Fully Developed Anisotropic Hydromagnetic Turbulence in Interplanetary Space

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The solar-wind magnetohydrodynamic turbulence is observed to be mainly made of Alfvénic fluctuations propagating away from the sun. It is shown that such an asymmetric state is a general consequence of the evolution of developed magnetohydrodynamic turbulence, which, starting from an initial asymmetry between modes with cross helicity +1 and −1, tends, as a consequence of nonlinear interactions, towards a state where the only modes left are those initially prevailing (with either cross helicity +1 or −1).

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Theoretical investigations of strong hydromagnetic turbulence have always dealt so far with the isotropic case[1,2] and most often with the case where the average magnetic field is zero.[3,4] In the latter case, Kraichnan[2] has derived, using dimensional arguments, a $k^{-3/2}$ power law for the spectrum of the magnetic and kinetic energy densities of the fluctuations in the stationary state. The difference in the spectral index with respect to that of the Kolmogorov spectrum of isotropic hydrodynamic turbulence is due to the presence, in the smaller scales, of Alfvén waves propagating in the magnetic field of the larger-scale eddies, thus impeding the energy transfer in this range of high wave numbers.

Observations of incompressible magnetohydrodynamic (MHD) turbulence in the magnetized plasma of interplanetary space[5-8] indicate, however,[9] that the existence of these Alfvén waves is not the only peculiar feature of MHD with respect to hydrodynamic turbulence.

On the one hand, the spectral energy density of magnetic fluctuations $F(k)$ defined by

$$\langle \delta B^2 \rangle / 4\pi\rho = \int_0^\infty F(k)\, dk \tag{1}$$

(ρ being the plasma mass density) seems to follow a power law $F(k) \propto k^{-\nu}$ with a spectral index ν ranging from 1.2 to 2, for frequencies between 10^{-1} and 10^{-4} Hz. Although the scatter of the observed values of ν precludes a definite identification with either a Kolmogorov or a Kraichnan spectrum, the observed power law is expected to result from a nonlinear energy cascade.

On the other hand, in the same domain of wave vectors, and mainly in the trailing edges of fast solar-wind streams, one observes a striking correlation between the velocity $\delta\vec{v}$ and magnetic fluctuations $\delta\vec{B}$ which satisfy to a good degree the relation

$$\delta\vec{v} = \pm\, \delta\vec{B}/(4\pi\rho)^{1/2} \tag{2}$$

the sign depending on the polarity of the average magnetic field and being such that only Alfvénic fluctuations propagating away from the sun are observed. Notice that, in terms of the so-called cross helicity of hydromagnetic turbulence,[10] the observational result (2) implies that the MHD turbulence in the solar wind is either in a state characterized by the value +1 for the cross helicity, or in a −1 state.

It is a simple matter to show that, if condition (2) is satisfied, there are no longer nonlinear interactions which is in apparent contrast with the presence of a spectrum. To see this, we write the equations for incompressible MHD fluctuations as[11]

$$\frac{\partial}{\partial t}\, \partial\vec{Z}^{\pm} \mp (\vec{C}_A \cdot \nabla)\delta\vec{Z}^{\pm} + (\delta\vec{Z}^{\mp} \cdot \nabla)\delta\vec{Z}^{\pm} = -\frac{1}{\rho}\nabla\left(p + \frac{B^2}{8\pi}\right), \tag{3}$$

where

$$\delta\vec{Z}^{\pm} = \delta\vec{v} \pm \delta\vec{B}/(4\pi\rho)^{1/2} \tag{4}$$

and $\vec{C}_A = \langle\vec{B}\rangle/(4\pi\rho)^{1/2}$ is the Alfvénic speed in the average field $\langle\vec{B}\rangle$. The above equations refer to a frame of reference moving with the fluid and they contain neither source nor dissipative terms as the present arguments will refer to the inertial range of the turbulence[2] where these terms are negligible. Indeed, in the case of the solar wind, the source terms are either of solar origin and affect scale lengths of the order of the dimensions of the fast streams themselves, or are of local origin and affect then essentially small wavelengths, of the order of the proton gyroradius. The most important dissipation mechanism is, on the other hand, presumably the collisionless absorption of energy at the ion cyclotron resonance.[5]

