ment because it has several desirable qualities. The mean free path of the incident electron is larger than in most other materials [at least for the (100) and (111) surfaces] giving more opportunity for photon emission. The top part of the *d* bands is unoccupied giving a range of high density of states to which the electron can make a transition, and providing a rich band structure on which to test whether direct transitions are observed. Tungsten also has a strong potential which increases the electron-photon interaction.

In conclusion, theoretical estimates all point to inverse photoemission being an easy experiment with plenty of signal. Experimental confirmation is essential for further progress.

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General Relation of Correlation Exponents and Spectral Properties of One-Dimensional Fermi Systems: Application to the Anisotropic $S = \frac{1}{2}$ Heisenberg Chain

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A relation between the spectrum and correlation exponents of the Luttinger model is argued to be a general property of a universality class called "Luttinger liquids." The spinless fermion model equivalent to the $S=\frac{1}{2}$ Heisenberg-Ising-XY chain in a field is argued to belong to this class, allowing for the first time the systematic calculation of its correlation exponents.

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Interacting quantum systems in one dimension (1D) have many distinctive features not present in higher dimensions, and which can be seen in various kinds of soluble models. In this Letter, I develop a new connection between two very different classes of soluble fermion models which in general have been studied by two separate sets of workers. These classes are (i) the Luttinger model,[1] a model with long-range (noncontact) forces, which has a conserved fermion current, and a gapless elementary excitation spectrum of free bosons with a linear spectrum, and (ii) models with contact interactions that can be solved by the Bethe *Ansatz*,[2] and which have a gapless linear excitation spectrum. It is now known that models soluble by the Bethe *Ansatz* have an S matrix that is nontrivial (i.e., their elementary excitations are *not* free particles) but *factorizable*.[3] This property reflects the existence of an infinite set of conserved quantities, but which does *not* include the current.

The idea underlying the result is simple, and is an extension of an idea implicit in the work of Luther and Peschel[4]: This is that the low-energy properties of the Luttinger model are characteristic of a larger universality class of systems, which includes most, and probably all, 1D fermion systems with a gapless linear spectrum. By analogy with the Fermi-gas/Fermi-liquid relation in higher dimensions, where the noninteracting elementary excitation spectrum of the Fermi gas provides the model for the low-energy spectrum of the Fermi liquid, I propose to call such systems "Luttinger liquids." Provided relevant

gap-inducing processes are absent, all such systems are equivalent to the Luttinger model at least to leading order in perturbation theory, since these terms depend only on the structure at the Fermi level.

The $T=0$ correlation functions of the spinless Luttinger model are characterized by power laws with coupling-constant dependent exponents, and the structure of the model means that they are all determined by a single parameter.[5] The idea of Luther and Peschel[4] is that such relations are generally valid in a larger universality class, among the members of which are certain models soluble by the Bethe *Ansatz*. In a particular limit of one of these models, they were able to identify one of the correlation exponents by an indirect route, and hence, from the universality argument, they were able to predict all the others. However, this identification of a correlation exponent is not generalizable, and, despite the fact that standard (numerical) techniques have been developed within the Bethe *Ansatz* framework for calculation of the excitation spectrum, the general calculation of correlation exponents has been a long-standing unsolved problem. In this Letter, I report a hitherto unrecognized relation between the correlation exponents and low-energy spectrum of the Luttinger model. When coupled with the argument that this relation too is a universal property of "Luttinger liquids," this allows the systematic identification of correlation exponents from calculated spectral properties.

As an example, I present the calculated phase diagrams of the exponent and spectral parameters of the $S=\frac{1}{2}$ Heisenberg-Ising-XY spin chain (i.e., the Heisenberg chain with uniaxially anisotropic nearest-neighbor exchange, or "XXZ" model[6]) in a field along the anisotropy axis (the model treated by Luther and Peschel was the zero-field limit of this). This Bethe-*Ansatz*–soluble model can be related to the Luttinger liquid as it can be represented as a spinless fermion model by use of the Jordan-Wigner transformation.[7] There is much rich structure in the phase diagram. The Hubbard chain,[8] a spin-$\frac{1}{2}$ fermion model, can also be described from a Luttinger-liquid standpoint, and a calculation of its phase diagram is in progress.

