Correlations for Reduced-Width Amplitudes in ^{49}V

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Measurement of the relative sign of inelastic proton-channel amplitudes permits the determination of amplitude correlations. Data were obtained for 45 $\frac{5}{2}^+$ resonances in ^{49}V. Although the reduced widths in each channel followed a Porter-Thomas distribution, large amplitude correlations were observed. The results are compared with the reduced-width–amplitude distribution of Krieger and Porter. This is the first direct test of the Krieger-Porter distribution.

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In the past decade improved experimental methods have been utilized to measure the spacing distributions of single-level populations. With a nearly pure set of neutron resonances[1] (very few missing or spurious levels), the existence of the expected long-range and short-range order[2] among spacings was established. This order was subsequently observed in other neutron measurements, as well as in proton resonance studies.[3] The situation was reviewed by Brody *et al.*[4]

After spacings, the most obvious quantities to consider are the strengths of the levels (reduced-width amplitude $\gamma_{\lambda c}$ for level λ, channel c, and the corresponding reduced width $\gamma_{\lambda c}^2$). There are many data sets which are in excellent agreement with the predicted distribution for γ^2 for one channel, the Porter-Thomas (PT) distribution.[5] However, this distribution is independent of subtle effects such as correlations between reduced-width amplitudes in different channels.[6] The reduced-width distribution is also relatively insensitive to the violation of time-reversal invariance.[7] Measurements of the magnitudes of width amplitudes (or γ^2), therefore, are of only limited value in the testing of statistical theories of spectra. Reduced-width amplitudes are much better suited for such detailed tests, but of course the measurement of the absolute sign of an amplitude is not possible. It appears that the most sensitive measurements possible involve interference effects in multichannel processes.

We have developed a method[8] which initially was used to establish properties of width amplitudes for common doorway states (fragmented analog states.)[9,10] In this approach the magnitude and the relative sign of reduced-width amplitudes are obtained (γ_c^2, $\gamma_{c'}^2$, and $\gamma_c\gamma_{c'}$) allowing the amplitude correlation to be determined. Combination of this method with our ultrahigh-resolution system[11] makes feasible the study of amplitude correlations for compound-nuclear resonances in medium-mass nuclei, permitting the study of sufficient resonances to test statistical concepts. In the present experiment, decay amplitudes are measured for $\frac{5}{2}^+$ resonances in ^{49}V, for which statistical behavior was expected. However, striking effects are observed—large correlations between the reduced-width amplitudes in different channels. A (slightly modified) version of the Krieger-Porter reduced-width distribution is in agreement with our experimental results. To our knowledge this provides the first such detailed test of the theory of width amplitude distributions. The physical origin of the observed correlations remains unexplained.

First, the method of measuring the interference effects is briefly outlined, and the experimental data presented. Then the theoretical width amplitude distribution is obtained and compared with experiment.

In these recent experiments on even-even nuclei, proton inelastic scattering to the first excited state is studied. The reactions are, therefore, $A(p,p')B^*$, $B^* \to A+\gamma$, where A and B correspond to 0^+ and 2^+ states, respectively. The reaction proceeds through an isolated compound-nuclear state with angular momentum and parity J^π; this state is formed by a unique orbital angular momentum l and channel spin $s=\frac{1}{2}$, but there are several possible decay modes. The relative signs between these exit-channel decay amplitudes are the quantities of interest. Our previous measurements involved $l=1$ and $l=3$ resonances; at low bombarding energies in the nuclear $1f$-$2p$ shell, there are many isobaric analogs of low-lying odd-parity states. Since there are very few even-parity analogs in this mass and energy region, $l=2$ resonances seem well suited for the study of statistical properties of compound-nuclear states.

We restrict our attention to $\frac{5}{2}^+$ resonances. These states can decay by $l'=0$, 2, and 4; if the $l'=4$ terms are neglected by penetrability arguments, there are three decay amplitudes: $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$. If the sign of the $s_{1/2}$ amplitude is arbitrarily chosen to be positive, the remaining unknowns may be written as two ratios of amplitudes, or mixing ratios. Since both proton and γ-ray angular distributions are of the form $1+A_2P_2(\theta)+A_4P_4(\theta)$, there are four experimental parameters, each of which is (at most) a quadratic function of the two mixing ratios. Thus, in principle the two mixing ratios are obtained for each resonance. Since the mixing ratios (δ) are infinitely ranged, we usually transform to a more convenient parameter, the mixing angle $\varphi \equiv \tan^{-1}\delta$.

