in Ref. 4.

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[10]The $u\bar{u}$ and $d\bar{d}$ channels involve more amplitudes and hence a different characteristic asymmetry.

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Zero Value for the Three-Loop β Function in $N=4$ Supersymmetric Yang-Mills Theory

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This Letter describes a calculation using superfield techniques, showing that the β function is zero to three loops in $N=4$ supersymmetric Yang-Mills theory. This result gives further indication that the theory is likely to be finite and conformally invariant order by order in perturbation theory.

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We present the results of a computation of the three-loop β function in $N=4$ supersymmetric Yang-Mills theory.[1] This theory is of special interest because it is a candidate for a finite four-dimensional quantum field theory, and the only known candidate for a conformally invariant one. It is known from previous calculations that the one- and two-loop β function is zero.[2,3] We have extended this result to the three-loop level using supergraph techniques.[3,4] We understand that the vanishing of the three-loop β function has also been established in a computer calculation by Tarasov with use of conventional Feynman graphs.[5]

The $N=4$ supersymmetric Yang-Mills model is a conventional theory containing the following physical fields: one Yang-Mills vector, four Majorana spinors, three scalars, and three pseudoscalars. All the fields are massless and in the adjoint representation of an arbitrary compact semisimple group. In addition to the gauge couplings, there are specific Yukawa and spin-0 self-interactions all governed by the same gauge coupling constant. The Lagrangian is[1]

$$\mathcal{L}=\mathrm{Tr}\{-\tfrac{1}{4}F_{\mu\nu}{}^2-\tfrac{1}{2}(D_\mu A_i)^2-\tfrac{1}{2}(D_\mu B_i)^2-\tfrac{1}{2}\bar{\lambda}_k \not{D}\lambda_k-\tfrac{1}{2}g\bar{\lambda}_k[\alpha_{kl}{}^j A_j+\gamma_5\beta_{kl}{}^j B_j,\lambda_l]$$
$$+\tfrac{1}{4}g^2([A_i,A_j]^2+[B_i,B_j]^2+2[A_i,B_j]^2)\}, \qquad (1)$$

where the α's and β's are 4×4 $SU(2)\times SU(2)$ matrices, $i=1,2,3$; $k=1,2,3,4$.[1]

The classical action is invariant under SU(4) superconformal transformations: four supersymmetry and four special supersymmetry transformations,[6] conformal transformations, and global SU(4) rotations. As a supersymmetric theory, it has a particularly simple description in terms of $N=1$ superfields[3,7]: one real superfield $V(x,\theta,\bar{\theta})$ (vector multiplet), and three chiral superfields $\varphi_i(x,\theta)$ (scalar multiplets). The action is

$$S=\mathrm{Tr}\Big(\int d^4x\, d^4\theta\, e^{-gV}\bar{\varphi}^i e^{gV}\varphi_i+(64g^2)^{-1}\int d^4x\, d^2\theta\, W^\alpha W_\alpha+\{(ig/3!)\int d^4x\, d^2\theta\,\epsilon_{ijk}\varphi_i[\varphi_j,\varphi_k]+\mathrm{H.c.}\}\Big), \qquad (2)$$

$$W_\alpha=\bar{D}^2(e^{-gV}D_\alpha e^{gV}) \qquad (3)$$

and is renormalizable by power counting and gauge Slavnov-Taylor identities.[8] Because the action (2) is written in terms of $N=1$ superfields, it is manifestly invariant only under global SU(3) rotations of the chiral fields and under one of the four supersymmetry transformations. The remaining three and global SU(4)/[SU(3) $\otimes$ U(1)] transformations are realized as follows:

$$\delta e^{gV} = ig(\chi_i \bar{\varphi}^i e^{gV} - e^{gV}\bar{\chi}^i \varphi_i),$$
$$\delta\varphi_i = (i/8g)W^\alpha D_\alpha \chi_i - \tfrac{1}{4}\epsilon_{ijk}\bar{D}^2\bar{\chi}^j e^{-gV}\bar{\varphi}^k e^{gV}.$$

The parameter $\chi_i(\theta)$ is a constant chiral superfield ($\bar{D}_\alpha \chi_i = \partial_a \chi_i = 0$) which induces central charge transformations at the θ-independent level, supersymmetry transformations at the linear level, and SU(4)/[SU(3) $\otimes$ U(1)] transformations at the quadratic level.

By superfield power counting the chiral three-point function $\langle T(\varphi_i \varphi_j \varphi_k)\rangle$ is finite, and hence the β function is determined entirely by the corrections to the propagator $\langle T(\bar{\varphi}_i \varphi_j)\rangle$.[4,9] As a result, the evaluation of the β function at the one- and two-loop level is easy, and at the three-loop level can be performed by hand. In contrast, component calculations separately examine the ghost-vector vertex function and the ghost and vector propagators (all of which are divergent already at the one-loop level) and are far more complicated.

