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Breakup-Fusion Description of Massive Transfer Reactions with Emission of Fast Light Particles

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It is shown that massive transfer reactions emitting energetic light particles can be described in terms of two-step processes, in which breakup of the projectile takes place first, followed by an absorption of the massive partner of the broken-up pair by the target.

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Twenty years ago, Britt and Quinton[1] noticed that very energetic α particles are emitted rather copiously, in reactions induced by heavy ions. They are energetic in that their velocities exceed that of the projectile. More recent experiments,[2,3] which measured the coincidence of light fast particles (having $z = 1$ and 2) with γ rays emitted from targetlike residues, seem to have revealed the fact that the emission of these fast particles is almost always accompanied by the fusion of the rest of the projectile into the target. Other types of coincidence experiments[4] also seem to lead to the same conclusion.

Take, as an example, the emission of fast α's observed in a bombardment of ^{159}Tb by ^{14}N.[3] If ^{10}B, the projectile minus α, is fused into ^{159}Tb, a highly excited compound nucleus ^{169}Yb is formed. This ^{169}Yb will first emit x number of neutrons, and the resultant $^{169-x}$Yb nuclei will then emit γ rays characteristic of each value of x. By measuring these γ rays in coincidence with the fast α's, one can extract the cross section of the reaction ^{159}Tb(^{14}N, α)^{169}Yb. It was found that this cross section is rather large,[3] being of the same order of magnitude as are the fast-α singles cross sections.[1] The authors of Ref. 3 called these reactions *massive transfer*. The above experimental fact shows that if once the massive transfer reaction is understood, so will be (the large part of) the emission of fast light particles.[1]

The purpose of the present Letter is to show that we can fit the data of the above experiments, based on a concept which may be termed a breakup-fusion (BF) mechanism. The name signifies that we describe the massive transfer reaction as a two-step process. Take again the above example. The first step is then the breakup of ^{14}N into $\alpha + {}^{10}$B. This is then followed by the second step, in which ^{10}B is fused into ^{159}Tb.

A general formulation of BF processes was made recently by Kerman and McVoy (KM).[5] The expression for the cross section [Eq. (27) of KM] may be written, with a slightly modified notation, as

$$d^2\sigma/dE_\alpha d\Omega_\alpha = [m_a m_\alpha/(2\pi\hbar^2)^2] \times (k_\alpha/k_a)\sum_l A_l \sum_m |\beta_{lm}(k_\alpha)|^2. \quad (1)$$

Here a (= ^{14}N) denotes the projectile, which is broken up into $\alpha + x$. Thus $\beta_{lm}(\vec{k}_\alpha)$ is the amplitude of the breakup process in which α is emitted with a momentum $\vec{k}_\alpha$, and x (= ^{10}B) with an angular momentum (lm) relative to the target A. The singles-α cross section associated with this (elastic) breakup process is given by (1), if we set $A_l = 1$ (all l). If we set $A_l = P_l/4$, on the other hand, (1) gives the additional contribution to the singles-α cross section due to the BF process.[5] It can also be interpreted as the cross section of the massive transfer reaction. Here P_l denotes the penetrability between x and A, and describes the absorption of x by A.

We have reformulated the KM work and found it desirable to use

$$A_l = P_l/|s_l|^2, \quad (2)$$

in place of $A_l = P_l/4$ of KM. Thus our A_l is $4/|s_l|^2$ times that of KM. This difference originates from the difference in the way we manipulate the Green's function that describes the propagation of x, between its creation (via breakup) and absorption (into the target). In obtaining $A_l = P_l/4$, KM took only the on-shell part of this Green's function, while we found that the off-shell part gives almost exactly the same amplitude (when the breakup is described as it was by Udagawa and co-workers[6,7]) as does the on-shell part. This is the origin of our extra factor of 4.

