Asymptotic Normalization Constants of ^{3}He and ^{3}H

T. Sasakawa
Department of Physics, Tohoku University, 980 Sendai, Japan

and

T. Sawada
Department of Applied Mathematics, Faculty of Engineering Science, Osaka University, 560 Toyonaka, Japan

and

Y. E. Kim
Department of Physics, Purdue University, West Lafayette, Indiana 47907, and Department of Physics, Seoul National University, Seoul 151, Korea
(Received 18 July 1980)

The s-wave asymptotic normalization constants $C_0(^3\mathrm{He})$ and $C_0(^3\mathrm{H})$ are calculated for the Reid soft-core potential. The Coulomb effect is taken into account for ^{3}He. $C_0(^3\mathrm{He}) = 1.765$ and $C_0(^3\mathrm{H}) = 1.706$ have been obtained. In ^{3}He (^{3}H), about 60% of the contribution is due to the transition from the pp (nn) $^1S_0$ state of a pair in one set of coordinates to the $np\,^3D_1$ state of another set.

PACS numbers: 21.10.Sf, 21.40.+d, 27.10.+h

The asymptotic normalization constant is a fundamental quantity for calculations of scattering amplitudes based on the analytic properties.[1] Since the constant $C(^3\mathrm{He})$ was used for an improved phase-shift analysis of the p-^{3}He scattering,[2] this constant has attracted a great deal of attention.[3-7] Since Knuston *et al.*[8] found that the tensor analyzing power of the (d,t) reaction is sensitive to the (spectator) d-state component of the triton wave function, the study of this constant has attracted a renewed attention.[9-11]

In calculating $C(^3\mathrm{H})$, we need the wave function of ^{3}H. (Up to the present time, many people have obtained the wave function of the triton for realistic potentials by various approaches.[12]) On the other hand, in calculating $C(^3\mathrm{He})$, we need the wave function of ^{3}He, in which the Coulomb interaction is taken into account. Since this interaction makes the calculations difficult, only a few groups[13-15] have so far obtained the wave function of ^{3}He. No calculations were done for $C(^3\mathrm{He})$. Only a conjecture about its magnitude has been made from the knowledge of $C(^3\mathrm{H})$.[7]

In the present paper, we report the calculations of $C(^3\mathrm{He})$ and $C(^3\mathrm{H})$, with recently obtained wave functions[15] of ^{3}He and ^{3}H. These wave functions have been calculated from the generalized Faddeev equation[16] for the Reid soft-core potential,[17] in which the Coulomb effect is fully taken into account for ^{3}He. These are obtained in coordinate space in a manner that the spectator is subject to the Coulomb interaction when it is charged. As a result, these wave functions are suitable for calculations of $C(^3\mathrm{He})$ as well as $C(^3\mathrm{H})$. As another advantage, we need not extrapolate the wave function to the unphysical pole.[3] At the present stage, each Faddeev component of our wave function is restricted to the $^1S_0$ and $^3S_1 + {}^3D_1$ states for the interacting pair and the s state for the third particle. Therefore, the calculations are limited to the s-wave asymptotic normalization constants $C_0(^3\mathrm{He})$ and $C_0(^3\mathrm{H})$. To our knowledge, this is the first calculation of $C_0(^3\mathrm{He})$ in which the Coulomb interaction is taken into account exactly.

With the notation we used previously,[18] we label $|{}^1S_0, l=0)|\zeta_a''\rangle_{12,3}$ as state (1), $|{}^1S_0, l=0)|\zeta_b''\rangle_{12,3}$ as state (2), $|{}^3S_1, l=0)|\zeta'\rangle_{12,3}$ as state (3), and $|{}^3D_1, l=0)|\zeta'\rangle_{12,3}$ as state (4). Here, ζ_a'', ζ_b'', and ζ' are the normalized isospin functions in which the interacting particles are the np pair with $I=1$, pp (or nn) pair with $I=1$, and np pair with $I=0$, respectively.

