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Target-Mass Corrections in Quantum Chromodynamics

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The diagrammatic approach to scale breaking, which allows for the fact that the target quark in a hadron is necessarily offshell, is employed to demonstrate that Nachtmann moments do not generally absorb all $M_{\rm hadron}{}^2/Q^2$ corrections.

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Experimental testing of asymptotically free quantum chromodynamics (QCD) requires a momentum transfer Q^2 large enough to make an expansion in powers of $\alpha_s(Q^2)$ valid. Moreover, Q^2 must be large compared to all relevant masses. At the energies at which most tests have been carried out, however, the ratio M^2/Q^2, where M is the mass of the target, is not negligible. Such target-mass corrections greatly complicate the interpretation of deep-inelastic-scattering data at moderate energies.

Georgi and Politzer[1] inferred from the operator product expansion that the use of a new scaling variable ξ instead of the Bjorken x would, to a good approximation, absorb all target-mass corrections. In standard notation where the lepton momentum transfer is q, the target momentum is p, $Q^2 = -q^2$, and $p^2 = M^2$, the variable ξ is defined by

$$\xi = \frac{2x}{1 + (1 + 4M^2x^2/Q^2)^{1/2}}, \qquad (1)$$

where $x = Q^2/2p\cdot q$. In terms of moments of structure functions, the claim is that the use of Nachtmann moments[2] incorporates target-mass corrections. For example, in deep inelastic neutrino scattering the Nachtmann moment of the F_3 structure function is

$$M_3{}^n \equiv \int_0^1 (dx/x^2) F_3 \xi^{n+1}[1 + (n+1)(1 + 4M^2x^2/Q^2)^{1/2}]. \qquad (2)$$

There has been extensive discussion and some controversy in the literature as to the validity of the claim that the Nachtmann moments incorporate all target-mass corrections.[3] The diagrammatic method for the analysis of QCD[4] now provides a systematic approach in which this question can be answered, without making the approximation, used in some previous approaches, that the target quark is on-shell. This paper gives a brief summary of our results,[5] which are negative: we find target-mass corrections of the form M^2/Q^2 which are not incorporated by the use of Nachtmann moments.

The advantages and limitations of Nachtmann moments can already be seen from analysis of the simple class of diagrams shown in Fig. 1, where the deep inelastic probe strikes a quark, and where K represents the two-particle irreducible amplitude for finding this quark inside the target. Let us first illustrate the simplest features of the problem by considering a model in which all the particles, including the deep inelastic probe, are scalar. We write a spectral representation for K:

$$K(k,p)=\int d\sigma\rho(\sigma,k^2,p^2)/[(p-k)^2-\sigma]. \qquad (3)$$

The deep inelastic amplitude of Fig. 1 is then

$$T(p,q)=\int\frac{d^4k d\sigma\rho(\sigma,k^2,p^2)}{(2\pi)^4k^4(k-q)^2[(p-k)^2-\sigma]}. \qquad (4)$$

We will calculate T in the Euclidean domain, which is algebraically equivalent to taking p^2 and

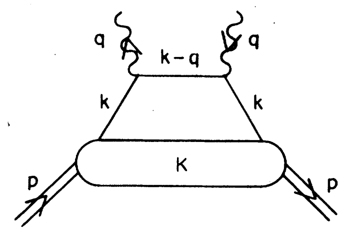

FIG. 1. General "handbag" diagram for deep inelastic scattering on a hadron target.

