conformally invariant theory required by their mechanism.

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New High-Accuracy Measurement of the Pionic Mass

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The pionic x-ray energies of the $4f$-$3d$ transition in π-P and the $5f$-$4d$ transition in π-Ti were measured with a bent-crystal spectrometer at the Nevis synchrocyclotron; and a new value of the pionic mass is deduced to be 139 567.5 ± 0.9 keV, leading to an improved value for the μ-neutrino mass of $m_{\nu_\mu}{}^2 = 0.102 \pm 0.119$ MeV^2; $m_{\nu_\mu} < 0.52$ MeV, at 90% confidence level.

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Using a high-resolution, large-aperture bent-crystal spectrometer, we have measured at the Nevis synchrocyclotron the pionic x-ray energies of the $4f$-$3d$ transition in π-P and the $5g$-$4f$ transition in π-Ti, from which we have deduced the π^- mass value, m_{π^-}, to an accuracy of ±6.4 ppm. These transitions were selected for the following reasons: (1) higher performance of the spectrometer near 40 keV, such as higher efficiency and better fractional energy resolution $\delta E/E$, (2) lower theoretical corrections in deducing m_{π^-}, as compared to transitions in higher-Z π-atoms, such as for vacuum polarization, strong interaction, and orbital electron screening, and (3) the availability of a good calibration γ ray. The experimental setup has been described in an earlier Letter,[1] but will be summarized again with additional relevant information. We refer to the previous Letter for more details and a figure of the setup.

The spectrometer was deployed in a modified Cauchois configuration, with the crystal viewing the x-ray target from 6.4 m through a narrow channel in a thick concrete wall. Using a 2-μA extracted proton beam and a novel target design, we have improved significantly the counting rate and the signal/background ratio over previous efforts.[2,3] The x-ray target consists of flat panels of metallic Ti or "red" phosphorus powder in flat cells with thin Al window, mounted directly above and below the π^- production target. The π^- target consists of 32 separate strips of Cu totaling 44.4 gm/cm^2 in the proton beam, and is shielded from direct view of the spectrometer by a horizontal flat plug at proton beam height.

The bent crystal is quartz, cut to the (3, 1, 0) orientation, and bent to a radius of 351 cm. By using a new bending technique,[4] the "intrinsic" angular resolution achieved over an open aperture of 9 cm × 9 cm is 6.6 sec of arc full width at half maximum (FWHM). The diffracted photons pass through a narrow slit at the image plane into a shielded Ge-detector assembly. To optimize the signal rate against the ambient neutron background in the detector, the slit was set at 170 μm wide and resulted in an effective resolution of $\delta E/E = 17$ eV FWHM/40 keV.

During data collection, the crystal was kept fixed in orientation, eliminating the need of elaborate turning devices and thereby improving stability against temperature changes and other mechanical complications. The slit was mounted on a linear motion stage of ± 0.5 μm accuracy, and scanned across the image plane under the control of a PDP-11/10 computer. In a typical 24-h run the spectrum was scanned 30 times in both directions to average out any intensity variations due to small shifts of the proton beam position on target. The beam dosage was monitored by integrated proton beam current for each slit position.

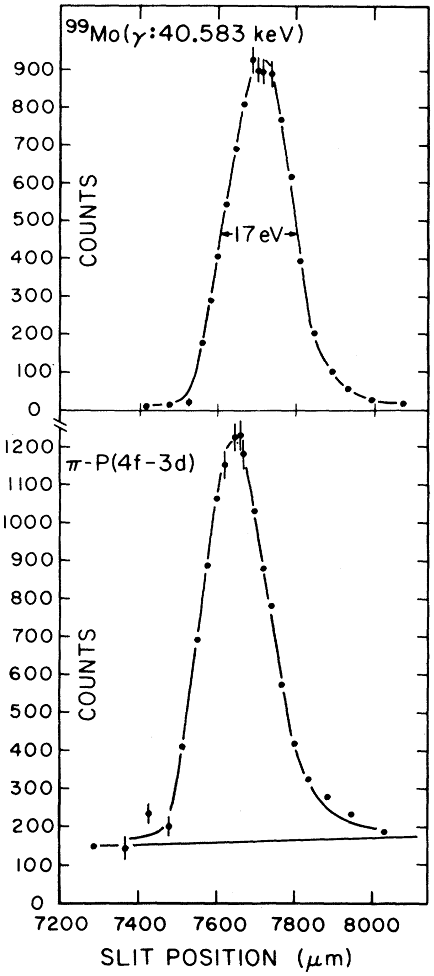

FIG. 1. Bent-crystal diffraction lines for the calibration γ ray from ^{99}Mo (upper panel) and the $4f$-$3d$ pionic x ray from π-P (lower panel).

