tated has been tested for a dilute disordered CO layer [about $\frac{1}{4}$ of the $c(2\times 2)$ coverage]. The C-O loss energy was found to remain almost constant over the range of $\vec{q}_{\parallel}$ explored; 258.0 meV at $\vec{q}_{\parallel}=0.04$ and 257.5 meV at $\vec{q}_{\parallel}=0.33$.

The solid curve in Fig. 3 is a dispersion relation calculated according to Eq. (8). $U(\vec{q}_{\parallel})$ was obtained as described above. In order to make a direct comparison with the polarizabilities derived from the cross section data we used $\alpha_e = 2.5$ Å^3 which gave $\alpha_v = 0.23$ Å^3 when the theory was fitted to the experimental data. This figure is compatible with $\alpha_v = 0.16$ Å^3 found above and we conclude that the C-O mode dispersion is dominated by dipole-dipole interactions among the adsorbed molecules.

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Ordering and Phase Transitions in Antiferromagnetic Potts Models

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Monte Carlo simulations and ϵ expansion techniques are used to analyze the behavior of antiferromagnetic Potts models and the Ashkin-Teller model. The Monte Carlo data show that the three- and four-state antiferromagnetic Potts models have an ordered low-temperature phase in three dimensions. Evidence is presented for the existence of a novel low-temperature, metastable, glassy "plastic crystal" phase in the four-state antiferromagnetic Potts model in three dimensions.

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There has been considerable recent work on the phase transition of the Potts model.[1] It is now believed that the ferromagnetic q-state Potts models with $q \geq 3$ exhibit first-order transitions in three dimensions.[2] The situation with antiferromagnetic (AF) Potts models is not as clear. Such systems have a highly degenerate ground state for $q \geq 3$. Studies of the Ashkin-Teller model,[3] in three dimensions, in the neighborhood of the four-state AF Potts point have also shown rich and unexpected behavior.[4] More recently, Berker and Kadanoff[5] have applied one-parameter-rescaling arguments to such systems and have suggested that they can exhibit a distinctive low-temperature phase in which correlations decay algebraically with distance.

In this Letter, we present results of Monte Carlo and ϵ-expansion analyses of the AF Potts models and the Ashkin-Teller model. Our Monte Carlo computations indicate that the AF Potts

model with nearest-neighbor interactions on a simple-cubic lattice is ordered at low temperatures both for $q=3$ and $q=4$. However, on quenching suddenly to zero temperature from high temperatures ($T>T_c$), we find evidence for a novel glassy "plastic crystal" phase[6] for $q=4$. This phase is similar to the one suggested by Berker and Kadanoff[5] with one important difference. We find that the glassy phase is metastable and on heating, we find an analog of the glass transition temperature above which the low-temperature phase orders.

Our ϵ-expansion analysis results taken together with the Monte Carlo computation indicate that both $q=3$ and $q=4$ Potts models with antiferromagnetic nearest-neighbor coupling exhibit continuous transitions in three dimensions and are in the $n=2$ and $n=3$ universality classes, respectively, where n is the number of components of the order parameter. The results of our ϵ expansion analysis for the Ashkin-Teller model permit the structure of the phase diagram obtained by Ditzian *et al.*[4] and further clarify the nature of the various transitions.

Our Monte Carlo[7] calculations have been carried out on finite lattices (typically $14\times14\times14$) with periodic boundary conditions. Our results are subject to the usual qualifications of finite sizes smearing out the transition and finite times of the simulation perhaps not leading to true equilibration. We have, however, carried out a number of runs starting from widely different starting configurations and have confirmed the equilibrium nature of the final states. Our simulations have also made use of a single spin-flip sequence with both the trial spin and the final possible state of the spin being determined randomly.

The AF Potts models are described by the Hamiltonian

$$-\mathcal{H}/kT=(J/kT)\sum_{\langle ij\rangle}\delta_{S_i,S_j},\quad J<0, \tag{1}$$

where $S_i=a,b,c,\ldots$ is in one of q states, δ_{S_i,S_j} is the Kronecker δ function and the sum is over pairs of nearest-neighbor spins. The hypercubic lattice can be divided into two sublattices so that a site on one sublattice has its nearest-neighbor sites on the other sublattice. To determine the ground state, we started with the q states of the Potts spins assigned randomly on the lattice sites (corresponding to $T=\infty$) and lowered the temperature in small steps allowing the system to equilibriate at each temperature. The system acquired a spontaneous staggered magnetization below T_c and ordered "antiferromagnetically." For the four-state Potts model, the ordering can be visualized as having two of the four states distributed randomly on one sublattice and the other two states distributed randomly on the other sublattice. This type of ordering leads to a ground-state entropy per site of $\ln 2$. Similarly, for the three-state Potts model, a simplified view of the ordering has one of the states on the first sublattice and the other two states distributed randomly on the other sublattice leading to a ground state entropy per site of $\frac{1}{2}\ln 2$.