In Ref. 3, we have shown that all one-loop propagator corrections vanish, and have calculated the one-loop $\langle T(\varphi_i \varphi_j \varphi_k)\rangle$ and $\langle T(\bar{\varphi}_i V \varphi_j)\rangle$ effective vertices, as well as the two-loop contributions to $\langle T(\bar{\varphi}_i \varphi_j)\rangle$. All of these quantities are finite, and we have shown that *all other two-loop Green functions are also finite.*

The determination of the three-loop β function involves the evaluation of the divergent part of the supergraphs in Figs. 1 and 2 (because of group theory, nonplanar graphs are zero; others are also zero because of their structure in θ space). In Ref. 3 we have given the Feynman rules for this model. In particular we have shown that the propagators are δ functions in θ space while vertices give covariant spinor derivative D_α, $\bar{D}_{\dot\alpha}$ acting on these δ functions. Using the methods of Refs. 3 and 4 we can perform, by a sequence of integrations by parts, the integration over internal θ variable. The algebra of the D's introduces momentum factors in the numerators of the Feynman integrals which cancel some of the propagator denominators. Thus the evaluation of divergent contributions of the supergraphs in Figs. 1 and 2 is reduced to that of scalar integrals corresponding to the ordinary graphs depicted in Fig. 3. We have calculated only the ultraviolet-divergent parts of the supergraphs; in graphs containing both ultraviolet and infrared divergences we have taken care to extract the former. Details of our calculations will be described in a separate publication.

Supergraphs listed in Fig. 1 in general contribute to all the scalar integrals in Fig. 3, whereas the supergraphs in Fig. 2 contribute only to Fig. 3(e). We note that the integrals in Figs. 3(a)–3(d) contain subdivergences and hence will give higher-order poles if evaluated by dimensional regular-

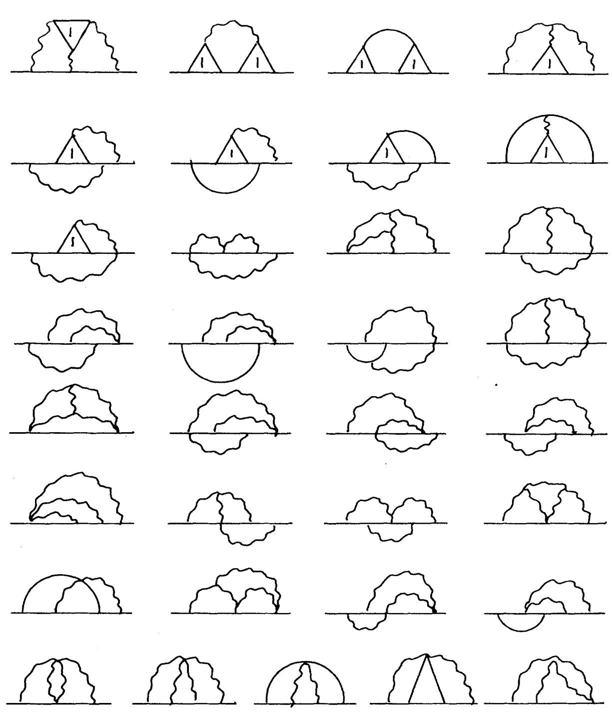

FIG. 1. Some contributions to the three-loop chiral self-energy corrections. The solid lines represent chiral propagators, the wavy lines, vector propagators. The number 1 inside triangle subgraphs represent total one-loop effective vertices.

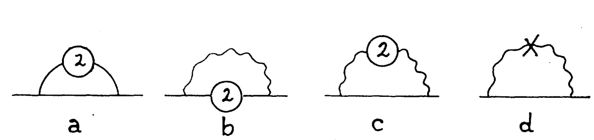

FIG. 2. Two-loop self-energy contributions to the three-loop chiral self-energy corrections. (d) Diagram corresponding to a finite renormalization of the gauge-fixing parameter needed to keep the two-loop vector propagator in Feynman gauge.

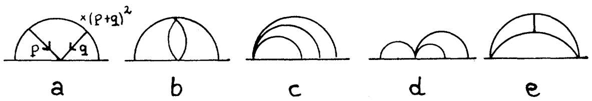

FIG. 3. Scalar integral diagrams resulting from those of Figs. 1 and 2 after θ integration.

ization; by contrast Fig. 3(e) has only $1/\epsilon$ poles. However, since the β function vanishes at one and two loops, such poles must be absent and hence the total contribution to these diagrams should cancel. It is a gratifying check of our calculations that these integrals receive the following contributions:

$$\tfrac{1}{32}-\tfrac{1}{8}+\tfrac{1}{16}-\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{16}-\tfrac{1}{32}\ \text{[of Fig. 3(a)]},$$

$$-\tfrac{1}{16}-\tfrac{5}{6}-\tfrac{1}{2}+\tfrac{1}{24}-\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{4}+2-\tfrac{1}{8}+\tfrac{1}{4}-\tfrac{3}{8}-\tfrac{1}{4}-\tfrac{5}{24}+\tfrac{1}{16}\ \text{[of Fig. 3(b)]},$$

$$-\tfrac{1}{16}+1-\tfrac{1}{4}+\tfrac{1}{48}-\tfrac{1}{4}-\tfrac{1}{8}-\tfrac{1}{4}-\tfrac{1}{12}\ \text{[of Fig. 3(c)]},$$

$$-\tfrac{5}{6}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{12}\ \text{[of Fig. 3(d)]}.$$