In manipulating the Green's function, one also needs to introduce a complete set of states, one way or another. If the relative motion between x and A is described, as we do it, through introduction of an optical potential, this complete set must be biorthogonal. Then a factor $1/|s_l|^2$ emerges inevitably.[8,9] (KM appear to have ignored this fact.) Since s_l is the S-matrix element for the elastic scattering between x and A, $1/|s_l|^2$ can be rather large (for smaller l). We show below that the presence of this factor $1/|s_l|^2$ is vital in making our BF cross section sufficiently large.

Previously[6,7] we showed that, if the breakup is described as an inelastic excitation of a into its continuum a' $(=\alpha + x)$, its amplitude T can be calculated rather easily. With this description, T is most naturally obtained as a function of two relative momenta $\vec{k}_a'$ and $\vec{k}_x'$, which are, respectively, between a' and A, and between α and x. The relative momenta more convenient to describe the BF process, on the other hand, are $\vec{k}_\alpha$, introduced in (1), and $\vec{k}_x$ between x and A (or more precisely $\vec{k}_\alpha$ and $\hat{k}_x$, because of energy conservation). Nevertheless, the pairs $(\vec{k}_a', \vec{k}_x')$ and $(\vec{k}_\alpha, \vec{k}_x)$ are simply related, and thus there is not much difficulty in obtaining the $\beta_{lm}(\vec{k}_\alpha)$ of (1) as

$$\beta_{lm}(\vec{k}) = \int T(\vec{k}_a', \vec{k}_x') Y_{lm}^*(\hat{k}_x)\, d\hat{k}_x. \tag{3}$$

The quantities P_l and s_l are evaluated easily, once an optical potential, U_{xA}, is introduced explicitly between x and A. In evaluating β_{lm} via (3), however, U_{xA} was not used. Yet in (1), β_{lm} and s_l appear in a combination of the form $|\beta_{lm}|^2/|s_l|^2$. This causes a subtle problem in the evaluation of P_l and s_l, in particular of the latter.

Physically, the appearance of the factor $|\beta_{lm}|^2/|s_l|^2$ in (1) is very reasonable. The β_{lm} describes the amplitude with which both α and x survive the absorption (by A). Because of the presence[6,7] of the strong absorption between a and A (and a' and A), β_{lm} is small for lower l. However, a large fraction of this absorption is caused by the capture of x by A, the cross section for which we intend to calculate. Therefore, we must have first a breakup amplitude before this absorption (to be described later via the factor P_l) takes place. The new amplitude β_{lm}/s_l, replacing β_{lm}, is precisely this amplitude.

The above argument shows that it is desirable to obtain β_{lm} and s_l on an equal footing, possibly based on the use of a common U_{xA}. In principle it is possible to do this, by describing the breakup as a stripping of x (from a) onto A (to form $x+A$ in continuum). In practice, however, such a calculation is almost impossible to carry out. It means performing an EFR-DWBA (exact-finite-range, distorted-wave Born approximation) calculation, in which an angular momentum l as large as 30 is transferred.[10] Faced with this situation, we decided to proceed as follows.

It is well known that $|s_l|$ can be very well represented analytically as

$$|s_l| = s_0 + (1 - s_0)\{1 + \exp[-(l - l_s)/\Delta]\}^{-1}. \tag{4}$$

Here $s_0 = |s_{l=0}|$. It is also well known that $s_0 \ll 1$, but $s_0 \neq 0$. The fact that $s_0 \neq 0$ is very important, because $|\beta_{lm}| \neq 0$ even for $l = 0$.

We extracted first the parameters s_0 and Δ of (4), by introducing an U_{xA}, with a reasonable set of parameters, and carrying out elastic-scattering calculations between x and A. The parameter l_s was also extracted in the course of the calculation, but it was found that it did not necessarily agree with l_β, which is the value of l at which $|\beta_{lm}|$ takes a maximum value. If an l_s that differs from l_β were used, it can happen that several of the $|\beta_{lm}|^2/|s_l|^2$ factors get very large, making the cross section of (1) behave rather erratically. We thus decided to use $l_s = l_\beta$ in (4). There is every reason to believe that we would get $l_s = l_\beta$, had β_{lm} and s_l been calculated consistently, as described above. Also the choice of parameters in U_{xA} is not unambiguous.