Previous high-resolution measurements have established the existence of some 112 $l=2$ resonances in ^{49}V between $E_p=2.2$ and 3.1 MeV.[12] We studied all of these resonances with the high-resolution system on the Triangle Universities Nuclear Laboratory 3-MV accelerator; the overall resolution was about 350 eV. For 80 resonances with sufficient yield, spin assignments were obtained: 45 resonances were assigned $\frac{5}{2}^+$ and 35 resonances were assigned $\frac{3}{2}^+$. For the $\frac{5}{2}^+$ resonances the average $l'=2$ strength was about 10% of the total inelastic strength; this proved sufficient to permit determination of the two mixing ratios.

The measured reduced widths for the three inelastic channels are shown as histograms in Fig. 1, with a Porter-Thomas distribution superimposed on each histogram. The agreement is excellent. In addition, no anomalies are observed in the energy dependence of the reduced widths in the elastic and three inelastic channels. One, therefore, expects statistical behavior for the reduced widths and the reduced-width amplitudes. Data for the two independent mixing angles are presented in Fig. 2. These results violently contradict the extreme statistical model: There are 43 resonances with φ_2 positive and only two with φ_2 negative.

Krieger and Porter[6] adopted a general approach to the problem of the reduced-width–amplitude distribution, in which they utilized "the principles of level independence and of invariance with respect to orthogonal transformations of the basis state vectors of the compound nuclei" to derive a multivariate reduced-width–amplitude distribution. For the two-channel case they obtained the

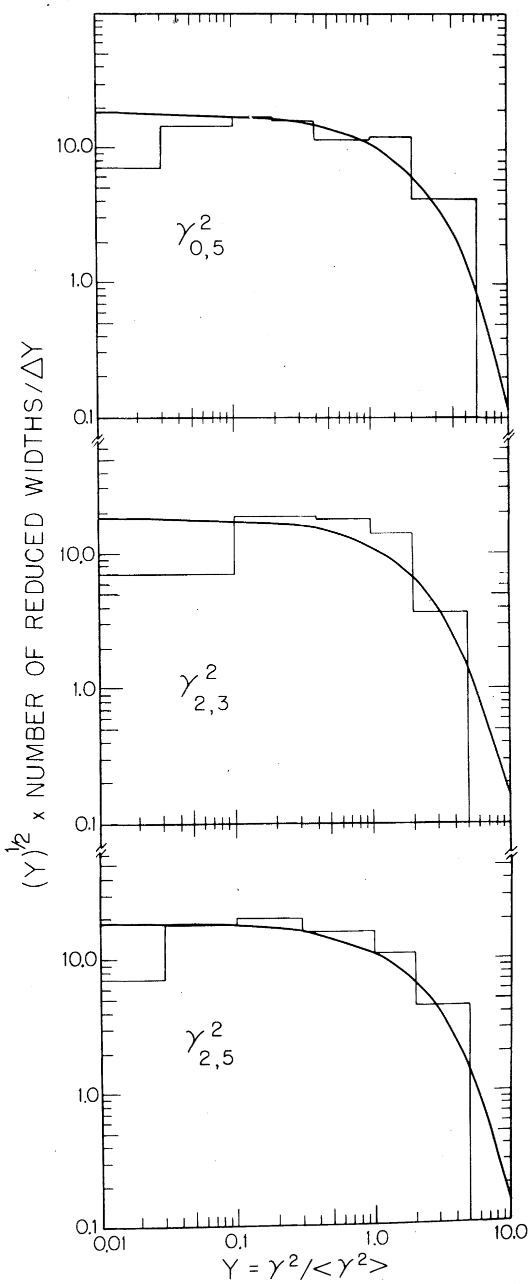

FIG. 1. Reduced-width distributions for three inelastic channels for $\frac{5}{2}^+$ resonances in ^{49}V, where the subscripts on the reduced widths represent l', $2s'$. The data are shown as histograms, while the solid curve in each case is the normalized Porter-Thomas distribution for 45 levels.

