and that correlated events, together with incomplete m mixing for strong collisions, play an important role. One of the most encouraging aspects of the results is that a considerable amount of extra information about the dynamics of an atomic collision is needed to describe far-wing depolarization and, conversely, a great deal of information may be obtained from its study. They confirm[1,3] that scattering of light *cannot* be characterized by simple absorption (or emission profiles).

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Ion Thermal Conductivity in a Helical Toroid

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Numerical calculations of the ion thermal conductivity in helical toroidal (torsatron) magnetic configurations show the presence of a plateau regime extending over two orders of magnitude in collision frequency, ν. The value of the ion thermal conductivity is approximately equal to the neoclassical plateau value for an equivalent torus without helical modulation. The predicted adverse $1/\nu$ behavior due to ripple trapping is not seen.

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The family of plasma confinement devices that includes stellarators, torsatrons, and heliotrons is of considerable interest for controlled-fusion applications. These are toroidal devices in which closed, nested magnetic surfaces are generated in the vacuum magnetic field by a helical configuration of the external windings. A major feature of this family is the presence of a strong helical modulation, or ripple, of the field strength on the flux surfaces. Until now, neoclassical transport theory has associated large transport coefficients with this modulation, due to particles which are trapped in the helical magnetic wells.[1,2] These particles are subject to a vertical drift resulting from the toroidal curvature of the magnetic field and can, in some circumstances, make large excursions from their initial flux surfaces.

An estimate of the ion thermal conductivity resulting from this mechanism has been given for stellarators by Connor and Hastie[2]:

$$\chi_i \simeq 11.6\epsilon_h^{3/2}\rho_i^2 v_{ti}^2/\nu_{ii}R^2 \tag{1}$$

(in SI units), where ϵ_h is the helical modulation of the field, ρ_i is the ion gyroradius, v_{ti} is the ion thermal velocity, R is the major radius, and ν_{ii} is the ion-ion collision frequency. This expression is presumed to be valid when the collision frequency is small enough for particles to complete bounce orbits in the helical modulation but large enough that these trapped particles do not complete their poloidal drift orbits.

This result is of particular concern for plasma conditions appropriate to fusion reactors. In fact, the $1/\nu$ dependence of the transport coefficients has led recent torsatron reactor design studies to concentrate on low-temperature, high-density plasma regimes where collisionality is large and χ_i is acceptably small.[3,4]

Extensive orbit calculations for particles in model fields show that the orbits are very much more complicated than assumed in neoclassical theoretical models for rippled fields.[5] Furthermore, recent experimental results indicate that the expected $1/\nu$ behavior may not be occurring.[6,7] In order to test the validity of the existing theory, we developed a computer program which follows test particle orbits, including all relevant collisional effects, in complex magnetic fields. By examining statistically significant numbers of such orbits, we can determine the thermal conductivity for various plasma conditions.

The calculations reported here were performed for a torsatron magnetic field configuration; the torsatron is representative of the heliotron/stellarator family and was chosen because of its particular suitability for scaling to a reactor size. The magnetic field used was that generated by a set of specified external conductors. This method was used, instead of recourse to a model field, in order to ensure that no significant oversimplifying assumptions regarding field configurations were made. The flux surfaces obtained by following field lines are labeled with the toroidal flux enclosed (ψ_t).

In order to follow particle orbits we use guiding-center equations accurate to second order in μ. Collisional effects are included through energy scattering, pitch-angle scattering, and drag terms dependent on the background density and temperature (which can be functions of ψ_t). In order to evaluate χ_i, n and T were chosen to be independent of ψ_t. To simplify further the calculation, we follow test ions and include only ion-ion scattering effects. This simplification is possible when thermal conductivity is being calculated; it would not be correct in calculations of particle transport. For each run, 360 test ions were launched on a given flux surface, ψ_{t0}. These test particles had a pitch angle and energy distribution appropriate to an isotropic Maxwellian with temperature equal to that of the background ions. They were uniformly spaced poloidally, and weighted according to the flux-surface area each particle represented. Each test particle was followed for 30 msec, with its energy and flux position recorded every 3 msec. The tracking time was long compared to the collisional detrapping time ($\equiv 1/\nu_{\rm eff}$), the time in which the test particle equilibrium distribution forms. This distribution was found to be an isotropic Maxwellian. The data comprise a test-particle distribution function $f_i(x,v,t)$, with $x = \langle r_{\rm sep}\rangle(\psi_t/\psi_{t\,\rm sep})^{1/2}$, where $\psi_{t\,\rm sep}$ is the flux function at the separatrix and $\langle r_{\rm sep}\rangle$ is the average radius of the separatrix.

The ion thermal conductivity can be determined from the test particle distribution by

$$\chi_i = \frac{t^{-1}\int dx\, \frac{1}{2}(x-x_0)^2 u_i(x,t)}{\int dx\, u_i(x,t)}, \tag{2}$$

where $u_i(x,t) = \int d^3v\, f_i(x,v,t)\frac{1}{2}m_i v^2$ is the kinetic energy density of the test-particle distribution.