The line shape of the spectrometer, obtained with the 40 583.468-eV γ ray[5] from a 1.5-Ci source of ^{99}Mo which had the same shape as that of the pionic x-ray targets, is shown in Fig. 1. Because of the large vertical aperture used, the line profile shows a tail on the low-energy (long-λ) side. This tail is very well accounted for by the fact that for a finite vertical aperture, the image at the slit plane generated by each point in the source is a hyperbolic arc curving toward the low-energy side. The line shape of a typical run for the π-P($4f$-$3d$) transition is also shown in Fig. 1. Aside from a slanting background, it has practically the same line profile as the calibration line. In obtaining this line profile, the ambient neutron background in the detector was subtracted by the same procedure as described in our earlier Letter.[1] The signal/background ratio before subtraction was 2.2/1. The remaining slanting background after the ambient-background subtraction is due to a diffracted soft-photon continuum mostly due to scattering from the target and the channel walls. Data for the π-Ti($5g$-$4f$) transition can be found in the earlier Letter.[1] The peak signal counting rate per proton charge on target for the π-P measurement was 12/μA · min and that of the π-Ti, 3/μA · min.

X-ray transition energies were determined by measuring directly the line-center positions relative to the calibration line center, and then adding the calibration line energy to the energy differences measured. To extract the line-center positions, we approximated the line shape by a Gaussian with an exponential tail integrated over the slit width. This line shape is dominatly instrumental since both the x ray and the γ ray have comparatively negligible natural line widths. For the 15 sets of x ray and γ-ray calibration data, χ^2 values averaging 1.1 for each degree of freedom were obtained, with only one case reaching 2 for each degree of freedom. Since the lines are quite close together, any effects due to the line-shape approximation essentially cancel out when the difference in line center positions are computed. Systematic shifts due to the relatively small and slowly varying residual background of the x-ray line (see Fig. 1) are quite negligible, as has been verified with Monte Carlo simulations.

For measuring the small energy differences between the lines, the energy scale factor of the spectrometer needs to be known only to a modest accuracy, since even a sizable percentage error of the small energy difference would only result in an insignificant absolute error of the total transition energy. This has also facilitated the maintenance of the spectrometer stability under relatively harsh conditions at an accelerator. In this experiment, the scale factor (~0.0890 eV/μm at 40 keV) was obtained from the line positions of electronic x-rays, Sm $K\alpha_1$ (40.1181 keV ±7 ppm),[6] and Eu $K\alpha_2$ (40.9010 keV ± 7 ppm),[6] to an accuracy far better than required. The stability of the spectrometer was checked repeatedly by observing the ^{99}Mo γ-ray line position which was found to be stable to ±1.5 μm. Our measured energy differences relative to the ^{99}Mo γ ray are

$$E(\pi\text{-P}(4f\text{-}3d)) - E(\text{Mo}, \gamma) = 4.84 \pm 0.20 \text{ eV},$$

$$E(\pi\text{-Ti}(5g\text{-}4f)) - E(\text{Mo}, \gamma) = -118.4 \pm 0.28 \text{ eV},$$

including a correction of ~0.8 eV for the vertical-aperture effect in accordance with standard procedure.[7] The errors for the energy differences are a combination of statistical errors from the line-shape–centroid fits and systematic errors associated with the vertical-aperture correction.

To deduce total transitional energies from these energy differences, the reference γ ray from ^{99}Mo was measured with a double-flat-crystal spectrometer at the National Bureau of Standards[5] to be 40 583.468 ± 0.17 eV. This measurement correlates our π-mass measurement directly with the wavelength and energy standards used at National Bureau of Standards.[8] Because of low counting rates, the accuracy of this measurement was not as good as one might expect from the capability of the double crystals. A new measurement is contemplated which could improve the accuracy to ~±0.05 eV, and the accuracy of the pionic mass value to ~5.5 ppm with use of our present pionic x-ray data.

