

The interesting new physics occurs at the $N=3$ level where the following eight multiplets are expected: $[56, 3^-]$, $[56, 1^-]$, $[70, 3^-]$, $[70, 2^-]$, $[70, 1^-]$, $[70, 1^-]$, $[20, 3^-]$, $[20, 1^-]$. Turning on the perturbation splits these multiplets and introduces a new parameter δ , specific to the $N=3$ levels. For example, for the $[70, 3^-]$ we obtain¹³

$$E[70, 3^-] = E_0 + 3\Omega - \frac{7}{10}\Delta + \frac{1}{2}\delta.$$

However, the important feature to note here is that the masses of three of the $N=3$ multiplets are independent of δ ; *their masses are entirely determined by the $N=2$ level parameters:*

$$E[70, 2^-] = E_0 + 3\Omega - \frac{2}{5}\Delta,$$

$$E[56, 3^-] = E_0 + 3\Omega - \frac{2}{5}\Delta,$$

$$E[56, 1^-] = E_0 + 3\Omega - \frac{11}{10}\Delta.$$

Reasonable phenomenology for the $N=0, 1,$ and 2 levels was obtained³ with $E_0 \approx 1150$ MeV and $\Delta \approx \Omega \approx 440$ MeV. Using these values, we predict the mass spectrum of Fig. 2. The mean mass of the $[56, 1^-]$ is around 1985 MeV—close to the mass of the $\Delta_{D_{85}}$ at 1930 ± 20 MeV. Given the simplicity of the model and our neglect of the hyperfine interactions, this is startlingly good agreement. We conclude that, in contrast to previous claims, the $\Delta_{D_{85}}(1930)$ does *not* represent unambiguous evidence for new degrees of freedom inside baryons.

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Chiral-Symmetry Breakdown in Large- N Chromodynamics

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Chromodynamics with n flavors of massless quarks is invariant under chiral $U(n) \otimes U(n)$. It is shown that in the limit of a large number of colors, under reasonable assumptions, this symmetry group must spontaneously break down to diagonal $U(n)$.

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In nature, the gauge group of chromodynamics is $SU(3)$, and quarks are color triplets. Nevertheless, it is useful to consider generalizations in which the gauge group is $SU(N)$ and quarks are color N -uplets. There are many observed properties of meson dynamics (e.g., Zweig's

rule) that can be argued to be exact in the large- N limit; it is tempting to believe that this indicates that large- N chromodynamics is in some sense a good approximation to the real world.¹

In this note we study large- N chromodynamics with n massless quark N -uplets. This theory is

invariant² under the chiral symmetry group $U(n) \otimes U(n)$. This group contains many inequivalent subgroups; thus group theory allows many possible patterns of spontaneous symmetry breakdown. We shall argue here that in the large- N limit, under reasonable assumptions, the pattern of chiral symmetry breakdown is uniquely fixed: Chiral $U(n) \otimes U(n)$ necessarily breaks down to diagonal $U(n)$. Hearteningly, this is the pattern observed in nature.

Our assumptions are as follows: (1) We assume that the large- N limit exists, that chromodynamics has an asymptotic expansion in powers of $1/N$. (2) We assume that chromodynamics yields confinement for arbitrarily large N . (3) We assume that the breakdown of chiral symmetry is characterized by a nonzero value of some order parameter which is bilinear in the quark fields and which transforms according to the representation $(n, n^*) \oplus (n^*, n)$ of the chiral group. (4) We assume that the ground states of the theory are found by minimizing some effective potential, V , an invariant function of the order parameter, constructed in the standard way by summing (an infinite number of) connected Feynman graphs. (5) We assume that in the large- N limit, the effective potential does not display accidental degeneracy, that any of its minima can be obtained from any other by the action of the chiral group.

Assumptions (1), (2), (4), and (5) are more or less standard. Assumption (3), though, requires comment, because it restricts the pattern of symmetry breakdown even before we invoke large- N dynamics. Let us label the order parameter by a (not necessarily Hermitian) $n \times n$ matrix, M . For example, the simplest candidate for M is

$$M_j^i = \langle \bar{\psi}^i (1 + \gamma_5) \psi_j \rangle, \quad (1)$$

where i and j are flavor indices, the brackets indicate vacuum expectation value, and the sum over (suppressed) color indices is implied. We stress that this is just an example; for our purposes some nonlocal or smeared-out version of this will do as well. All we need are the chiral transformation properties of M ,

$$(u, v): M \rightarrow u M v^\dagger, \quad u, v \in U(n). \quad (2)$$

It is easy to show that by a transformation of this form we can always make M real, diagonal, and nonnegative. The squares of the diagonal entries are the eigenvalues of $M^\dagger M$ (or, equivalently, of MM^\dagger). Thus V can depend only on these eigenvalues, and the pattern of chiral symmetry break-

down is determined by the pattern of eigenvalues at the minimum of V . For example, if all the eigenvalues vanish at the minimum, there is no symmetry breakdown; if they are all equal but nonzero, the symmetry breaks down to diagonal $U(n)$; if they are all unequal and nonzero, it breaks down to $U(1)^n$, etc. Note that under our assumption, breakdown beyond $U(1)^n$ is impossible. If we had assumed two order parameters, M and M' , or if we had assumed different chiral transformation properties for the order parameter, further breakdown would have been allowed.

This concludes our introductory discussion. The remainder of this note is the proof of the announced result.