$q^2>0$ and changing the sign of σ, and expand

$$T(p,q)=\sum_{n=0}^{\infty}M_n(p^2,q^2)\left(\frac{p}{q}\right)^n C_n(\hat{p}\cdot\hat{q}), \qquad (5)$$

where the $C_n\equiv C_n{}^1$ are Gegenbauer polynomials. If the M_n so defined are continued to the physical region one finds that they are the Nachtmann moments appropriate to the spinless problem,

$$M_n(p^2,q^2)=\int_0^1(dx/x^2)\xi^{n+1}(1/\pi)\,\mathrm{Im}T(p,q). \qquad (6)$$

One can perform the angular integration in (4) by expanding the denominators[6]

$$\frac{1}{(p-k)^2+\sigma}=\sum_{n=0}^{\infty}\frac{(Z_{pk}{}^{\sigma})^{n+1}}{pk}C_n(\hat{p}\cdot\hat{k}), \qquad (7)$$

where

$$Z_{pk}{}^{\sigma}\equiv\frac{p^2+k^2+\sigma-[(p^2+k^2+\sigma)^2-4p^2k^2]^{1/2}}{2pk}. \qquad (8)$$

Using standard properties of the C_n one finds

$$M_n(p^2,q^2)=\frac{1}{q^2(n+1)}\int_0^{\infty}\frac{dk^2}{16\pi^2k^4}\int d\sigma\rho(\sigma,k^2,p^2)\left(\frac{k}{p}Z_{pk}{}^{\sigma}\right)^{n+1}\left[1+\left(\frac{q^2}{k^2}\right)^{n+1}\theta(k^2-q^2)\right]. \qquad (9)$$

If the function ρ is highly convergent the second term in the bracket is suppressed and (continuing to $p^2=-M^2, q^2=Q^2$)

$$M_n\simeq(1/Q^2)A_n(M^2). \qquad (10)$$

In contrast an ordinary x moment, in this same model, would have a series of M^2/Q^2 corrections.[7]

In general, we can write a Nachtmann moment as

$$M_n=f_n{}^{(0)}(Q^2)A_n{}^{(0)}(M^2)+f_n{}^{(1)}(Q^2)(M^2/Q^2)A_n{}^{(1)}(M^2)+\ldots. \qquad (11)$$

The question we are examining is whether the $A_n{}^{(i)}$ are negligible for $i\geqslant1$. One might hope, for example, that they are of order $\alpha_s(Q^2)$ or at least $\alpha_s(M^2)$, relative to $A_n{}^{(0)}(M^2)$. Or, one might argue, following De Rujula *et al.*,[3] that the correction terms would be characterized by a scale λ^2/Q^2 where λ is characteristic of the inverse hadron size rather than its mass. In fact we have shown that these are the only possibilities in the scalar model with strongly convergent ρ.

Exactly how many of the $A_n{}^{(i\geq1)}$ vanish depends on the explicit form of ρ. For fixed σ, and ρ of the power-law form

$$\rho(k^2)=[\lambda/(k^2+\lambda)]^2, \qquad (12)$$

we find $A_n{}^{(1)}=A_n{}^{(2)}=0$ and that explicit M^2/Q^2 corrections begin with $A_n{}^{(3)}\neq0$. The Nachtmann moments continue to absorb the leading M^2/Q^2 corrections. We shall show that this simplicity of the scalar model does not persist in models with more realistic spin assignments; explicit M^2/Q^2

corrections appear at all orders.

Perhaps the cleanest case is that of a virtual photon target,[8] where the lowest-order diagrams are shown in Fig. 2. We compute the Feynman integrals as before, and look for leading $\ln q^2$ terms. Here we give the result only for the simplest Nachtmann moment of the O(4)-spin-zero amplitude

$$T_0 = g_{\mu\nu} T^{\mu\nu\alpha\beta} g_{\alpha\beta} . \tag{13}$$

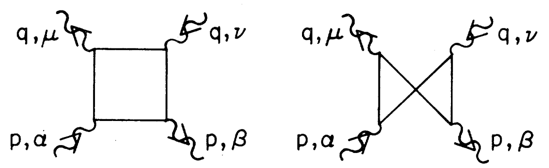

FIG. 2. Diagrams for deep inelastic scattering on a photon target. Both are required for gauge invariance.