By referring to the case of small-amplitude fluctuations, it is seen that the physical interpretation of the quantities $\delta\vec{Z}^{\pm}$, defined in (4), is that of the two possible Alfvénic waves propagating away from and toward the sun. According to the property (2), however, one of the two amplitudes δZ^{+} or δZ^{-} is zero, and, therefore, the nonlinear terms in the equations (3) disappear. To the apparently contradictory observations of (1) the presence of a power spectrum of the magnetic fluctuations and, (2) the absence of nonlinear interactions, two interpretations can be given. The first one is that all the observed waves are of solar origin and they propagate without interacting in interplanetary space. However, in that case one would expect to find in the spectrum some features characteristic of the solar atmosphere.[12]

The second interpretation is that this property (i.e., the absence of nonlinear interactions) is not a particular one of the turbulence in the solar wind but is a general outcome of the relaxation of an initially excited MHD turbulence, provided that this initial excitation is asymmetric, i.e., favors one sense of propagation of the Alfvénic fluctuations.

We will now substantiate this last interpretation, by means of a dimensional analysis of equations of asymmetric MHD turbulence. We assume that the interactions of Alfvénic fluctuations are local in the wave-number space.[2] Then consider the interaction between Alfvénic fluctuations (or eddies) of the same scale $l \sim k^{-1}$. One can define two different time scales for such an interaction. The first one,

$$\tau_{ia} \simeq l/C_A, \tag{5}$$

is determined by the transport velocity C_A, i.e., the velocity of propagation of the fluctuations in the average magnetic field or, in the absence of a background magnetic field, in the magnetic field of the largest eddies (scales $\gg l$). The second time,

$$\tau_i^{\pm} \sim l/\delta Z^{\mp}, \tag{6}$$

corresponds to the lifetime of the fluctuating eddies and, because of the structure of the nonlinear interactions in (3), is different for the two types of modes. Note that $\tau_{ia} < \tau_i^{\pm}$ in the case of a relatively weak turbulence and that $\tau_i^{\pm} > \tau_{ia}$ for strongly developed MHD fluctuations (in the following we shall consider only the two extreme cases for the whole inertial range). The variation $dZ^{\pm}$ in amplitude of a given vortex ($\delta Z^{\pm}$) due to its interaction with another vortex of the other type (i.e., $\delta Z^{\mp}$) in one interaction time τ_i, to be later identified with the shorter of (5) and (6), is, from Eqs. (3), in order of magnitude,

$$dZ^{\pm} \sim \tau_i \, \delta Z^{\pm} \delta Z^{\mp}/l. \tag{7}$$

In N such interactions, because of their stochastic nature, the amplitude variation will be $\Delta Z^{\pm} \sim \sqrt{N} \times dZ^{\pm}$. Then the number of interactions $N^{\pm}$ it takes to obtain a variation, for a given vortex $\delta Z^{\pm}$, equal to its initial amplitude, i.e., $\Delta Z^{\pm} \sim \delta Z^{\pm}$, will be given by

$$N^{\pm} \sim [l^2/(\delta Z^{\mp})^2]\tau_i^{-2}. \tag{8}$$

The corresponding time $T^{\pm}$ is

$$T^{\pm} \sim (\tau_i^{\pm})^2/\tau_i \tag{9}$$

and can obviously be considered as the time after which a substantial modification of the amplitude of $\delta Z^{\pm}$, at the scale l, has occurred.

For fully developed turbulence, and all scales excited, the shorter time of (5) and (6), in the smallest scales, is the Alfvénic time so that, from (9), we obtain

$$T^{\pm} \sim C_A l/(\delta Z^{\mp})^2. \tag{10}$$

Consider now a stationary state for the turbulence. For this, we must suppose to have, outside the inertial range, a source of turbulence at low wave numbers on the one hand and, on the other, a dissipation sink at high wave numbers, sufficiently effective to ensure in fact the possibility of a stationary state.