Equation (1) depends much less sensitively on P_l, than it does on s_l. We used for P_l the following analytic form; $P_l = 1/\{1 + \exp[(l - l_{cr})/\Delta]\}$, where l_{cr} is the critical angular momentum associated with fusion, and is calculated by using the formula given by Lefort.[11]

Numerical calculations based on the theory developed above have been carried out to analyze the data of two reactions, $^{159}Tb(^{14}N, \alpha xn)$ (Ref. 3) and $^{181}Ta(^{14}N, \alpha)$,[12] both with $E_L(^{14}N) = 115$ MeV, and the results are presented in Figs. 1 and 2. In obtaining these results, we used $s_0 = 0.01$ and $\Delta = 1.9$, obtained in the way described above. Further, we used 18 MeV for Γ_Q, which is the width

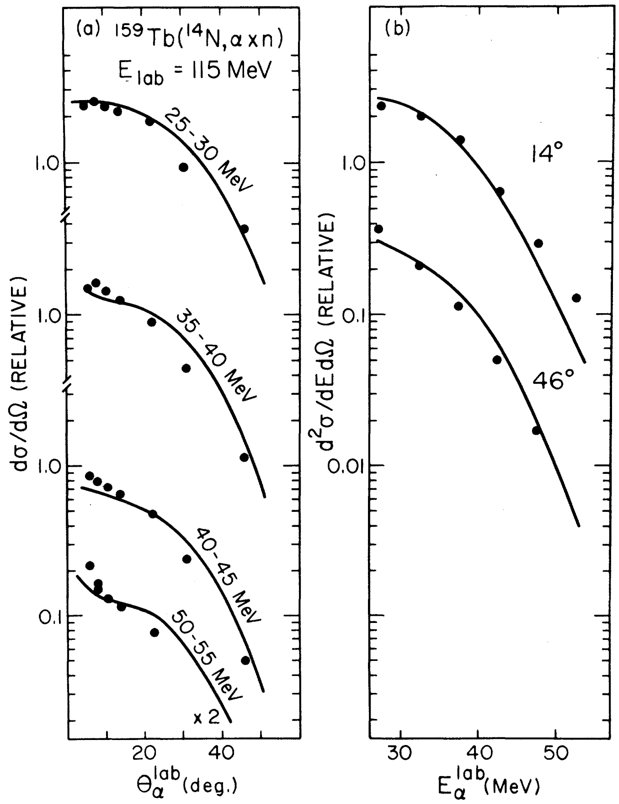

FIG. 1. Fit to data of the reaction ^{159}Tb(^{14}N,αxn). Theoretical cross sections are represented by solid lines, while the data of Ref. 3 are by full circles.