This amplitude has a Gegenbauer expansion and Nachtmann moments just the same as the scalar case given by (5) and (6).

We find

$$M_0^{\,n} = -\frac{1}{8\pi^2} \ln q^2 \left[\frac{2}{n+2} - \frac{2}{n+1} + \frac{1}{n} + \frac{p^2}{q^2}\left(\frac{6}{n+2} - \frac{2}{n+4} - \frac{2}{n+1} \right) + \sum_{k=2}^{\infty} \left(\frac{p^2}{q^2}\right)^k (-1)^k \left(\frac{2}{n+2k-2} + \frac{2}{n-2k+2} - \frac{4}{n+2k} \right) \right] . \tag{14}$$

Explicit p^2/q^2 corrections are present, and Nachtmann moments do not have simple scaling properties for this photon-target case. Similar results are obtained for other terms in the QCD ladder series. Moreover, the higher-order terms in the series cannot cancel the p^2/q^2 corrections in (14) because each term contains a different number of intermediate-state particles, and these contribute additively to the imaginary part.

Thus for the photon target, the additional structure introduced by spin and by the inevitable presence of the crossed graph [from which all the $(p^2/q^2)^k, k \geq 2$, terms arise] results in target-mass corrections which are not incorporated simply by using Nachtmann moments or ξ scaling. One might still hope, however, that the trouble lies in the anomalous character of the photon target —the pointlike part of the photon contains quarks with very high transverse momentum. However, even for a hadronic target, the spin structure and the presence of nonleading graphs (related to gauge invariance) turn out to be essential complications; the damping provided by the hadron wave function will not generally suppress the target-mass dependence by replacing it with λ/Q^2 where λ is a parameter characterizing the average value of k^2 of the quark in the hadron. In particular, we have examined this question using two different models of hadrons, and find that target-mass corrections persist if the hadronic wave function falls off like a power at high k^2, as in QCD.[9]

We shall illustrate here only the simplest model we have considered, in which we calculate the amplitude F_3 for neutrino scattering on a massive, charged, spin-one-half bound state (a "proton") with one, charged, spin-one-half constituent of zero mass and one, uncharged, spin-zero, constituent of mass σ, shown in Fig. 3. At each proton vertex we supply a factor $[\rho(k^2)]^{1/2}$ to represent the suppression of high k^2 by the proton wave function. The result of a calculation similar to the ones already described is[10]

$$M_3^{\,n} = \frac{1}{32\pi} \int \frac{dk^2}{k^2} \rho(k^2) \left(\frac{k}{p} Z_{pk}^{\sigma} \right)^n \left\{ \frac{n+2}{n} - \frac{k^2}{q^2}\frac{n+2}{n+1} - \frac{p^2}{k^2}\frac{n+2}{n+1}\left(\frac{k}{p} Z_{pk}^{\sigma} \right) + \frac{p^2}{q^2}\frac{n}{n+1}\left(\frac{k}{p} Z_{pk}^{\sigma} \right)\left[1 + \frac{1}{n+2}\left(\frac{k}{p} Z_{pk}^{\sigma} \right) \right] \right\} . \tag{15}$$

Again, one sees explicit p^2/q^2 terms, which originate from the spin structure. In general, the phenomenological importance of these terms is difficult to evaluate, since they depend on σ and on the form of $\rho(k^2)$. We can, however, examine them for the form of ρ given in Eq. (12) which at least has the high-k^2 behavior of QCD. Since the large-k^2 region controls the large-n behavior we concentrate on that limit, where we find

$$M_3^{\,n} \underset{n\to\infty}{\sim} \frac{1}{32\pi}\left(\frac{\lambda}{\sigma}\right)^2 \frac{1}{n^3}\left(4 + 2\frac{M^2}{\sigma} - n\frac{M^2}{Q^2} - 3n\frac{\sigma}{Q^2} \right). \tag{16}$$