We now discuss the contributions to the dangerous scalar integral of Fig. 3(e). Those of Figs. 1, 2(a), and 2(b) are $-\frac{3}{8}+1-\frac{1}{2}$. Figure 2(c) does not contribute: The longitudinal part of the $\langle T(VV)\rangle$ propagator receives no corrections as follows from Slavnov-Taylor identities, and the transverse part gives only ultraviolet-finite contributions. To avoid infrared complications (p^{-4} terms in the vector propagator), we have done all our calculations in Fermi-Feynman gauge; in order to stay in that gauge, we must renormalize the gauge-fixing parameter by a (finite) amount determined by the two-loop corrections to the vector propagator. This gives rise to Fig. 2(d) and a $-\frac{1}{8}$ contribution to the scalar integral in Fig. 3(e). The total contribution vanishes, the two-point function is finite, and therefore the three-loop β function is zero.

It seems very likely that the vanishing of the β function will persist to higher orders and it would be desirable to give a general proof of this. We expect that in a formulation in terms of $N=4$ superfields the finiteness of the theory would be manifest. Our $N=1$ superfield formulation obscures the manifest $N=4$ invariance and replaces it by the nonlinear transformation laws of Eq. (4). These laws do relate the *finite* $\langle T(\varphi_i\varphi_j\varphi_k)\rangle$ vertex to the other sectors of the theory and could, in principle, establish the finiteness of the theory. However, due to the nonlinearity of these transformations, and our inability to find an $N=4$ invariant Yang-Mills gauge-fixing term, we were unable to extract any useful information from the corresponding Ward identities.

Since $N=4$ symmetry is not manifestly maintained, the theory described by the classical action (2) could actually have two independent β functions (in the component approach the situation is worse, and there could be at least three β functions). We present the following argument that both our β functions are equal: In a Wess-Zumino Yang-Mills gauge, the classical action becomes that of Eq. (1) and contains Yukawa couplings of the component fields, some of which arise from the gauge superfield couplings, while others arise from the trilinear chiral superfield coupling. In this gauge these particular couplings are related by the global (linear) SU(4) transformations[1] which are unaffected by gauge fixing. Hence the corresponding β functions must be equal. Since β functions are gauge invariant, they must remain equal in our supersymmetric gauge.

Our results indicate that many (and perhaps all) Green's functions are finite up to the three-loop level. Therefore the superconformal invariance of the classical action survives quantization. It is possible that this and the other invariances of the theory are sufficiently strong to permit a complete solution of the model. One could then envisage treating related theories by means of a perturbation expansion about this solution.[10]

Recently, one of us has pointed out[11] that the trace anomaly vanishes in some versions of $N=8$ supergravity as well. Since both are maximally extended supersymmetric gauge theories, one may hope that $N=8$ supergravity shares the higher-loop finiteness properties of $N=4$ Yang-Mills theory. If, however, extended supergravity theories fail to be finite,[12] then perhaps gravity can be described by bound states of $N=4$ supersymmetric Yang-Mills fields. This is the case in dual models,[13] where the graviton (closed string) sector arises as bound states of the Yang-Mills (open string) sector. Indeed the $N=4$ Yang-Mills theory is the zero-slope limit of a unitary dual model which includes fermions, and consequently one would expect an effective $N=4$ supergravity theory to arise.[1] We also note the possibility of a mechanism suggested by Englert, Gastmans, and Truffin[14] whereby dynamical breakdown of conformal symmetry leads to the generation of gravitons. The $N=4$ supersymmetric Yang-Mills theory is an obvious candidate for the conformally invariant theory required by their mechanism.

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New High-Accuracy Measurement of the Pionic Mass

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The pionic x-ray energies of the $4f$-$3d$ transition in π-P and the $5f$-$4d$ transition in π-Ti were measured with a bent-crystal spectrometer at the Nevis synchrocyclotron; and a new value of the pionic mass is deduced to be 139 567.5 ± 0.9 keV, leading to an improved value for the μ-neutrino mass of $m_{\nu_\mu}{}^2 = 0.102 \pm 0.119$ MeV^2; $m_{\nu_\mu} < 0.52$ MeV, at 90% confidence level.

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Using a high-resolution, large-aperture bent-crystal spectrometer, we have measured at the Nevis synchrocyclotron the pionic x-ray energies of the $4f$-$3d$ transition in π-P and the $5g$-$4f$ transition in π-Ti, from which we have deduced the π^- mass value, m_{π^-}, to an accuracy of ±6.4 ppm. These transitions were selected for the following reasons: (1) higher performance of the spectrometer near 40 keV, such as higher efficiency and better fractional energy resolution $\delta E/E$, (2) lower theoretical corrections in deducing m_{π^-}, as compared to transitions in higher-Z π-atoms, such as for vacuum polarization, strong interaction, and orbital electron screening, and (3) the availability of a good calibration γ ray. The experimental setup has been described in an earlier Letter,[1] but will be summarized again with additional relevant information. We refer to the previous Letter for more details and a figure of the setup.

