in Fig. 1 on the time scale of our calculation.

In addition to several calculations of the type reported here, many calculations on 432 particles were also made. In this case, starting with V_{LJ} and a bcc structure, one obtains a close-packed structure with stacking faults. Repeated heating and cooling of the faulty structure finally gives a perfect fcc ordering. This implies that our dynamical equations do allow the system to monitor in configuration space the subtle local minima which correspond to stacking faults in a close-packed system.

The exploratory calculations we have reported here are an example of the way our dynamical equations make it possible to relate particle interaction to particle arrangements in ordered structures. Many other applications seem possible. Generalizing from uniform p_{ext} in Eq. (3) to a general external stress tensor, the recent work of Milstein and Farber[9] on the fcc-bcc transition under (100) tensile loading can be investigated as a function of temperature and of the characteristics of the pair potential. We are at present investigating this problem.

Finally, the low-temperature phase transitions in light alkali metals can also be investigated, with the dependence of the pair potential on the density of the system taken into account.[3]

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Effect of the Quasiparticle Mean Free Path on Poiseuille Flow in Normal Liquid ^3He

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Direct observations of the effect of quasiparticle mean free path on the hydrodynamics of normal liquid ^3He are presented. Both the viscosity and the mean free path are found to vary as T^{-2} down to 1.5 mK. The relevance of these observations for other ^3He experiments is mentioned.

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In recent years considerable interest has been focused on the properties of liquid ^3He at very low temperatures.[1] Measurements are often interpreted using hydrodynamic theory which implicitly assumes that a continuum picture suitably describes the system. In ^3He, however, the quasiparticle mean free path, λ, grows as T^{-2}. Below about 10 mK, λ can become nonnegligible compared to relevant experimental dimensions. Therefore, continuum hydrodynamics may lead to erroneous interpretation of low-temperature ^3He measurements. In this paper we show how the nonzero mean free path causes departures from ordinary Poiseuille flow. Analysis of flow data allows us to determine both the viscosity and the first-order mean free path correction to the flow resistance. Without this correction one would deduce that the viscosity, η, does not scale as T^{-2} in apparent contrast to the Landau Fermi-liquid theory.[2]

In the usual treatment of Poiseuille flow through a channel one has $\dot{V} = \Delta P / Z\eta$, where ΔP is the pressure difference across the channel, $\dot{V}$ the volume flow rate, and η the shear viscosity. The flow impedance Z is a purely geometric factor,[3] being $128L/\pi d^4$ for a tube of length L and diameter d.

When the mean free path of the particles in the fluid approaches the diameter we might write

$$\dot{V}/\Delta P = (Z\eta)^{-1} f(\lambda/d), \tag{1}$$

where f is some function of the ratio of the mean free path to the diameter d. For small λ/d the function f must approach unity. Over a certain range of λ/d we may expect f to be sufficiently approximated by the first two terms of its Taylor expansion

$$\dot{V}/\Delta P = (Z\eta)^{-1}(1 + c\lambda/d) \tag{2}$$

c being a dimensionless constant.

Several models exist to predict the value of c. In the "partial-slip" model of gas flow[4] the boundary condition is changed to allow a nonzero velocity at the tube walls. The magnitude of this "slip" velocity is the product of ξ, the slip length, and the gradient of the nonslip velocity profile near the wall. For the classical gas ξ is approximately[4,6] equal to λ. This model replaces λ by ξ in Eq. (2) with $c = 8$.

Boltzmann equation calculations for a Fermi liquid confined between parallel planes have been done by Jaffe[5] and Jensen *et al.*[6] To first order their results are the same as Eq. (2). Jensen *et al.*[6] find a slip length ξ of approximately $0.58\,\lambda$ for flow near a plane surface. This ratio of slip length to mean free path is the same in a cylindrical geometry,[7] at least to first order in λ/d.

In our experiment to study the departure from Poiseuille flow we employ a U-tube geometry consisting of two identical reservoirs of cross-sectional area A connected by a cylindrical flow tube. A small level difference is initially established and subsequently released, the levels relaxing under the influence of gravity. Since the system is heavily overdamped, the relaxation is exponential with time constant τ determined by Eq. (2) to be

$$\tau^{-1} = (2\rho g/Z\eta A)(1 + c\lambda/d), \tag{3}$$

where ρ is the fluid density and g the acceleration due to gravity.