Referring first to a symmetric case $\delta Z^{+} \sim \delta Z^{-} \sim \delta Z$, we impose that the energy $\Pi(l)$ transferred per unit time from all vortices of scale $>l$ to vortices of scale $<l$ due to the nonlinear interactions must assume a value ϵ independent from l. We will have $\Pi \sim (\delta Z)^2/T$ and, using (10) for the time T of a significant energy transfer, we obtain ϵ

$\sim(\delta Z)^4/C_A l$.

Writing $(\delta Z)^2 \sim kE_k$, where E_k is the energy density in the mode k ($k\sim 1/l$), we then derive

$$E_k \sim (\epsilon C_A)^{1/2} k^{-3/2} \qquad (11)$$

and therefore recover (as ϵ and C_A are independent of k) Kraichnan's[2] power law for the spectrum of MHD turbulence. This turns out to be shallower than the Kolmogorov spectrum because the shorter time of (5) and (6) is the Alfvénic time, which is equivalent to say that the interactions are weak.

In the second place, we want to consider the possibility of a stationary state with an asymmetry $\delta Z^+ \neq \delta Z^-$ produced at the source. Conservation of total energy, $\propto[(\delta Z^+)^2 + (\delta Z^-)^2]$, and of total cross helicity, $\propto[(\delta Z^+)^2 - (\delta Z^-)^2]$, imply that we can impose, for a stationary state, the constancy of energy transfer, across the inertial range, separately for the modes + and −. If we call $\Pi^\pm(l)$ the transfer rates for $(\delta Z^\pm)^2$, respectively, we must write

$$\Pi^\pm(l) \sim (\delta Z^\pm)^2/T^\pm. \qquad (12)$$

Using in (12) the time (10), we obtain

$$\Pi^\pm(l) \sim l^{-1} C_A^{-1} (\delta Z^+)^2 (\delta Z^-)^2. \qquad (13)$$

As, for a stationary state, $\Pi^\pm(l) \sim \epsilon^\pm$ (with the $\epsilon^\pm$ independent from l), we see, from (13), that it is impossible to have an asymmetric stationary state ($\epsilon^+ \neq \epsilon^-$). For a stationary state to be possible, it is required to have symmetry between the two types of modes at source.

Let us finally consider the case of the relaxation of an Alfvénic turbulence (disconnected from the source), with a given asymmetry between modes + and − at time $t=0$. We continue to postulate the existence of a dissipation sink. For example, let us suppose that, at $t=0$, $\delta Z^+ > \delta Z^-$. Then, using (10) as a typical time, we obtain

$$T^+/T^- \sim (\delta Z^+)^2/(\delta Z^-)^2 > 1 \qquad (14)$$

which means that the typical energy-transfer time for modes δZ^- is shorter than that for the modes δZ^+. The energy-transfer rate $\Pi^\pm$ continues in fact to be equal for both δZ^+ and δZ^-, as indicated by (13). However, because of the initial unbalance $\delta Z^+ > \delta Z^-$ and the fact that both modes dissipate at the short wavelengths, we necessarily end up with at situation where practically all the energy available remains in the mode δZ^+. Notice that this final state is not a stationary one (this would require the continuous presence of the source) but rather a static state. With only modes of type δZ^+ (or δZ^-) present there are in fact no longer nonlinear interactions and no energy transfer across the spectrum. It is important to remark that this tendency toward an asymmetric state does take place also in the absence of a background magnetic field the effect of which is then replaced by the magnetic field of the largest-scale turbulent eddies.