We find, however, that the system does not completely order in the above fashion. On occasion, states of the Potts spins are on the "wrong" sublattice if the surrounding neighbors permit it. This leads to an *unsaturated* magnetic ordering even at zero temperature. Our results indicate that the ordering, while having a very high degeneracy, does *not* seem to have the high degree of complexity needed for the arguments of Berker and Kadanoff.[5] The algebraic order in the low-temperature phase suggested by them may not be realized in AF Potts models in three dimensions, but the Monte Carlo results cannot rule it out categorically.

To carry out the ϵ-expansion analysis, we have made the Kac-Baker-Hubbard transformation[8] to obtain a continuous-spin Hamiltonian. Terms beyond fourth order are irrelevant and, as in conventional analysis, are neglected in determining the critical behavior (irrelevant variables can, however, influence the nature of the ordered phases). The AF Potts models are distinctly different from their ferromagnetic counterparts in that the usual third-order terms do not appear because of the sublattice symmetry. The $q=3$ case maps into a $n=2$ continuous-spin Hamiltonian with the x-y-like stable fixed point being accessible from the starting Hamiltonian.[9] The four-state Potts model, on the other hand, maps into a $n=3$ Hamiltonian. While the isotropic fixed point is stable in this case, the starting Hamiltonian has approximately the symmetry of the Ising-like tricritical point.[9] The prediction of the ϵ-expansion analysis in this case is that *if* the transition is continuous, it is in the $n=3$ universality class. The transition, of course, may be first order because of the inaccessibility of the stable fixed point.

Monte Carlo simulations suggest continuous transitions for both the $q=3$ and $q=4$ cases in three dimensions. However, the possibility of very weak first-order transitions cannot be ex-

cluded. The order parameter of the q-state AF Potts model may be defined to be

$$M=\{|\sum_{i\subset I}\delta_{S_i,a}-\sum_{i\subset II}\delta_{S_i,a}|+|\sum_{i\subset I}\delta_{S_i,b}-\sum_{i\subset II}\delta_{S_i,b}|+|\sum_{i\subset I}\delta_{S_i,c}-\sum_{i\subset II}\delta_{S_i,c}|+\cdots\}/N. \quad (2)$$

a, b, and c are the q states of the Potts spin, I and II are the two sublattices, and N is the total number of spins. The dependence of the order parameter on the temperature is shown in Fig. 1. Figure 2 shows a plot of the specific heat for both the $q=3$ and $q=4$ cases. The cusps in the specific heats near the critical point are consistent with the renormalization-group–theory predictions of small negative α.[10,11] We have also done Monte Carlo simulations of the five-state AF Potts model in three dimensions. Our results suggest that the system is paramagnetic at all temperatures.

The Ashkin-Teller model[3] can be considered to be a simple-cubic lattice with two Ising spins σ and S sitting on each lattice site coupled by the Hamiltonian

$$-\mathcal{H}/kT=(J_2/kT)\sum_{\langle ij\rangle}(\sigma_i\sigma_j+S_iS_j)+(J_4/kT)\sum_{\langle ij\rangle}\sigma_i\sigma_jS_iS_j=K_2\sum_{\langle ij\rangle}(\sigma_i\sigma_j+S_iS_j)+K_4\sum_{\langle ij\rangle}\sigma_i\sigma_jS_iS_j \quad (3)$$

with J_2 chosen to be ferromagnetic.

The phase diagram of the Ashkin-Teller model was recently determined with use of series analysis and Monte Carlo simulations by Ditzian *et al.*[4] and their results are shown in Fig. 3. By transforming to a continuous-spin Hamiltonian,[8] we have carried out an ϵ-expansion analysis of the Ashkin-Teller model. As suggested by Ditzian *et al.*,[4] AG is a line of first-order transitions (the point P represents the ferromagnetic four-state Potts point) and FB, GC, and KD describe Ising transitions. The point K (at $K_2=-K_4$) is the AF four-state Potts point and is a $n=3$ bicritical point. The line between the decoupled Ising-style tricritical point at $K_4=0$, A, and the bicritical point K is a line of XY-like transitions with the point H being a XY-like bicritical point. The lines KE and HI are lines of first-order transitions, where the nature of the ordered state changes. The scaling predictions of Fisher and Nelson[12] for the Heisenberg bicritical point are valid at the point K. The analysis predicts smooth behavior of the line AK near the XY bicritical point.