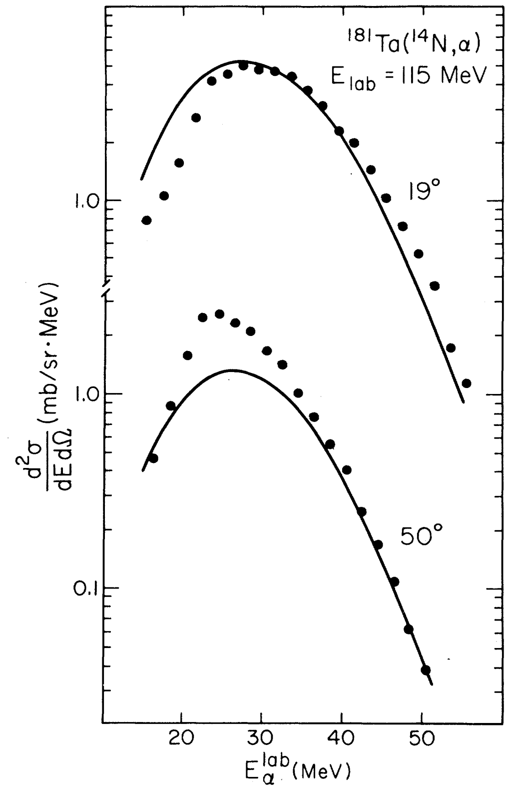

FIG. 2. Same as in Fig. 1, expect that it is for reaction ^{181}Ta(^{14}N,α) of Ref. 12.

of the Q window embodied by the breakup amplitude T that appears in the integrand of (3). We first obtained $\Gamma_Q = 12$ MeV, calculating T in the way as described earlier,[6,7] by further assuming that the $\alpha + {}^{10}$B system lies in an $L = 0$ relative state. Had the $L \neq 0$ states been considered as well, a larger Γ_Q should have resulted. The choice of $\Gamma_Q = 18$ MeV, which was made somewhat arbitrarily, is thus more reasonable than that of 12 MeV. The lack of knowledge of spectroscopic factors for splitting ^{14}N into $\alpha + {}^{10}$B with $L \neq 0$ (and with ^{10}B in excited states) makes it all but meaningless to try to obtain Γ_Q more accurately, at least at this stage.

As seen in Fig. 1, the theoretical prediction of both angular and energy distributions of the α particles in the reaction ^{159}Tb(^{14}N,αxn) agrees almost perfectly with experiment.[3] Unfortunately, the absolute magnitude of the experimental cross sections is not known very accurately yet, except that it is rather large.[3] Nevertheless, excellent fits to relative cross sections in Fig. 1 may allow us to conclude that the mechanism of the massive transfer reaction is now well understood.

The absolute cross section of the reaction ^{181}Ta(^{14}N,α) is known,[12] and in plotting the theoretical cross sections in Fig. 2, we multiplied the calculated values with an overall factor of $N_0 = 6.4$. Note that we have considered only $L = 0$ state for $\alpha + {}^{10}$B. Had the $L \neq 0$ states also been taken into account, we might have obtained N_0 very close to unity. Thus the good fit to data obtained in Fig. 2 may be considered to include that of the absolute magnitude. We remark that we would have obtained N_0 exceeding 100, had we used $A_l = P_l/4$, instead of A_l of (2).

The theoretical spectra in Fig. 2 disagree somewhat with the experiment for $E_\alpha < 25$ MeV. This may be due to the fact that our method obtaining the β_{lm} by using (3) did not take into account sufficiently well the Coulomb repulsion between α and A. Also experiment may include the evaporation α's, which we do not include in our calculations.

In conclusion, it is gratifying to see that a reasonable answer to the long-standing question,[1] of how to explain the large cross section of fast light particles in heavy-ion-induced reactions, has now been given. It, at the same time, explains the massive-transfer reactions. [We may further remark here that the factor $P_l|\beta_{lm}|^2/|s_l|^2$ in (1) is peaked, in the above examples, at $l=26$, and has a full width at half maximum (FWHM) equal to about 7. The smallness of the FWHM explains the absence of side feeding which has been noticed experimentally.[3]] Of course, our theory involves, as explained above, a few steps which need further refinements. Part of the needed refinement is presently underway.

Extension of the present work to the emission of fast $z=1$ particles (p, d, and t) would be very interesting. We emphasize here, however, that the theoretical framework presented above is not limited to BF processes. The BF is only an example of a much wider variety of processes, which might be called DRF (direct reaction followed by fusion). Here DR means a direct-reaction process which ends up with a three-body system. This first step is to be followed by a fusion of one of the pairs together. It will not be difficult to think of an almost inexhaustible number of examples that fall under the category of DRF. One may even think of its application to transitions leading to discrete states.

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