The apparatus consists of several identical reservoirs connected to a central reservior via separate cylindrical flow tubes. The tubes are of different diameters and lengths. The channels were constructed by casting epoxy around steel wire and then pulling the wire out after the epoxy had cured. Figure 1 schematically illustrates one arm of this device. At a given time only the central and one outer reservoir are partially filled with liquid while the other reservoirs are kept completely full via surface tension. Without warming above a few tenths of a degree we can change the active flow channel by applying heat to the reservior of the desired channel. This not only partially empties this reservior by breaking the surface tension clamp but also totally fills the reservior previously used. Hence we can study flow in one channel at a time and switch to another with minimum delay.

The fluid reservoirs are actually the annular gaps of concentric cylinder capacitors, each of length 1 cm, gap 0.020 cm, inner diameter 0.518 cm, and capacitance about 7.5 pF. One capacitor serves as a liquid-level detector while the other is used to generate level differences which increase quadratically with applied dc potential.

The wires used to contruct the flow channels for this experiment have diameters 454, 354, and 252 μm, each with an uncertainty of about 3 μm. The first two are 1.00 cm long while the last is 0.50 cm. As a check on the tube diameters we observed the flow of superfluid ^{4}He in the U tubes, measuring the oscillation frequencies. This gave agreement, to within 3%, with the wire diameter used to make the channels. To the eye, the channels appear quite circular. It should be noted that the d^{-4} dependence of the flow impedance makes uncertainty in the diameters a major source of systematic error.

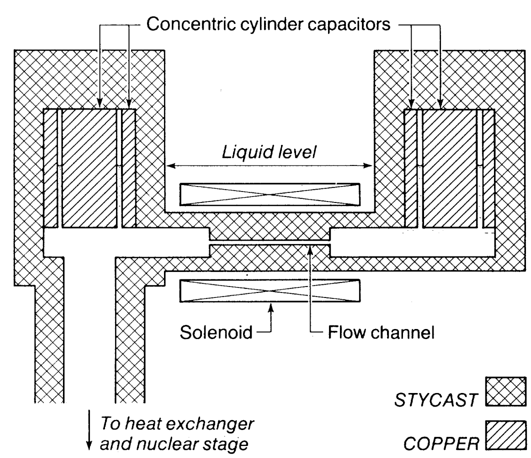

FIG. 1. Schematic of one arm of the U-tube device. The solenoid was not used in this experiment. The thermometer is not shown.

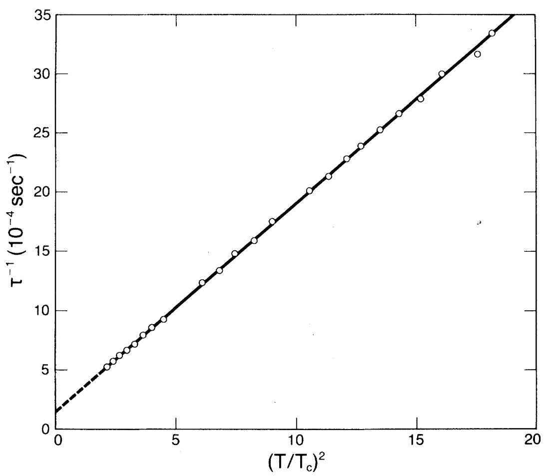

FIG. 2. Plot of inverse time constant vs $(T/T_c)^2$ for the 454-μm-diam tube.

The cell is epoxied onto the lid of a sintered silver heat exchanger connected to a conventional nuclear demagnetization refrigerator. This refrigerator is capable of cooling the ^{3}He to below 0.5 mK. The ^{3}He thermally relaxes to the nuclear stage in less than 3 min at 1 mK. The flow channels present thermal resistances of less than 0.5 mK/nW at 1 mK and we estimate[8] the heat flux along the channels to be less than 50 pW.

Our thermometry is based upon pulsed NMR in ^{195}Pt and the assumption of a Curie-law susceptibility. We calibrate against the superfluid transition T_c where a marked change in the U-tube flow resistance occurs. We reference all our final results to the value of T_c.[9]

The Landau Fermi-liquid theory[2] predicts that both η and λ vary with temperature as T^{-2}. Defining $\alpha = \eta T^2$ and $\beta = \lambda T^2$ Eq. (3) becomes

$$\tau^{-1} = (2\rho g/ZA\alpha)(T^2 + c\beta/d). \tag{4}$$

Thus, as a function of T^2, τ^{-1} is linear with a temperature-independent offset. Figure 2 shows a plot of the measured inverse time constant versus $(T/T_c)^2$ for the 454-μm tube. The solid line is a least-square fit to a straight line $\tau^{-1} = M(T/T_c)^2 + B$. It is clear that the first-order correction adequately describes the data, and the offset caused by the mean free path is readily detectable.