As discussed previously, the three- and four-state AF Potts model order in the low-temperature phase. One can, however, realize a glassy "plastic crystal" phase[6] in the Ashkin-Teller model in $d=3$ at $K_4=-K_2$ or its $q=4$ AF Potts analog by quenching suddenly to zero temperature from the paramagnetic phase. It is convenient to consider the Ashkin-Teller model since the order parameter is unambiguous[4] and is given by either $\langle\sigma\rangle$, $\langle S\rangle$, or $\langle\sigma S\rangle_{AF}$. The Ashkin-Teller model ($K_4=-K_2$) was quenched from $T=\infty$ to $T=0$ ($kT_c/|J_2|\simeq 2.9$). Single spins were then flipped to lower the internal energy. This was continued until a "ground state" was reached. The ground state was characterized by short-range order but there was no semblance of any long-range ordering. On making thousands of passes through the lattice the system did not show any ordering in the sense previously discussed.

For all the glasses studied, when the temperature was raised suddenly from 0.0 to $kT|J_2|^{-1}$

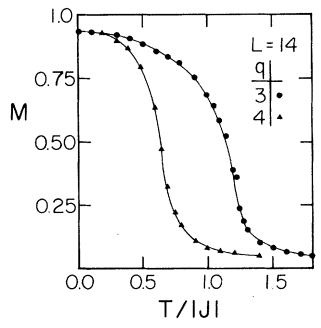

FIG. 1. The order parameter of the three- and four-state AF Potts model ($d=3$) plotted as a function of the reduced temperature.

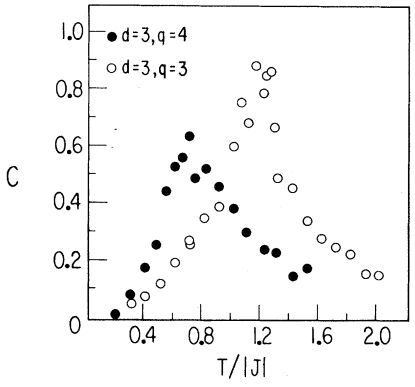

FIG. 2. Specific heats of the q-state antiferromagnetic Potts models ($q=3$ and 4; $d=3$) as a function of the temperature.

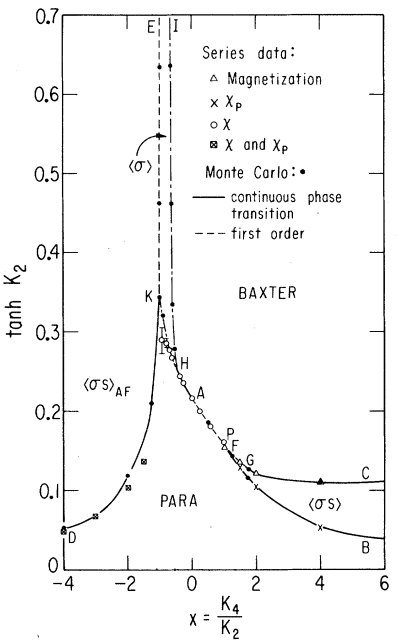

FIG. 3. Phase diagram of the Ashkin-Teller model in three dimensions obtained from series analysis and Monte Carlo simulations (from Ref. 4). The Baxter phase has $\langle\sigma\rangle$, $\langle S\rangle$, and $\langle\sigma S\rangle$ nonzero. The $\langle\sigma S\rangle$ and $\langle\sigma S\rangle_{AF}$ phases have the (σS) product ordered ferromagnetically and antiferromagnetically, respectively, whereas the σ and S spins are disordered. In the $\langle\sigma\rangle$ phase, the symmetry between the σ and S spins is broken spontaneously and only one of them is ordered ferromagnetically. The product (σS) is disordered in the $\langle\sigma\rangle$ phase.

$\gtrsim 0.8$, one of the three order parameters quickly (300 to 500 passes) acquired its nonzero equilibrium value while the other two remained zero. Suddenly raising the temperature of the same glassy state from 0.0 to $kT \lesssim 0.8|J_2|$ resulted in the system remaining in the glassy state for longer times. For example, at $kT|J_2|^{-1}=0.6$, the system ordered after 2000 passes, whereas at $kT|J_2|^{-1} \lesssim 0.5$, the system did not order during the time span of the simulation ($\sim$ 3000 passes).