joint probability distribution of the width amplitudes,

$$P_{cc'} = \prod_\lambda (|M|^{1/2}/2\pi) \exp[-\tfrac{1}{2}(\gamma_\lambda, M\gamma_\lambda)], \qquad (1)$$

where M is a real, 2×2 symmetic matrix and $M^{-1} = \langle \gamma_\lambda \times \gamma_\lambda \rangle$, $|M|$ is the determinant of M, and γ_λ is a two-component vector. Unless M is diagonal $P_{cc'}$ is not independent (factorable) with respect to channels, and thus correlations may exist between amplitudes in different channels.[6] Explicitly,

$$P(\gamma_1, \gamma_2) = (|M|^{1/2}/\pi) \exp[-\tfrac{1}{2}M_{11}\gamma_1^2 - \tfrac{1}{2}M_{22}\gamma_2^2 - M_{12}\gamma_1\gamma_2] \qquad (2)$$

where a factor of 2 arises from the restriction $\gamma_1 \geq 0$. Transforming to a polar system (r, φ), where $r = (\gamma_1^2 + \gamma_2^2)^{1/2}$ and $\varphi = \tan^{-1}(\gamma_2/\gamma_1)$, one obtains

$$P(r, \varphi) = (|M|^{1/2}/\pi) r \exp(-\tfrac{1}{2}r^2)(M_{11}\cos^2\varphi + M_{22}\sin^2\varphi + M_{12}\sin 2\varphi). \qquad (3)$$

Integrating over r one obtains the marginal probability density function (pdf)

$$P(\varphi) = (|M|^{1/2}/\pi)(M_{11}\cos^2\varphi + M_{22}\sin^2\varphi + M_{12}\sin 2\varphi)^{-1}, \qquad (4)$$

where $|M|^{-1} = \langle\gamma_1^2\rangle\langle\gamma_2^2\rangle - \langle\gamma_1\gamma_2\rangle^2$, $M_{11} = \langle\gamma_2^2\rangle|M|$, $M_{22} = \langle\gamma_1^2\rangle|M|$, and $M_{12} = -\langle\gamma_1\gamma_2\rangle|M|$. There are some interesting special cases contained in Eq. (4). If there are no correlations, then $M_{12} = 0$, and $P(\varphi)$ is symmetric about $\varphi = 0$. In the special case $M_{11} = M_{22}$, the marginal pdf is uniform. Although our data involve three channels, it is easy to show[13] that the appropriate three-channel pdf (function of r, θ, φ, say) reduces to Eq. (4) after integration over r and θ, and that each pair of channels may be considered separately.

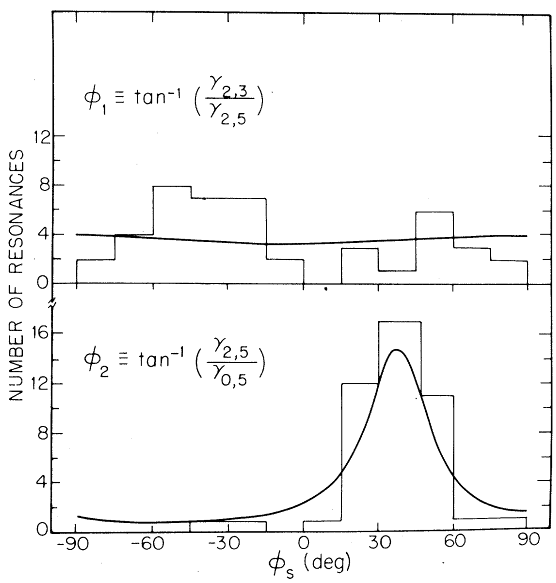

FIG. 2. Plots of the two independent mixing angles for inelastic decay from $\frac{5}{2}^+$ resonances in ^{49}V. The data are shown as histograms, while the solid curves are calculated from the marginal probability density function $P(\varphi)$ obtained in the text. These data are in the channel spin representation; similar results are obtained in the particle angular-momentum representation.