Numerically χ_i was found by performing a least-squares fit to

$$A + \chi_i t_j = \frac{\sum_k \frac{1}{2}(x_k - x_0)^2 u_{kj}}{\sum_k u_{kj}} \equiv y(t_j), \tag{3}$$

where the index j denotes the time and the index k denotes the spatial interval. Although all test particles in a particular case are started on the same flux surface, there is a spreading of the test particle distribution because the collisionless drift surfaces differ from the flux surfaces. This spreading results in a statistical fluctuation in $y(t)$ and an uncertainty in the fit for χ_i. This method for calculating χ_i was tested in axisymmetric tokamaks and was found to reproduce neoclassical results[8] to within 10% throughout the banana and plateau regimes.

In Fig. 1 we plot the results of a scan of χ_i vs ν_{ii} for a reactor-sized torsatron with $R_0/a = 12$. Both the test and background ions have tempera-

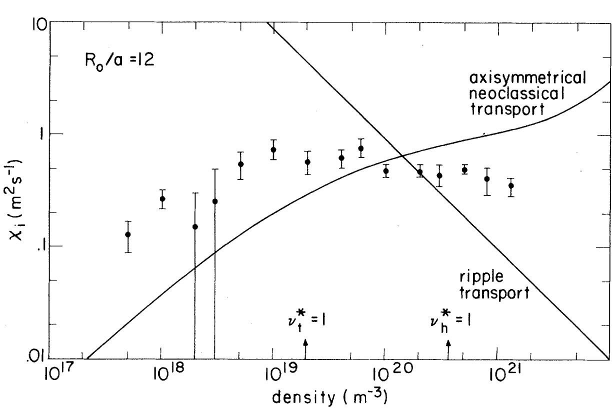

FIG. 1. Ion thermal conductivity as a function of plasma density for a torsatron with the following parameters: $R_0 = 48$ m; $a_c = 4$ m; $l = 3$; $N = 32$; B_z (axis) $= 5.5$ T; $\psi_{\rm sep} = 82.8$ Wb; and $\langle r_{\rm sep}\rangle = 2.1$ m. Curves correspond to the theoretical conductivity predicted by axisymmetric neoclassical transport and ripple transport. ν_h^* and ν_t^* are, respectively, the ratio of collisional detrapping frequency to bounce frequency due to the helical and toroidal modulations in B. The indicated errors on the conductivity measurements correspond to 1 standard deviation in a least-squares fit to Eq. (3).

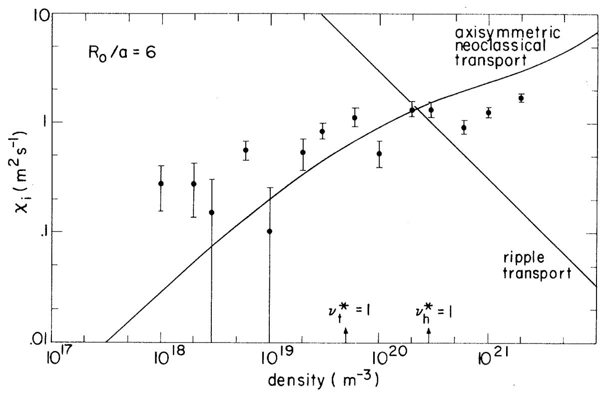

FIG. 2. χ_i versus plasma density for aspect ratio 6.

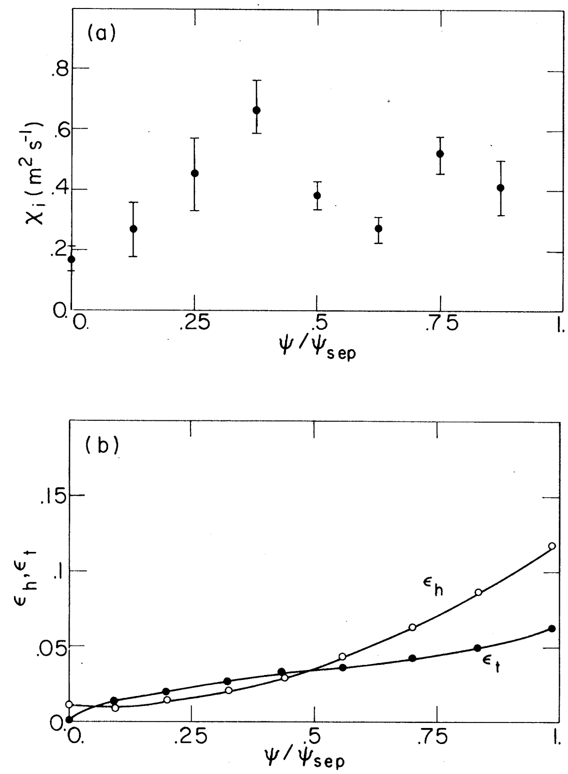

FIG. 3. (a) Ion thermal conductivity vs flux surface position for aspect ratio 12 and $n = 3\times10^{20}$ m^{-3}. (b) Peak-to-average magnetic ripple vs flux surface position. Open circles correspond to helical ripple, and solid circles correspond to toroidal ripple.