The behavior of the plastic crystal phase is consistent with that of ordinary glasses. The essential difference is that whereas in a real glass, the configuration of atoms is such that the glass is in a metastable state of higher internal energy, the plastic crystal phase is in a state of lower entropy than the "ordered" phase, which is of no consequence at $T=0$. For nonzero T less than T_c the system equilibrates to a state with finite staggered magnetization. At $T=0$, we have found that there is a barrier to nucleation of the ordered phase. A system created with half of it ordered and the other half in a plastic crystal phase essentially remains unchanged after several hundred passes through the lattice. It seems possible that the distinctive low-temperature phase obtained in the one-parameter scaling analysis of Berker and Kadanoff[5] is the plastic crystal phase.

The three-state Potts model is not a good glass-former in $d=3$. When the system is quenched from $T=\infty$ to $T=0$, after only a few hundred passes through the lattice, the system acquires a well-defined "antiferrromagnetic ordering" of the type described previously.

These unusual properties of AF Potts models may be related to the fact that domain walls in such models do not cost any internal energy. However, their construction entails a loss of entropy proportional to the area of the domain walls. One may speculate that properties of the plastic crystal phase may be understood in such terms. In particular, it is likely that a multispin-flip Monte Carlo sequence will lead to a rapid decay of metastable states facilitated by the motion of domain walls.

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Hydrodynamic Theory of Mutual Friction in He II

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Theoretical values of the mutual-friction parameters B and B' are derived from a purely hydrodynamic model, in close agreement with experimental data in the temperature range from 1.7 to 2.1 K.

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In this paper we wish to consider the small two-dimensional motion $\vec{r}_L(t)$ of a vortex line subject to the normal and superfluid velocity fields $\vec{v}_{n1}$ and $\vec{v}_{s1}$ of a second-sound wave. Its velocity $\vec{v}_L$ will be given by[1]

$$d\vec{r}_L/dt = \vec{v}_L$$

$$= \vec{v}_{s1} - \frac{B\rho_n}{2\rho}\vec{\nu}\times(\vec{v}_{s1}-\vec{v}_{n1}) - \frac{B'\rho_n}{2\rho}(\vec{v}_{s1}-\vec{v}_{n1}), \quad (1)$$

where ρ is the helium density, ρ_n the normal fluid density, and $\vec{\nu}$ is a unit vector along the vortex line. Equation (1) may be regarded as the very definition of the two mutual-friction coefficients B and B', first introduced by Hall and Vinen. In their original paper,[2] Hall and Vinen (H-V) worked out a detailed microscopic model for B and B'. This model combines a kinetic treatment of the rotons-vortex collisions with purely hydrodynamic arguments (Magnus effect, dragging of the normal fluid), and leads to intricate relationships giving B and B' as a function of the effective collision diameters $\sigma_\parallel$ and $\sigma_\perp$ describing the scattering of rotons by vortices.[3] In a range of temperature around 1.9 K (i.e., 1.9 ± 0.2 K), where it is known from experiment[4] that $B'/B \lesssim 0.1$, the H-V expression of B can be simplified, and then B written, with accuracy better than 1%, as a complex quantity of the form

$$B = B_1 + iB_2 = \left[A - \frac{\rho_n\rho_s K}{16\pi\eta\rho}\left(\ln\omega - \frac{i\pi}{2}\right)\right]^{-1}. \quad (2)$$

Here $\rho_s = \rho - \rho_n$, η is the coefficient of normal viscosity, K the quantum of circulation, and $\omega/2\pi$ is the second-sound frequency [according to notations of Ref. 3, $B = (2\rho/\rho_n\rho_s K)X^{-1}$, if quantities of the order of $(B'/B)^2 = (Y/X)^2 \lesssim 10^{-2}$ are neglected]. The main term A in Eq. (2)—$A \sim 1$—is a real and frequency-independent quantity, appearing in explicit form as a rather complicated expression involving $\sigma_\parallel$ and $\sigma_\perp$, the roton-roton mean free path L, and also the normal viscosity η.[3] The other two explicit terms ($\sim 5\times10^{-2}$ typically) represent small corrections usually ignored when analyzing experimental results. In particular, the sound frequency ω is not systematically specified in collected data B vs T. However, it has been pointed out by Mehl *et al.*[5] that the slight frequency dependence of the attenuation of second sound, observed in a rotating cavity, and the apparent variation due to rotation of the second-sound velocity are well accounted for by the $\ln\omega$ term and the imaginary term in (2), respectively. Improving the accuracy of measurements still further and enlarging the worked frequency range, we ourselves have studied B_1 and B_2 vs ω at 1.9 K.[6] Our results corroborate those of Mehl *et al.*,