If M^2 and σ are comparable (as expected for stability of the "proton") then the terms of the form nM^2/Q^2 and $n\sigma/Q^2$ can be very important phenomenologically.[11] Abbott and Barnett[12] have shown

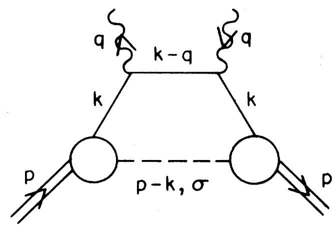

FIG. 3. Deep inelastic scattering on a spin-$\frac{1}{2}$ "proton" composed of a charged spin-$\frac{1}{2}$ massless "quark" and a charged spin-zero constituent of mass σ.

that such terms, with an appropriate coefficient, can account for much if not all of the observed nonscaling behavior usually attributed to QCD anomalous dimensions. We find the numerical coefficients of these terms to be model dependent. (In particular, nonleading graphs, related to gauge invariance, typically change the sign of the nM^2/Q^2 and $n\sigma/Q^2$ terms.) Nevertheless, it is interesting that they arise in such simple diagrams as Fig. 3.

To summarize, we have found—rigorously for a photon target and in models for a hadron target—that explicit target-mass corrections of the form M^2/Q^2 arising from spin structure and nonleading diagrams are *not* taken into account simply by use of Nachtmann moments or ξ scaling. It may still turn out that Nachtmann moments or ξ scaling provide improved fits to the data, but this is an experimental and phenomenological question.

We conclude with a comment on a limit in which consideration of target-mass corrections simplifies; namely, the limit

$$p^2, q^2 \to \infty; \quad r = q^2/p^2 > 1, \text{ fixed}. \tag{17}$$

In this limit the QCD log development collapses. For instance if we compare the zero-gluon and one-gluon terms of the standard axial-gauge ladder series we have (omitting wave-function renormalization factors inessential to argument) in the limit (17)

$$\text{0-gluon} + \text{1-gluon} = 1 + \gamma_n \int_{p^2}^{q^2} \frac{d\ln k^2}{\ln k^2}$$

$$\sim 1 + \gamma_n \frac{\ln r}{\ln q^2} \tag{18}$$

and we see that the one-gluon contribution is suppressed by a factor $\alpha_s(Q^2)$. Thus the one-loop diagrams we have been calculating dominate in this limit. In fact, it is only in this limit that the p^2/q^2 corrections can be more important than the higher-order α_s effects,

$$p^2/q^2 > 1/\ln q^2. \tag{19}$$

In the usual fixed p^2, $q^2 \to \infty$ limit, the sense of the inequality is reversed and higher-order α_s effects should be more important than target-mass corrections.

It is even possible that the low-q^2 region of deep inelastic data on a nucleon target is closer to a fixed q^2/p^2 regime than to the regime in which the leading log series of QCD, appropriate to the asymptotic $q^2 \to \infty$ limit, has a chance to develop. In that case, simple one-loop diagrams such as we have been calculating should describe the data, with the scaling violation resulting entirely from M^2/Q^2 corrections!

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$Q^2) \sim (1/Q^2)(\ln Q^2/\Lambda^2)^{-\gamma n} A_n(M^2)$ but no new M^2/Q^2 correction terms develop.

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[10]The diagram in Fig. 3 gives rise to a gauge-invariant calculation of F_3 even without including other diagrams in which the hadronic vertex is probed. Such diagrams do, however, add to the corrections given in (17). See Ref. 5.

[11]In the limit $\sigma \ll M^2$ the x dependence of the structure function is highly unphysical. In any case a reanalysis of (15) in this limit yields multiple M^2/Q^2 corrections to M_3^n for $n \geq 3$ (see Ref. 5).

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Gluon Condensation from Trace Anomaly in Quantum Chromodynamics

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Condensation of the operator $(G_{\mu\nu}{}^a)^2$ in quantum chromodynamics is shown by constructing the effective potential through the trace anomaly equation. Effects of Wilson loop on the condensation are also studied.