From the fitted slope M and our best estimate of the flow impedance[10] we can determine α/T_c^2, the viscosity at T_c. These values are shown in Table I for all three tubes used along with estimates of the total errors. These error estimates include errors in tube diameters, reservoir cross-sectional area, T_c determination, and statistical errors. The weighted mean is 2.44 P and taking[9] T_c to be 1.04 mK gives $\eta T^2 = 2.64$ P mK^2 in agreement with the previous work of Parpia *et al.*,[11] who find 2.55 P mK^2 at saturated vapor pressure, subject to an estimated 25% systematic uncertainty.[12] We estimate our systematic uncertainty in α/T_c^2 to be 5%. Our result is in disagreement with the result of Bertinat *et al.*[13]

The existence of the offset in Fig. 2 is clear evidence for the mean free path altering the flow resistance. It should be emphasized that the flow resistance deviates from the bulk T^{-2} dependence by as much as 30%, for our lowest-temperature datum point in the 252-μm-diam tube, and yet this is not due to any failure of $\eta T^2 = $ const.

According to Eq. (4) and the definition of the fit parameters, Bd/M should be the same for all three tubes. Table I indicates that while the results for the two smaller tubes are consistent to within error, the largest tube is out of step. This is at least partly due to the impedance presented by the capacitors themselves. For this largest tube they contribute about 9% to the impedance and have a substantially smaller narrow dimension. They therefore experience a larger percentage deviation due to the mean free path than the flow channel itself. Using the results of Jensen *et al.*[6] and assuming all quasiparticle-wall colli-

TABLE I. Numerical results for all three tubes. As explained in the text, the last column is the mean free path at T_c using the predictions of Jensen *et al.* (Ref. 6).

d (μm)	M (10^{-5} sec^{-1})	B (10^{-5} sec^{-1})	Bd/M (μm)	αT_c^{-2} (P)	βT_c^{-2} (μm)
454	17.52	15.38	399 ± 19	2.47 ± 0.10	82 ± 4
354	7.49	6.56	310 ± 28	2.28 ± 0.11	67 ± 6
252	3.32	3.75	285 ± 23	2.64 ± 0.14	62 ± 5

sions are diffuse[13] we can make a quantitative correction for this effect. In the 454-μm-diam tube this amounts to about 5% of B/M while in the two narrower tubes it is substantially less than 1%. In these two tubes the capacitors contribute less than 2.5% to the total impedance.

Assuming that all wall collisions are diffuse and that the slip length ξ is 0.58λ as Jensen *et al.*[6] estimate we can determine βT_c^{-2}, i.e., the mean free path at T_c. Table I lists these values for the various tubes including a correction for the effect mentioned above. Jensen *et al.*[6] indicate that this "viscous" mean free path is directly comparable to that determined from the gas-kinetic formula $\eta = \frac{1}{5} p_F n \lambda$ which gives $\lambda \simeq 90$ μm at T_c with use of our viscosity measurement. Here p_F is the Fermi momentum and n the fluid number density.

In conclusion, we have shown that apparent low-temperature departures of the viscosity from the Fermi-liquid result ηT^2 =const are due to the nonzero mean free path of the quasiparticles. It is thus the inadequacy of the hydrodynamic description rather than of the Landau theory that generates these deviations. Presumably all transport measurements made in restricted geometries are subject to similar effects. In the superfluid phase the Cooper pairing will cause the mean free path to increase rapidly below T_c causing even larger effects of the type investigated here. Recent measurements[14] in the B phase show a rapidly decreasing viscosity with no evidence of the theoretically predicted plateau.[15] Fourth-sound measurements[16] have given lower Q factors than expected from simple theory. Slippage of the normal component at the boundaries may help to explain these results. Recent sound-attenuation experiments[17] required an estimate of the mean free path in their analysis. Even equilibrium measurements such as heat capacity[18] might be subject to systematic errors related to the coupling of a probe (such as a heater wire) to the liquid ^{3}He.

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