We express the radial wave functions for the S and D states of the deuteron as u_d and w_d. We introduce a function

$$\bar{\varphi}_d = [u_d|{}^3S_1, l=0) + w_d|{}^3D_1, l=0)]|\zeta'\rangle_{12,3}. \quad (1)$$

The wave function of a three-nucleon system is expressed as a sum of three Faddeev components,[15]

$$\Psi = \Phi(12,3) + \Phi(23,1) + \Phi(31,2). \quad (2)$$

We designate the Faddeev component $\Phi(12,3)$ projected onto the state $[|{}^3S_1, l=0) + |{}^3D_1, l=0)]|\zeta'\rangle_{12,3}$ by $\Phi[{}^3S_1 + {}^3D_1, l=0; (12,3)]$. Then the s-wave asymptotic normalization constants $C_0(^3\mathrm{H})$ and $C_0(^3\mathrm{He})$ are defined by[3]

$$C_0(^3\mathrm{H}) = \frac{1}{\sqrt{\beta}} \lim_{y\to\infty} \frac{\langle \overline{\varphi}_d | \Phi^{^3\mathrm{H}}[^3S_1 + {}^3D_1,\ l=0;(12,3)]\rangle}{y^{-1}\exp(-\beta y)} \tag{3}$$

and similarly for $C_0(^3\mathrm{He})$ with $\Phi^{^3\mathrm{H}}$ being replaced by $\Phi^{^3\mathrm{He}}$ and an extra factor $(2\beta y)^{-2Me^2/3\hbar^2\beta}$ in the denominator. Here y is the distance between the center of mass of the pair 1, 2 and particle 3, and β is given in terms of binding energies of the deuteron $|E_d|$ and the three-nucleon system $|E|$ by

$$\beta = \left[\left(\frac{4M}{3\hbar^2}\right)(|E| - |E_d|)\right]^{1/2} = \left(\frac{4M}{3\hbar^2}\right)^{1/2} \times \begin{cases}(6.400 - 2.221)^{1/2} = 0.4207\ \mathrm{fm}^{-1} \text{ for } {}^3\mathrm{H}, \\ (5.775 - 2.221)^{1/2} = 0.3880\ \mathrm{fm}^{-1} \text{ for } {}^3\mathrm{He}\end{cases} \tag{4}$$

where we have used the calculated binding energies.[15]

Equation (3) indicates that the overlap of $\overline{\varphi}_d$ with $\Phi[^3S_1 + {}^3D_1,\ l=0;(12,3)]$ at a sufficiently large distance y becomes proportional to $y^{-1}\exp(-\beta y)$. However, to obtain such a y dependence numerically is extremely difficult. This difficulty is avoided if we use the following expression for $C_0(^3\mathrm{H})$ readily derived from the Faddeev equation[19,20]:

$$C_0(^3\mathrm{H}) = \left(\frac{4M}{3\hbar^2}\right)\int_0^\infty y\,dy \frac{F(\beta,y)}{W}\int_0^\infty x^2\,dx \langle u_d, w_d | \begin{pmatrix} (^3S_1|V|^3S_1) & (^3S_1|V|^3D_1) \\ (^3D_1|V|^3S_1) & (^3D_1|V|^3D_1)\end{pmatrix} \begin{vmatrix} \langle ^3S_1,\ l=0|_{12,3}\langle \zeta' | P\Phi^3\mathrm{H}\rangle \\ \langle ^3D_1,\ l=0|_{12,3}\langle \zeta' | P\Phi^3\mathrm{H}\rangle \end{vmatrix}, \tag{5}$$

where $P\Phi^{^3\mathrm{H}}$ denotes the sum of Faddeev components

$$P\Phi^{^3\mathrm{H}} \equiv P\Phi^{^3\mathrm{H}}(12,3) = \Phi^{^3\mathrm{H}}(23,1) + \Phi^{^3\mathrm{H}}(31,2), \tag{6}$$

$F(\beta,y) = \sinh\beta y$, and $W = -\beta$.