If we expand V in powers of M and M^\dagger , we will encounter terms like $\text{Tr}(MM^\dagger)^r$, $\text{Tr}(MM^\dagger)^r \times \text{Tr}(MM^\dagger)^s$, etc. Because traces of quark operators arise in Feynman graphs from sums over quark loops, the terms of the first kind come from graphs with one quark loop, those of the second kind from graphs with two quark loops, etc. However, it is known¹ that in the large- N limit, connected graphs with L quark loops are $O(N^{2-L})$. Thus, the dominant graphs are those with only one quark loop, and, to leading order in $1/N$,

$$V = N \text{Tr} F(MM^\dagger), \quad (3)$$

where F is some N -independent function. If we denote the eigenvalues of MM^\dagger by λ_i , $i = 1, \dots, n$, then

$$V = \sum_i N F(\lambda_i). \quad (4)$$

Since the eigenvalues are independent variables, to minimize this sum is to minimize each term. Each eigenvalue must be at the minimum of F , and thus the eigenvalues are either all zero (no symmetry breakdown) or all equal and nonzero [breakdown to $U(n)$].

We shall now eliminate the first alternative. We shall apply a method of analysis recently devised by 't Hooft,³ based on the Adler-Bell-Jackiw anomaly.⁴ The simplicity of the large- N theory makes the application particularly clean; there is no need of the supplementary assumptions required in the examples considered by 't Hooft.

Let us consider a chiral current

$$j_\mu = \bar{\psi} A (1 + \gamma_5) \gamma_\mu \psi, \quad (5)$$

where A is an $n \times n$ Hermitian matrix, and let

us define the three-current Green's function by

$$\Gamma_{\mu\nu\lambda}(p, q, r) = \int d^4x d^4y e^{ip \cdot x} e^{iq \cdot y} \times T \langle j_\mu(x) j_\nu(y) j_\lambda(0) \rangle, \quad (6)$$

where r is $-(p+q)$. Γ is symmetric under simultaneous permutations of (p, q, r) and (μ, ν, λ) . The anomaly equation^{4,2} states that

$$r^\lambda \Gamma_{\mu\nu\lambda} = (N/\pi^2) (\text{Tr } A^3) \epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma. \quad (7)$$

We will choose A such that $\text{Tr } A^3$ is not zero.

Equation (7) implies that Γ cannot be analytic at $p=q=r=0$. Proof: If Γ is analytic, it has a Taylor expansion, and the right-hand side of Eq. (7) must come from a first-order term in this expansion. If we neglect the permutation symmetry of Γ , there are two independent first-order pseudotensors, $\epsilon_{\mu\nu\lambda\sigma} p^\sigma$ and $\epsilon_{\mu\nu\lambda\sigma} q^\sigma$. However, when we symmetrize these, each becomes $\epsilon_{\mu\nu\lambda\sigma} (p+q+r)^\sigma = 0$, Q.E.D.

It is known that in leading order in $1/N$, the only singularities in Green's functions made of strings of quark bilinears are poles.¹ For a three-bilinear Green's function, like Γ , these poles are at values of p^2 , q^2 , and/or r^2 equal to the masses of the particles made by applying the individual bilinears to the vacuum. Because Γ is not analytic at $p=q=r=0$, j_μ must create at least one massless particle when applied to the vacuum. If we were dealing with massive particles, a vector current could create either vector or scalar particles. For massless particles, though, Lorentz invariance forbids the creation of vector particles; only scalar particles are allowed.⁵ But for a conserved current like j_μ , this is the Goldstone alternative: The current creates a massless scalar particle from the vacuum if and only if the associated symmetry suffers spontaneous breakdown.

Added comments.—(1) Our reasoning can be extended to theories in which the quarks transform according to the fundamental representation of color $\text{SO}(N)$ [or $\text{Sp}(N)$]. These representations are equivalent to their conjugates, and so quark and antiquark together transform like a vector under chiral $\text{U}(2n)$, while the analog of M transforms like a symmetric tensor. A trivial rerun of our arguments then implies that, in the large- N limit, chiral $\text{U}(2n)$ breaks down to $\text{O}(2n)$.

(2) If we abandon assumptions (3) to (5), the part of our argument based on the anomaly equation survives, and leads to a weaker result, but one that is still nontrivial: In the large- N limit, the chiral group must break down to an anomaly-

free subgroup. In particular, this implies that there must be *some* chiral symmetry breakdown.

(3) If we consider variant theories in which the quarks transform like a rank-two tensor under the color group, graphs with a single quark loop no longer dominate. Thus assumptions (3) to (5) are useless, and the only conclusion we can draw is that of the preceding comment.

(4) Regrettably, in no case do our arguments give any insight at all into the mechanism of symmetry breakdown.

Thus the first of our two alternatives, no symmetry breakdown at all, is excluded, and only the second, breakdown to diagonal $\text{U}(n)$, remains. This completes the argument.

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²The chromodynamic anomaly in the $\text{U}(1)$ axial current is irrelevant in the large- N limit; its effects first appear in next-to-leading order in $1/N$. See E. Witten, Nucl. Phys. **B156**, 269 (1979); P. Di Vecchia, Phys. Lett. **85B**, 357 (1979); G. Veneziano, Nucl. Phys. **B159**, 213 (1979). These papers all assume that the observed pattern of chiral symmetry breakdown persists in the large- N limit. The analysis given above replaces this assumption with a much weaker one.

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⁵The little group of a null vector, \mathbf{k} , is isomorphic to the two-dimensional Euclidean group. Under this group, the single helicity state of a scalar particle transforms according to the trivial representation, while the two helicity states of a vector particle each transform according to nontrivial one-dimensional representations. On the other hand, of the four components of the current, only the one aligned with \mathbf{k} transforms according to a one-dimensional representation, the trivial representation.