ture 8 keV, giving $\nu_{ii} \simeq 125(n_i/10^{20}\ \mathrm{m}^{-3})\ \mathrm{sec}^{-1}$ for ions with mass 2.5. The test ions for this case were started at $\psi_{t0} = 0.25\ \psi_{t\ \mathrm{sep}}$ where $\epsilon_t = 0.02$, $\epsilon_h = 0.015$, and $\iota = 1/q = 0.25$; ϵ_t is the toroidal modulation given by the local value of the inverse aspect ratio and ι is the local rotational transform. In the same figure we plot for comparison the theoretical ion thermal conductivity for an axisymmetric torus of otherwise identical parameters,[8] and the result of the neoclassical calculation including helical trapping [Eq. (1)]. In Fig. 2 we plot the same quantities for a torsatron with $R_0 = 24$, $N = 16$. In Fig. 3(a) we plot for the conditions of Fig. 1, χ_i vs ψ_{t0} for $\nu_{ii} = 370\ \mathrm{sec}^{-1}$. In Fig. 3(b), the values of ϵ_t and ϵ_h are shown. Apparently, the thermal transport for fixed collisionality is not sensitive to the exact value of the modulation in the parameter range examined.

The principal result of these computations is that the predicted $1/\nu$ behavior does *not* occur; instead ion thermal conductivity is nearly independent of collision frequency over a wide range of collisionality. This transport coefficient is approximately equal to that derived for the neoclassical axisymmetric plateau regime, and maintains this value over at least two orders of magnitude in density (or collision frequency) variation. The "plateau" character of the thermal conductivity is confirmed by the $1/R$ dependence seen in Fig. 4. This result has favorable implications for reactor design, as it indicates that the plasma temperature, in moderate aspect ratio helical systems, may be raised to 15 keV (the value for minimum $n\tau$ at ignition) without suffering from increased loss due to ripple transport.

Although the reason for the absence of the predicted $1/\nu$ behavior is not clear, and detailed calculations are beyond the scope of this paper, a number of observations can be made. First, we suggest that ripple transport theory is not applicable to the torsatron geometry. The usual calculation of transport due to ripple trapping relies on the assumption that a significant fraction ($\sim \epsilon_h^{1/2}$) of the particles in the system are trapped in the helical ripples, and that the deviation of their collisionless drift orbits from their initial flux surface position is determined primarily by the vertical drift caused by the toroidal $1/R$ magnetic field gradient. However, detailed calculations of collisionless particle orbits in a wide variety of helical toroidal configurations indicate that this assumption does not hold. As in axisymmetric systems, most particles are never reflected, make small excursions from flux surfaces, and do not contribute significantly to transport in low- to moderate-collisionality regimes. In helical systems, the rest of the particles undergo very complex motions. They can make frequent transitions between quasicirculating

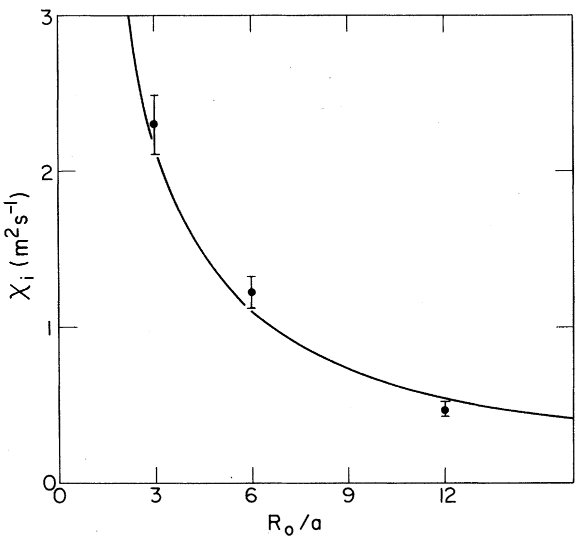

FIG. 4. χ_i vs aspect ratio. The solid curve is the best fit to the equation $C_0(a/R_0) = \chi_i$.

("blocked") and helically trapped orbits.[5] Even for large aspect ratios ($\epsilon_h > \epsilon_t$) the contribution from particles with simple, helically trapped orbits is negligible. The transition particles comprise a fraction of the total which is the larger of $\epsilon_t^{1/2}$ or $\epsilon_h^{1/2}$.

We suspect the observed transport results from orbit resonance between motion in the helical modulation and bounce motion in the toroidal modulation of the field.[9,10] Particles subject to these resonances do not conserve the adiabatic invariant, J, because their orbits depend critically on the phase of the bounce motion near the transition points. This behavior leads to a transport coefficient which is independent of ν. Furthermore, we suggest that in large-aspect-ratio systems ($\epsilon_h \gtrsim \epsilon_t$), it is not appropriate to consider the helical field structure as a perturbation on an axisymmetric torus, which is the ordering common to existing theories, but rather that one should consider orbits and transport in a two-dimensional (straight) helix perturbed by a weak toroidal curvature.

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