The Luttinger model[1] is a model of interacting 1D electrons and positrons with a linear dispersion; I will restrict the discussion to the spinless fermion case, on a ring of finite length L. It is well known that the elementary excitations are noninteracting bosons,[9] which are quantized density-wave excitations of the Fermi gas. However, there is an additional nonbosonic part of the spectrum,[10,11] which has generally been ignored in the literature. These terms are the energies associated with states of nonzero charge (with respect to a Fermi sea filled up to a Fermi vector k_F) and current. Charge is, of course, conserved, and since the Luttinger model has a linear fermion dispersion and no backscattering terms, so is the current. The charge and current are not affected by the presence of boson sound-wave excitations. In its diagonal form, the spectrum is[9-11]

$$H=\sum_q \omega_q b_q^\dagger b_q+\tfrac{1}{2}(\pi/L)(v_N N^2+v_J J^2). \quad (1)$$

The boson dispersion is linear as $q\to 0$: $\omega_q\sim v_S|q|$; the boson index q takes values $2\pi n/L$, $n=\pm 1$, $\pm 2,\ldots$, but there is no $q=0$ mode. Instead, there are the integer quantum numbers N (the charge) and J, which is subject to the selection rule $(-1)^J=(-1)^N$. The momentum is given by $P=[k_F+\pi(N/L)]J+\sum_q q b_q^\dagger b_q$, so J is increased by 2 for each $2k_F$ excitation. Note the dependence on the "true" Fermi vector $[k_F+\pi(N/L)]$, rather than the nominal value k_F which defines the zero-charge state. The low-energy spectrum depends on *three* distinct parameters, all with dimensions of velocity. In the case of the noninteracting model, $v_S=v_N=v_J$, but this is not generally the case.

A key aspect of the theory of the Luttinger model is the representation of the fundamental electron and positron fields in the basis of eigenstates of (1). Again, the commonly quoted forms[5,12] are incomplete or ambiguous in their treatment of the nonbosonic part, but precise forms, valid without qualification on a ring of finite length, are available.[10,11] The fermions are described by subsidiary "right-" and "left-moving" fermion fields $\psi_p^\dagger(x), p=\pm 1$, parametrized by a parameter $\varphi_q\equiv\varphi g(|q|R)$:

$$\psi_p^\dagger(x)=\bar{N}L^{-\nu}R^{\nu-1/2}e^{i\phi_p^\dagger(x)}U_p e^{i\phi_p(x)}, \quad (2)$$

where

$$\phi_p(x,\varphi_q)=\Big[\tfrac{1}{2}p(k_F+\pi N/L)x+\tfrac{1}{2}\pi(J/L)x + i\sum_q \alpha(pq,-\varphi_q)\exp(-iqx)b_q\Big],$$

and so (2) is normal ordered in the bosons, and $\alpha(q,\varphi_q)=(2\pi/L|q|)^{1/2}[\theta(q)\cosh\varphi_q+\theta(-q)\sinh\varphi_q]$; the exponent ν is $\frac{1}{2}\cosh(2\varphi)$. The normalization constant $\bar{N}$ depends only on φ and the function $g(y)$, where $g(0)=1$, and $y^{1/2}g(y)\to 0$ as $y\to\infty$.[13] R is a characteristic length associated with the range of interactions. The unitary operators U_p

are ladder operators that change the integers N and J: $[N,U_p]=U_p$, $[J,U_p]=pU_p$. Reflecting the Fermi statistics, U_p and U_{-p} anticommute, and $U_p|N,J\rangle=p^N|N+1,J+p\rangle$. Finally, U_p, like N and J, commutes with the bosons. The construction of U_p in terms of the bare fermions has been described in Ref. 11.

The model is fully parametrized by L, k_F, ω_q, φ_q, v_N, and v_J. However, these last two are not free parameters, and are given by

$$v_N=v_S\exp(-2\varphi);\ v_J=v_S\exp(2\varphi). \tag{3}$$

These relations can be obtained in two ways: (i) By expressing their components in terms of the bare parameters of the model, or (ii) by using (2) to obtain the density operators

$$\rho_p(x)=\lim_{a\to 0}[\psi_p^\dagger(x+a)\psi_p(x)-\langle\psi_p^\dagger(x+a)\psi_p(x)\rangle_0],$$

and comparing their response functions (a) for $q=0$ (a calculation involving only nonbosonic terms) and (b) for $q\to 0$ (a calculation involving only boson terms). Similarly, (2) can be used to establish that the mean current j is $v_J(J/L)$, so v_J is the renormalized Fermi velocity. v_S is, of course, the sound velocity, and v_N relates changes in chemical potential to those in k_F: $\delta\mu=v_N\delta k_F$. The correlation exponents depend only on the parameter φ: Various exponents are tabulated in terms of $\theta=\frac{1}{2}\exp(-2\varphi)$ in Ref. 4.