For ^{3}He, we find from the generalized Faddeev equation[16] an expression which is similar to Eq. (5) but with the following modifications: $\varphi^{^3\mathrm{H}}$ is replaced by $\varphi^{^3\mathrm{He}}$. The function $F(\beta,y)$ is the negative-energy regular solution of the Schrödinger equation with a modified Coulomb interaction $e^2y^{-1}(1-e^{-\lambda y})$. The constant W is the Wronskian of the regular and irregular solutions of this Schrödinger equation.

The calculated results of $C_0(^3\mathrm{He})$ and $C_0(^3\mathrm{H})$ are given in Tables I and II, respectively.

The present value 1.7055 for ^{3}H, which we designate by $C_0^{\mathrm{SSK}}(^3\mathrm{H})$, should be compared with 1.776 [$\equiv C_0^{\mathrm{KM}}(^3\mathrm{H})$] by Kim and Nuslim.[7] In Ref. 7, β was taken as 0.390 ($\equiv \beta^{\mathrm{KM}}$). We see that this difference in $C_0(^3\mathrm{H})$ is due to the difference in β,

$$C_0^{\mathrm{KM}}(^3\mathrm{H})/C_0^{\mathrm{SSK}}(^3\mathrm{H}) \simeq (\beta^{\mathrm{SSK}}/\beta^{\mathrm{KM}})^{1/2} \simeq 1.04. \tag{7}$$

The relative magnitudes of contributions from various states are mostly understood from the spin-isospin transformation coefficient

$$N_{\alpha\alpha'} \equiv \langle I, I_z; \tfrac{1}{2}, m_t; M_T(12,3) | I', I_z'; \tfrac{1}{2}, m_t'; M_T(23,1)\rangle \langle (S,\tfrac{1}{2})S_0 M_{S_0}(12,3) | (S',\tfrac{1}{2})S_0 M_{S_0}(23,1)\rangle. \tag{8}$$

The values of this coefficient are given in Table III. If we use these coefficients explicitly in the Faddeev equation, we find that the radial function of state (2) is exactly $\sqrt{2}$ times as large as that of state (1). Further more, $N_{32} = \sqrt{2}N_{31}$. As a result, we see that the contributions from state (2), which we designate by $C_0^u(2)$ [or $C_0^w(2)$], are twice as large as $C_0^u(1)$ [or $C_0^w(1)$]. This property is confirmed

TABLE I. Asymptotic normalization constant of ^{3}He. The state number indicates the state involved in $\varphi^{^3\mathrm{He}}$ replacing $\varphi^{^3\mathrm{H}}$ of Eq. (5). C_0^u and C_0^w are the contributions to C_0 from u_d and w_d terms in Eq. (5), respectively. We obtained the s-wave asymptotic normalization constant $C_0(^3\mathrm{He}) = 1.7654$.

State	C_0^u	C_0^w
(1)	−1.556 (−1)	5.149 (−1)
(2)	−3.184 (−1)	1.0482 (0)
(3)	−2.711 (−1)	9.581 (−1)
(4)	−1.278 (−2)	2.069 (−3)
Sum	−7.579 (−1)	2.5233 (0)

TABLE II. Asymptotic normalization constant of ^{3}H. See the caption of Table I. We obtained the s-wave asymptotic normalization constant $C_0(^3\mathrm{H}) = 1.7055$.

State	C_0^u	C_0^w
(1)	−1.536 (−1)	5.053 (−1)
(2)	−3.071 (−1)	1.0104 (0)
(3)	−2.646 (−1)	9.267 (−1)
(4)	−1.385 (−2)	2.279 (−3)
Sum	−7.392 (−1)	2.4447 (0)

TABLE III. Isospin-spin transformation coefficient $N_{\alpha\alpha'}$ of Eq. (8).

State	(1)	(2)	(3)	(4)
(1)	$-1/4$	$1/2\sqrt{2}$	$-\sqrt{3}/4$	0
(2)	$1/2\sqrt{2}$	0	$-\sqrt{3}/2\sqrt{2}$	0
(3)	$-\sqrt{3}/4$	$-\sqrt{3}/2\sqrt{2}$	$1/4$	0
(4)	0	0	0	$-1/2$

in Table II.