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Certainly one of the most important problems in low-energy quantum chromodynamics (QCD) is the determination of its correct ground state. Various pictures have been offered[1] as to how it might be formed, but at the moment we do not yet have secure knowledge of its structure and the primary mechanism(s) responsible for it. In this note we study the structure of the vacuum in QCD with massless quarks by constructing the effective potential for the gauge-invariant gluonic operator $\hat{\varphi} \equiv \frac{1}{4}\int d^4x [G_{\mu\nu}{}^a(x)]^2$, where $\hat{G}_{\mu\nu}{}^a \equiv \partial_\mu \hat{A}_\nu{}^a - \partial_\nu \hat{A}_\mu{}^a + g f^{abc} \hat{A}_\mu{}^b \hat{A}_\nu{}^c$.[2] [We use a caret to denote operators and the internal symmetry group is taken to be SU(N).]

Our central machinery is the trace anomaly equation for the energy-momentum tensor $\hat{\varphi}_{\mu\nu}$ of the theory with a constant source J coupled to $\hat{\varphi}$. From it, through a Legendre transform, a nonlinear differential equation for the effective potential $V(\varphi)$ is derived [see Eq. (10)], and is solved for small coupling. The solution leads us to conclude that there exists a unique stable vacuum in which $\hat{\varphi}$ condenses with positive sign, relative to the perturbative value, which agrees with that deduced from experiments.[3] It should be emphasized that our discussion does not rely on any assumption of the dominance of certain field configurations nor on the large-N limit.

We then introduce the Wilson loop $\psi(c)$ into the condensed vacuum and derive an exact renormalized equation [see (11)], which states that the area dependence of $\psi(c)$ is determined by how the condensation $\langle\hat{\varphi}\rangle$ changes due to the presence of the loop. The salient feature is that the condensation is broken near the loop.

An elegant derivation of the trace anomaly in QCD[4] has been given by Collins, Duncan, and Joglekar.[5] A new situation arises, however, when one introduces the source J for a "hard" operator $\hat{\varphi}$ and wishes to study the J dependence; one needs to renormalize the theory in such a way that multiple insertions of $\hat{\varphi}$ become finite. This must be fully discussed before we can utilize the method of Ref. 5. Below we shall present the discussion without quarks and later indicate a change to be made when we include them.

Our starting point is the generating functional Z, in an axial gauge, given by

$$Z \equiv \exp(iW) = \int \mathfrak{D}\hat{A}_0 \delta[\eta_\mu \hat{A}_0{}^\mu(x)] \exp\left(i\int d^d x\{-\tfrac{1}{4}(1+J_0)[\hat{G}_0{}^{\mu\nu}(x)]^2 + j_0{}^\mu(x)\hat{A}_{\mu 0}(x)\}\right), \quad (1)$$

where "0" indicates dimensionally regularized bare quantities and d is the dimensionality of space-time. For the purpose of renormalization it is convenient to go to an alternative representation of Z. By making a scale transformation $g_{J0}{}^2 \equiv g_0{}^2/(1+J_0)$, $\hat{A}_{J0}{}^\mu \equiv (1+J_0)^{1/2} A_0{}^\mu$, and $j_{J0}{}^\mu \equiv (1+J_0)^{-1/2} j_0{}^\mu$, Z takes the form (in dimensional regularization the Jacobian is unity),

$$Z = \exp(iW) = \int \mathfrak{D}\hat{A}_{J0} \delta[\eta_\mu \hat{A}_{J0}{}^\mu(x)] \exp\left(i\int d^d x\{-\tfrac{1}{4}[\hat{G}_{J0}{}^{\mu\nu}(x)]^2 + j_{J0}{}^\mu(x)\hat{A}_{J\mu}{}^0(x)\}\right), \quad (2)$$