I now propose that (3) and the relation between $\exp(-2\varphi)$ and the correlation exponents are generally valid properties of 1D conductors, which I propose to call "Luttinger liquids." One test, briefly reported here, is provided by models soluble by the Bethe *Ansatz*, for which v_S and v_N, and by inference $\exp(-2\varphi)$, can be obtained. Features of these models, particularly those associated with Umklapp processes, are found[14] to conform in precise detail to those expected from the analysis of Luttinger-type models. A second piece of evidence, reported elsewhere,[14] is a direct demonstration (by perturbation expansion around the Luttinger model) that introduction of nonlinearity into the bare fermion dispersion changes neither (3) nor the relation between $\exp(-2\varphi)$ and the correlation exponents. The new features that appear are an irreducible boson-boson interaction term, making the model no longer soluble, and dependence of the fundamental parameters v_S and $\exp(-2\varphi)$ on ground-state occupancy. The model becomes completely general if current-nonconserving terms involving $U_pU_{-p}^{-1}$ are added: A scaling analysis[14] shows that unless a gap is induced, the model retains Luttinger-liquid character at low energies.

I have tested these ideas on the spinless fermion representation of the anisotropic Heisenberg chain, with anisotropy parameter $\Delta<1$ (the region $\Delta\geq 1$ is ferromagnetic, and does not have Luttinger-liquid character). $\Delta=0$ corresponds to a free-fermion system, and $\Delta=-1$ is the isotropic antiferromagnet. I performed the calculation in finite field numerically, using standard techniques.[6,15] In zero-field (corresponding to a half-filled-band fermion system) analytic results in the literature are in precise accord with the Luttinger-liquid predictions: When parametrized by $\cos\bar{\mu}=-\Delta$, v_S is given[16] by $(\pi/\bar{\mu})\sin\bar{\mu}$, v_N by[6] $2(\pi/\bar{\mu}-1)\sin\bar{\mu}$. Equation (3) then predicts $\exp(-2\varphi)=2(1-\bar{\mu}/\pi)$: This result has in fact been obtained previously by Luther and Peschel[4] using entirely different methods.

The calculated phase diagrams are parametrized by Δ and the fermion occupation per site n: The magnetization per site S^z in the spin picture is $n-\frac{1}{2}$, and the field h along the anisotropy axis that produces this is given in Fig. 1. The spectrum has Luttinger-liquid form[17] for $\Delta<1$, $0<n<1$, except along $n=\frac{1}{2}$, $\Delta<-1$, where Umklapp processes induce a gap. Figures 2(a)–2(d) show the calculated v_S and v_N, and the derived

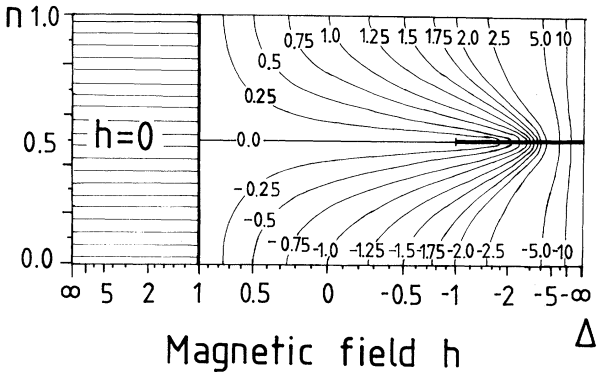

FIG. 1. Ground-state phase diagram of the $S=\frac{1}{2}$ Heisenberg-Ising-XY chain, giving the relation between magnetic field h along the anisotropy axis (in units of basal plane exchange $|J_\perp|$) and magnetization per site $S^z=n-\frac{1}{2}$, where $\Delta=J_\parallel/|J_\perp|$, plotted on a nonlinear scale $\propto\tan^{-1}(-\Delta)$. In the fermion description, $-\Delta$ is the coupling, and n the occupation per site. The region $\Delta\geq 1$ is ferromagnetic. In the region $\Delta<1$, $0<n<1$ the excitation spectrum is of gapless Luttinger-liquid type, except along the line $\Delta<-1$, $n=\frac{1}{2}$, representing a pinned density wave with doubled unit cell (i.e., the easy-axis antiferromagnet with $S^z=0$).