From Ref. 15, we find that the relative magnitude of radial functions of states (3) and (1) for $\varphi(12,3)$ is approximately $|\varphi(3)| \simeq 3|\varphi(1)|$. Since $N_{31} \simeq \sqrt{3}N_{33}$, we obtain the estimate $C_0^u(3)/C_0^u(1) \simeq C_0^w(3)/C_0^w(1) \simeq \sqrt{3}$. Table II shows that these relations are approximately satisfied.

We see from Table II that the contribution from any state to C_0^w is approximately three times as large as C_0^u for the same state. This result indicates the importance of the tensor coupling.

In conclusion, we find that the difference between $C_0(^3\mathrm{He})$ and $C_0(^3\mathrm{H})$ due to the Coulomb interaction is small (3–4%), but still measurable by an accurate experiment and that about 60% of contributions are due to the transition from the pp (nn) $^1S_0$ state of a pair in one set of coodinates to the np $^3D_1$ state of another set through the tensor coupling.

We acknowledge the Research Center for Nuclear Physics at Osaka University and the Computer Center at the Research Institute for Plasma Physics, Nagoya University, for the use of their computing facilities. We wish to express our thanks to Mrs. N. Tichit, Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay for her numerical assistances. One of the authors (Y. E. K.) acknowledges the financial support by the Seoul National University–Agency for International Development project for his stay at Seoul National University during the summers of 1979 and 1980. This work was supported by the U. S. National Science Foundation and the Japan Society for Promotion of Science as a part of the Japan–U. S. Scientific Cooperative Research Program.

[1]See, M. P. Locher and T. Mizutani, Phys. Rep. 46, 43 (1978), for histroy, concept, and review of this subject in nuclear physics.

[2]M. Bolstrli and G. Hale, Phys. Rev. Lett. 28, 1285 (1972).

[3]Y. E. Kim and T. Tubis, Phys. Rev. Lett. 29, 1017 (1972).

[4]Y. E. Kim and T. Tubis, Annu. Phys. Nucl. Sci. 24, 69 (1974).

[5]B. A. Girard and M. G. Fuda, Phys. Rev. C 19, 583 (1979).

[6]E. P. Harper, D. R. Lehman, and F. Prats, Phys. Rev. Lett. 44, 237 (1980).

[7]Y. E. Kim and Muslim, Phys. Rev. Lett. 42, 1328 (1979).

[8]L. D. Knutson, B. P. Hichiwa, A. Barroso, A. M. Eiró, F. D. Santos, and R. C. Johnson, Phys. Rev. Lett. 35, 1570 (1975).

[9]S. Roman, A. K. Basak, J. B. A. England, J. M. Nelson, N. E. Sanderson, F. D. Santos, and A. M. Eiró, Nucl. Phys. A289, 269 (1977).

[10]F. D. Santos, A. M. Eiró, and A. Barroso, Phys. Rev. C 19, 238 (1979).

[11]L. D. Knutson and S. N. Yang, Phys. Rev. C 20, 1631 (1979).

[12]See references in T. Sasakawa and T. Sawada, Phys. Rev. C 19, 2035 (1979).

[13]C. Gignoux and A. Lavern, in *Few Particle Problems in the Nuclear Interaction*, edited by I. Slaus *et al.* (North-Holland, Amsterdam, 1972), p. 411.

[14]G. L. Payne, B. F. Gibson, and J. L. Friar, LASL Report No. LA-UR-80-52 (to be published).

[15]T. Sasakawa, H. Okuno, and T. Sawada, to be published.

[16]T. Sasakawa and T. Sawada, Phys. Rev. C 20, 1954 (1979).

[17]R. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[18]T. Sasakawa, T. Sawada, and Y. E. Kim, to be published.

[19]Equation (24) of D. R. Lehman and B. F. Gibson, Phys. Rev. C 13, 35 (1976).

[20]Equation (17) of Y. E. Kim, C. Sander, and A. Tubis, Phys. Rev. C 14, 2008 (1976).