma(\omega_1 r_1)^a t(0)$, whence by (5)

$$a=1+\frac{\log[T_1(0)/t(0)]}{\log\gamma(\omega_1 r_1)}. \qquad (7)$$

If (7) does not always give the same value of a, then an empirical determination of the temperature transformation is still in principle available from (6) $t(v)=\gamma_1(r)T_1(0)$.

[1] Ø. Grøn, Nuovo Cimento 17B, 141 (1973).
[2] I. Paiva-Veretennicoff, Physica (Utrecht) 75, 194 (1974).
[3] D. Eimerl, Am. Phys. 91, 481 (1975).
[4] N. Agmon, Found. Phys. 7, 331 (1977).
[5] H. J. Treder, Ann. Phys. (Leipzig) 34, 23 (1977), and 35, 77 (1978).
[6] J. E. Kriznan, Phys. Lett. 71A, 174 (1979).
[7] R. G. Newburgh, Nuovo Cimento 52B, 219 (1979).
[8] P. T. Landsberg, in *A Critical Review of Thermodynamics*, edited by E. B. Stuart, B. Gal-Dr, and A. J. Brainard (Mono Book, Baltimore, 1970), p. 253.
[9] P. T. Landsberg and K. A. Johns, Proc. Roy. Soc. London, Ser. A 306, 477 (1968).
[10] G. Horwitz and J. Katz, Ann. Phys. (N. Y.) 76, 301 (1973).
[11] P. T. Landsberg, *Thermodynamics and Statistical Mechanics* (Oxford Univ. Press, Oxford, 1978), p. 353.
[12] J. M. Bardeen, Astrophys. J. 162, 71 (1970).
[13] D. C. Champeney, G. R. Isaak, and A. M. Khan, Proc. Phys. Soc. London 85, 583 (1965).
[14] Ø. Grøn, Found. Phys. 9, 353 (1979).

Classical Quantization of a Hamiltonian with Ergodic Behavior

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Conservative Hamiltonian systems with two degrees of freedom are discussed where a typical trajectory fills the whole surface of constant energy. The trace of the quantum mechanical Green's function is approximated by a sum over classical periodic orbits. This leads directly to Selberg's trace formula for the motion of a particle on a surface of constant negative curvature, and, when applied to the anisotropic Kepler problem, yields excellent results for all the energy levels.

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The relations between classical and quantal mechanics are of central importance to the understanding of physics. Classical mechanics is believed to be a limit of quantum mechanics when Planck's constant is small. It is natural to use the classical motion as a starting point in order