v_J and $\exp(-2\varphi)$ (in principle v_J should also be directly calculable). A detailed discussion of the results will be given elsewhere. The behavior in the weak-coupling regions $\exp(-2\varphi) \sim 1$ agrees with perturbation expansions. For $\Delta = -\infty$, the Bethe *Ansatz* equations are analytically soluble[15]: I obtain[14] $\exp(-2\varphi) = (\frac{1}{2} + |n - \frac{1}{2}|)^{-2}$. The structure near $n = \frac{1}{2}$ is due to an Umklapp term[14]

$$H^1 = W \sum_p \int_0^L dx e^{-ipgx} (\psi_p^\dagger \nabla \psi_p^\dagger)(\psi_{-p} \nabla \psi_{-p}), \qquad (4)$$

where g is the reciprocal-lattice vector. When transcribed into "boson form," this term is analogous to that describing Umklapp effects in spin-$\frac{1}{2}$ fermion systems[19]: For $\exp(-2\varphi) > 2$, it is relevant (opens a gap) as $k_F \to \frac{1}{4}g$; for $\exp(-2\varphi) < 2$, φ has nonanalytic dependence on $|k_F - \frac{1}{4}g|$, $\varphi = \varphi_0 + O(|k_F - \frac{1}{4}g|^\epsilon)$, $\epsilon = [2\exp(2\varphi_0) - 1]$ (Ref. 14): The Heisenberg-chain results, including expansions in $|n - \frac{1}{2}|$ given in the literature,[6,20] when interpreted according to the Luttinger-liquid scheme, exactly mirror the scaling-theory predictions for the Luttinger model with (4), even in the details of the critical region $\Delta \sim -1$, $n \sim \frac{1}{2}$.[14]

When $\Delta < -1$, and $n = \frac{1}{2}$, the Umklapp process stabilizes a $2k_F$ pinned density wave. The behavior of the Luttinger-liquid parameters near this line can be interpreted in terms of a dilute weakly interacting gas of a new type of fermion—"solitons" with charge $\pm\frac{1}{2}$, and soliton Fermi vector $k_F^s = 2k_F - \frac{1}{2}g$. The limiting behavior $\exp(-2\varphi) \to 4$ characterizes the noninteracting limit of such a gas of spinless fermions.[14] Such a soliton gas has previously been predicted from studies of the $\Delta \sim -\infty$ limit of the Heisenberg chain.[15] Information on the soliton mass, size, and interaction can be extracted[14]: The linear decrease of $\exp(-2\varphi)$ at finite soliton density indicates attractive soliton-soliton coupling, hence repulsive soliton-antisoliton coupling, consistent with the absence of bound states in the gap.

In a treatment of the model by linearization[4] around $\Delta = 0$, v_J is unrenormalized[14]; Fig. 2(c) indicates the region of validity of such a scheme.