The π^- mass value, calculated from the above data, is $m_{\pi^-} = 139\,567.5 \pm 0.9$ keV (~6.4 ppm). Relevant quantities of the calculation are given in Table I. The first-order vacuum polarization and the strong interaction are calculated by a numerical solution of the Klein-Gordon equation with the appropriate potential.[9,10] Higher-order vacuum-polarization and relativistic-recoil corrections are calculated by perturbation theory.[11,12] To correct for orbital-electron screening, a cascade

TABLE I. Computation for pionic mass. Calibration energy used is $E(^{99}\text{Mo}, \gamma) = 40\,583.468 \pm 0.17$ eV.

Transition	π-P($4f$-$3d$)	π-Ti($5g$-$4f$)
Measured transition energy	40 588.31 ± 0.26 eV	40 465.07 ± 0.33 eV
Corrections for		
Vacuum polarization		
$\alpha(Z\alpha)$	−99.3 eV	−82.6 eV
$\alpha^2(Z\alpha)$	−0.7 eV	−0.6 eV
$\alpha(Z\alpha)^{3,5,7}$	+0.1 eV	+0.22 eV
Strong interaction	−0.23 ± 0.06 eV	0.0 eV
Electron screening	+0.98 ± 0.2 eV	+3.99 ± 0.2 eV
Klein-Gordon energy	40 489.16 ± 0.33 eV	40 386.06 ± 0.38 eV
Pionic mass	139 567.8 ± 1.1 keV	139 567.1 ± 1.3 keV
Average pionic mass	139 567.5 ± 0.9 keV	

calculation,[13] including both radiative and nonradiative refilling processes to the K, L, and M electron shells, was used to determine the electrons present during the pionic transition. For π-P because of the relatively slow M-shell refilling rates, the uncertainty in the population of the M shell at the start of the cascade calculation leads to a ±20% error for the correction. This M-shell population is extrapolated from that of Si which was deduced from fitting the cascade calculation to the experimentally determined K-shell depletion for the μ-Si($4f$-$3d$) transition.[14] For π-Ti the conduction electrons constitute part of the M electrons, so that the M-shell population uncertainty is relatively unimportant. The electron screening effect is then obtained from the subsequent K- and L-shell population by standard treatment.[15]

Our present mass value of the π^- can be compared with the three best previous measurements. For π^-, $m_{\pi^-} = 139\,566.7 \pm 2.4$ keV as measured by Marushenko *et al.*[2] with a bent-crystal spectrometer and adopted by the Particle Data Group,[16] and $m_{\pi^-} = 139\,568.6 \pm 2.0$ keV measured by Carter *et al.*[17] with Ge(Li) detectors; for π^+, $m_{\pi^+} = 139\,565.8 \pm 1.8$ keV by Daum *et al.*[18] from a measurement of the μ^+ momentum in $\pi^+ \to \mu^+ + \bar{\nu}_\mu$ decay, using the better known μ^+ mass,[16] and assuming the neutrino mass to be zero. Our measurement agrees very well with the average of these previous measurements if $m_{\pi^+} = m_{\pi^-}$. (World average $m_\pi = 139\,566.9 \pm 1.2$ keV.[16])

Alternatively, again assuming $m_{\pi^+} = m_{\pi^-}$, our new m_π value and the measured μ^+ momentum in π^+ decay,[18] $P_\mu = 29.787\,7 \pm 0.001\,4$ MeV/c, lead to a new calculated value of the square of the μ-neutrino mass, $m_{\nu_\mu}{}^2 = 0.102 \pm 0.119$ MeV^2; and an upper bound for the μ-neutrino mass, $m_{\nu_\mu} < 0.52$ MeV (at 90% confidence level). The improvements over previous values[18] of $m_{\nu_\mu}{}^2 = 0.13 \pm 0.14$ MeV^2, $m_{\nu_\mu} < 0.57$ MeV, are limited by the fact that the errors originate dominantly from the P_μ measurement. For the same reason there is essentially no difference if the present "world average" of the π^- mass, dominated by the present measurement, is used in the calculation.

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