The experimental averages of γ_c^2 and $\gamma_c\gamma_{c'}$ (in kiloelectronvolts) are $\gamma_{0,5}^2 = 0.343$, $\gamma_{2,3}^2 = 0.176$, $\gamma_{2,5}^2 = 0.216$, $\gamma_{0,5}\gamma_{2,3} = -0.014$, $\gamma_{2,3}\gamma_{2,5} = 0.002$, and $\gamma_{0,5}\gamma_{2,5} = 0.238$, where the subscripts are l', $2s'$. The calculated distributions from Eq. (4) are shown as solid curves in Fig. 2. For the uncorrelated case the prediction is symmetric about $\varphi = 0$, and differs from the uniform value because of the unequal average widths in the two channels. For one pair of channels ($l' = 0$, $s' = \frac{5}{2}$; $l' = 2$, $s' = \frac{5}{2}$) the correlation is extremely high—the amplitude correlation is +0.88! Considering that the correlations have been assumed constant over the entire energy range (only approximately true), and that the sample consists of only 45 resonances, the agreement between theory and experiment seems surprisingly good.

To our knowledge this is the first explicit test of reduced-width-*amplitude* distributions, as opposed to reduce-width distributions. The agreement demonstrates the generality of the results of Krieger and Porter. It is also worth reemphasizing that agreement of a reduced-width distribution with the Porter-Thomas distribution sheds no light on certain effects such as correlations between amplitudes.

The physical origin of this amplitude correlation is unknown. Since the large correlation observed is approximately constant over the energy range of the experiment (~1 MeV), one hesitates to apply the usual label "intermediate structure." We have observed a variety of nonstatistical effects in our studies of amplitude correlations,[14] but previously observed effects were more localized in energy (typically 200 or 300 keV). The effect observed in the present data would appear to relate to some more general feature of nuclear structure or of the reaction mechanism. We have a variety of experiments in progress to examine these and related questions.

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$K \neq 0$ Bands in $^{158}_{68}$Er

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$K \neq 0$ bands in ^{158}Er are obtained in Hartree-Fock calculations using a $-\lambda_K J^2$ term in the Hamiltonian. A single major shell for each proton and neutron and a surface δ interaction were used in the calculation. The large-K bands have oblate shape and their Hartree-Fock field does not have time-reversal symmetry. Some of the $K \neq 0$ bands have quite low excitation energy. Their structure and the possible isomerism of the bandheads are discussed.

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The low-spin spectra of even-even deformed nuclei are usually described by $K=0$ intrinsic states, obtained from the Nilsson model[1] and the deformed Hartree-Fock (HF) or Hartree-Fock-Bogoliubov approximation.[2] Such a description has limitations when one goes to the high-spin region. More complicated intrinsic configurations (such as aligned configurations[3]) have to be considered in the backbending region and beyond. Rotational bands built on $K \neq 0$ bandheads have been known in hafnium nuclei.[4] $K \neq 0$ intrinsic configurations are thus expected to play an important role in the high-spin region. In this Letter, I present the results of a self-consistent HF calculation for various $K \neq 0$ intrinsic states of ^{158}Er nucleus. I present their excitation energies and their configurations, and discuss the possibility of isomerism of the bandheads. The reason for choosing ^{158}Er is that it is one of the nuclei where isomers are likely to be found experimentally.[5]

I have used one major shell for each proton and neutron in the HF calculation. (Single-particle energies are in Table I.) A surface δ interaction with the force strength of -0.36 MeV was used. To get large-K bands, a term $-\lambda_K J^2$ was added to the Hamiltonian. The resulting HF equations, if one assumes axial symmetry, are[6]

$$[\epsilon_j - \lambda_K(j^2+j)]C_j^{\alpha m} + \sum_{j_1 j_2 j_4 m_2} V^A(j_1 m j_2 m_2; j m j_4 m_2)\rho_{j_4 m_2 j_2 m_2} C_{j_1}^{\alpha m} + \lambda_K \sum_{j'} ([j' m_-][j m_-])^{1/2} \rho_{j', m-1, j, m-1} C_{j'}^{\alpha m} + \lambda_K \sum_{j'} ([j' m_+][j m_+])^{1/2} \rho_{j', m+1, j, m+1} C_{j'}^{\alpha m} - 2\lambda_K m K C_j^{\alpha m} = e_{\alpha m} C_j^{\alpha m} \quad (1)$$