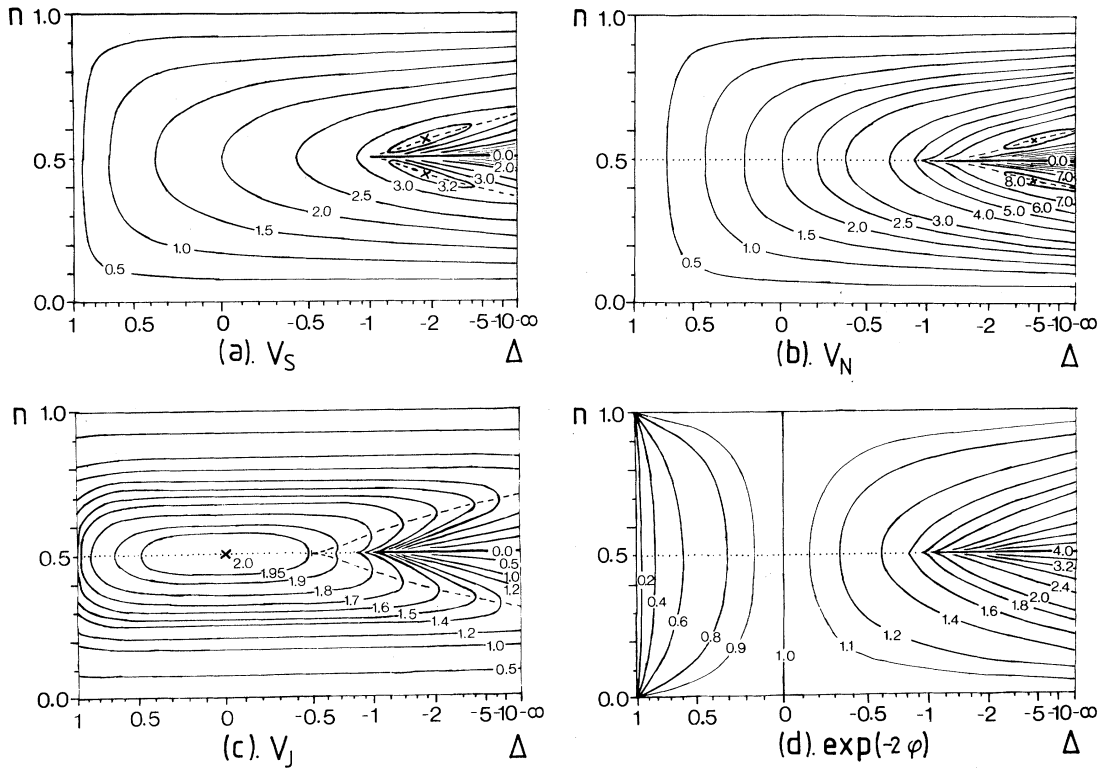

FIG. 2. Luttinger-liquid parameters: (a) The sound velocity v_S; (b) $v_N = d\mu/dk_F$; (c) the renormalized Fermi velocity v_J; (d) the interaction parameter $\exp(-2\varphi)$ that determines correlation exponents. The basal-plane spin-correlation exponent θ of Ref. 4 is $\frac{1}{2}\exp(-2\varphi)$. The velocities v_S, v_N, and v_J are given in units $|J_\perp|/2a$; a is the lattice spacing. The loci of the maxima of v_S, v_N, and v_J for fixed Δ are shown when not at $n = \frac{1}{2}$. Nonanalyticity of v_N, v_J, and $\exp(-2\varphi)$ across the line $n = \frac{1}{2}$, $|\Delta| < 1$ is indicated. The remaining Luttinger-liquid parameter k_F is $(\pi/a)n$. See also Ref. 18.

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Unusual Critical Behavior of the Diluted Uniaxial Dipolar Ferromagnet $LiTb_{0.3}Y_{0.7}F_4$

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The experimental critical behavior of the susceptibility of $LiTb_{0.3}Y_{0.7}F_4$ is described by the power law $\chi = \Gamma[(T-T_c)/T_c]^{-\gamma}$ with $T_c = 0.520 \pm 0.003$ K and $\gamma = 1.80 \pm 0.04$. This behavior is dramatically different from that previously observed in $LiTbF_4$ and is evidence of a departure from marginal dimensionality when magnetic ions have been randomly replaced by nonmagnetic ions. Series expansion of $(\chi T)^{-1}$ in powers of T^{-1} for a diluted Ising dipolar ferromagnet gives a good description of experimental results described in this Letter.

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The critical behavior of pure uniaxial dipolar ferromagnets is one of the best explained critical phenomena. In this case the marginal dimensionality is $d^* = 3$. In the close vicinity of the critical temperature T_c, the magnetic susceptibility is predicted[1] to have logarithmic corrections to the classical law so that it diverges as $t^{-1}|\ln t|^{1/3}$, where t is the reduced temperature $t = (T-T_c)/T_c$. The first higher-order term of the logarithmic corrections has been calculated for all the thermodynamic quantities in zero magnetic field[1] and in the whole critical region for a finite field.[2]

$LiTbF_4$ is a quasiuniaxial dipolar ferromagnet.[3,4] The experimental critical behavior of these crystalline pure compounds[5,6,7] is well described by the theoretically predicted classical behavior with logarithmic corrections.

In diluted uniaxial dipolar ferromagnets where some magnetic ions are randomly replaced by nonmagnetic ones, quite a different behavior was predicted by Aharony.[8] In this Letter we report measurement on the dilute ferromagnet $LiTb_{0.3}Y_{0.7}F_4$ which for the first time clearly shows departure from the behavior expected from a system at marginal dimensionality. The parallel susceptibility of $LiTb_{0.3}Y_{0.7}F_4$ corrected from demagnetizing effects cannot be described by the Aharony law $\chi = \Gamma t^{-1}\exp[D\ln(1/t)]^{1/2}$ with the universal parameter $D \sim 0.11795$ in the temperature range $10^{-3} < t < 10^{-1}$ even by substituting $1/t$ by t_0/t, taking approximately into account high-order terms. As the crossover between "pure" behavior to asymptotic "random" behavior is not well known, we have described our experimental results by a power law with the unusual large effective exponent $\gamma = 1.80$.