Turning now to the solar wind observations, the initial asymmetry between modes + and − necessary to arrive at a one-mode state is clearly given by the fact that the waves of solar origin observed at the orbit of Earth, having overtaken the critical Alfvénic point, must be necessarily all propagating outwards with respect to the sun. The local sources, on the other hand (related to the development of instabilities due to velocity shears[13] or nonthermal properties of the particle distribution,[14] are likely to produce + and − waves with roughly the same efficiency. The evolution of such fluctuations can indeed be regarded as a relaxation of an initially excited turbulence, since each fluid element, being convected with the average wind velocity, becomes rapidly detached from the original source, a situation similar to that of grid turbulence in a wind tunnel. Thus our arguments on the relaxation to a completely asymmetric state can be applied to the solar wind, provided the relevant nonlinear time is much shorter than a typical convection time. From (10) we can write the nonlinear time as

$$T^\pm \simeq T_w(\langle B\rangle/|\delta B^\mp|)^2$$

where $T_w \simeq (kC_A)^{-1}$ is the wave period and $|\delta B|/\langle B\rangle \sim 0.3$–$0.4$ for the solar wind turbulence. For wave periods up to a few hours, $T^\pm$ is indeed much shorter than the convection time (a few days) and the nonlinear interactions seem then to be able to generate both a nonlinear cascade and the asymmetry which is observed.

In conclusion, our dimensional analysis has indicated that an initially asymmetric MHD turbulence (in the presence of dissipation) relaxes toward a state characterized by the absence of one of the possible modes (δZ^+ or δZ^-) and, hence, without nonlinear interactions. This is true both in the presence of a background magnetic field and without. Furthermore, we have found that an asymmetric ($\delta Z^+ \neq \delta Z^-$) stationary state of fully developed turbulence is not possible.

Clearly, a more quantitative theory, with strong-turbulence techniques,[15] should, however, be developed for the rather large-amplitude fluctuations characterizing the interplanetary plasma. It is worth recalling that a theory based on the weak-turbulence approximation[16] has also shown a tendency for Alfvénic turbulence towards the asymmetry discussed here.

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ERRATA

ANGULAR DISTRIBUTIONS FOR THE REACTION $^{18}O(\pi^+, \pi^-)^{18}Ne$ AND PION DOUBLE-CHARGE-EXCHANGE FORM FACTORS. Kamal K. Seth, S. Iversen, H. Nann, M. Kaletka, J. Hird, and H. A. Thiessen [Phys. Rev. Lett. 43, 1574 (1979)].

It has been found that a counter inefficiency correction was inadvertently applied twice in determining the cross sections shown in Fig. 2. The correct measured cross sections with their statistical errors are as follows:

$\theta_{c.m.}$ (deg)	σ (μb/sr) 0^+	2^+
13.2	360(65)	98(30)
18.3	69(15)	155(20)
23.4	87(20)	248(35)
30.5	261(45)	330(50)
45.7	202(40)	87(25)

The additional error in absolute normalization is estimated to be $\leq \pm 15\%$. The summed cross sections for the excitation region 5–20 MeV (referred to in paragraph 4 of page 1575) accordingly decrease monotonically from 5.8 μb/sr at 13° to 2.5 μb/sr at 45°.

The above changes have no effect on any other results or conclusions of the paper.

WHAT CAN WE LEARN ABOUT NUCLEAR ELECTRIC DIPOLE MOMENTS FROM PARITY NONCONSERVATION IN ATOMIC TRANSITIONS? Geoffrey N. Epstein [Phys. Rev. Lett. 44, 905 (1980)].

There was an error in the evaluation of $\langle \beta_0 | V^{PT} \times | e_0 \rangle$. Remarkably this specific matrix element is zero even for relativistic wave functions. This has the effect that for both hydrogen and deuterium we must look for higher-order corrections which come from vacuum polarization, the electron anomalous magnetic moment, two-photon exchange processes, etc. The complete and careful evaluation of these pieces is now essential and will be discussed fully elsewhere. However, it is evident now that with the order α^2 correction being zero we must end up with an order α^3 correction. Therefore at worst we will lose two orders of magnitude of sensitivity to the proton and deuteron electric dipole moments (e.d.m.) in the ^1H and ^2H experiments discussed. This means that the deuteron e.d.m. limit will be improved by seven, not nine, orders of magnitude. The proton e.d.m. limit will be improved by five, not seven, orders of magnitude over that obtained