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Magnetic Monopoles in Grand Unified Theories

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It is suggested that the magnetic monopoles predicted by grand unified theories would not be produced in significant numbers if electromagnetic gauge invariance is spontaneously broken when the temperature T is greater than $T_c \gtrsim 1$ TeV. A model possessing this behavior is displayed and the cosmological implications are discussed.

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There has been considerable discussion[1-4] recently that if grand unified theories[5] are correct then an unacceptably large number of superheavy ($m_m \gtrsim 10^{16}$ GeV) magnetic monopoles[6] may have been produced immediately after the big bang. Magnetic monopoles of the 't Hooft–Polyakov type[6] can exist if a semisimple gauge group G is spontaneously broken down to a subgroup H which contains a U(1) factor. The monopole mass m_m is of order M_X/α_g, where M_X is a typical mass of a gauge boson associated with a broken generator, g is the gauge coupling, and $\alpha_g = g^2/4\pi$. In the Georgi-Glashow[5] SU(5) model $M_X \simeq 10^{14}$–10^{15} GeV and $m_m \simeq 10^{16}$ GeV.

It is likely that G was unbroken immediately after the big bang when the temperature T was large compared to M_X. As the universe cooled it presumably underwent one or more phase transitions, finally entering the phase in which G is broken down to H [containing the U(1) factor] at some temperature T_t. Preskill has argued[2] that the ratio $r(T) \equiv n_m(T)/n_\gamma(T)$ of monopole to photon density must have been less than 10^{-19} initially (i.e., when $T \lesssim T_t$). However, Preskill[2] and Einhorn, Stein, and Toussaint[3] have estimated that if the phase transition to the H phase is second order or weakly first order then $r(T_t) \simeq 10^{-6}$, thirteen orders of magnitude too large, unless unacceptably large values for the Higgs self-coupling are assumed. One attractive solution to this problem, suggested by Preskill,[2] Einhorn, Stein, and Toussaint[3] and Guth and Tye,[4] is that the phase transition at which the U(1) factor occurs is strongly first order, in which case it may be possible to have $r < 10^{-19}$.

In this paper we propose an alternative scenario for the suppression of monopoles, in which the universe undergoes two or more phase transitions (which can be second order)

$$G \underset{T_1}{\rightarrow} H_1 \underset{T_2}{\rightarrow} \cdots \underset{T_n}{\rightarrow} H_n \underset{T_c}{\rightarrow} \mathrm{SU}(3)^c \otimes \mathrm{U}(1)^{\mathrm{em}}, \qquad (1)$$

where $\mathrm{U}(1)^{\mathrm{em}}$ is not a subgroup of H_n. The critical temperature T_c at which $\mathrm{U}(1)^{\mathrm{em}}$ appears is $T_c \gtrsim 1$ TeV. Since $T_c \lll m_m \simeq 10^{16}$ GeV, no monopoles will be produced. For example, SU(5) could break down to $SU(3)^c$ at $T_1 \lesssim M_X$ and undergo a second phase transition to the higher symmetry group $SU(3)^c \otimes U(1)^{em}$ at $T_c \gtrsim 1$ TeV.

We consider a model which at $T=0$ is the standard Georgi-Glashow SU(5) model,[5] with SU(5) broken to $SU(3)^c \otimes SU(2) \otimes SU(1)$, by an adjoint Higgs representation and then to $SU(3)^c \otimes U(1)^{em}$ by one or more five-dimensional Higgs representations. We assume that a hierarchy exists, i.e., that $M_{W^\pm, Z} \lll M_{X,Y}$ and that the color-triplet components of the Higgs fields have masses $\lesssim M_X$. For $0 \leq T \ll M_X$ we need only consider the SU(2) $\otimes$ U(1) part of the model [we assume that $SU(3)^c$ is never broken].

Therefore, consider an $SU(2) \otimes U(1)$ model with n complex Higgs doublets φ_i. It will turn out that at least three doublets (or two doublets and a singlet) are required, so we will taken $n=3$. The Higgs potential (at $T=0$) is

$$V = \sum_{i=1}^{3} [-\mu_i^2 \varphi_i^\dagger \varphi_i + \lambda_i (\varphi_i^\dagger \varphi_i)^2] + \sum_{i<j} [\sigma_{ij} \varphi_i^\dagger \varphi_i \varphi_j^\dagger \varphi_j + \rho_{ij} \varphi_i^\dagger \varphi_j \varphi_j^\dagger \varphi_i + \eta_{ij} (\varphi_i^\dagger \varphi_j)^2 + \eta_{ij}^* (\varphi_j^\dagger \varphi_i)^2], \tag{2}$$

where we have imposed discrete symmetries under $\varphi_i \to -\varphi_i$ for simplicity. If the minimum of V occurs when only one doublet (e.g., φ_1) has a nonzero vacuum expectation value $\langle \varphi_i(0) \rangle$ (VEV), then $SU(2) \otimes U(1)$ is broken down to $U(1)^{em}$ and we can take $\langle \varphi_1(0) \rangle = (0\ v_1)^T/\sqrt{2}$. If two doublets φ_1 and φ_2 both have nonzero VEV's then either $\langle \varphi_2(0) \rangle = (v_2\ 0)^T/\sqrt{2}$ or $\langle \varphi_2(0) \rangle = (0\ v_2)^T/\sqrt{2}$ which occur for ρ_{12} greater or less than $2|\eta_{12}|$, respectively. $SU(2) \otimes U(1)$ is either completely broken or broken to $U(1)^{em}$ for these two cases, respectively. We want $U(1)^{em}$ to be unbroken at $T=0$ but broken for $T > T_c$. Therefore, we take $\rho_{ij} > 2|\eta_{ij}|$, so that the VEV's want to be orthogonal, but we will arrange the other parameters so that $\langle \varphi_2(0) \rangle = \langle \varphi_3(0) \rangle = 0$. This occurs for $\mu_1^2 > 0$, $\mu_{2,3}^2 < 0$, and

$$|\mu_i|^2 + \sigma_{1i} \mu_1^2 / 2\lambda_1 > 0, \quad i = 2, 3. \tag{3}$$

Then v_1^2 is given by $\mu_1^2/\lambda_1 = (\sqrt{2} G_F)^{-1}$. We also require

$$\lambda_i > 0, \qquad \sigma_{ij} > -(\lambda_i \lambda_j)^{1/2}, \tag{4}$$

which are sufficient conditions for V to be bounded from below.

For $T > 0$, the VEV's $\langle \varphi_i(0) \rangle$ must be replaced by ensemble averages[7-9] $\langle \varphi_i(T) \rangle$. It has been shown that the $\langle \varphi_i(T) \rangle$ can be obtained at least for sufficiently large T, by minimizing the effective potential

$$V(T) \equiv V + \sum_{i=1}^{3} \tfrac{1}{2} T^2 F_i \varphi_i^\dagger \varphi_i, \tag{5}$$

where

$$F_i = \tfrac{1}{8}(3g^2 + g'^2) + \lambda_i + \sum_{j \neq i} [\tfrac{1}{3}\sigma_{ij} + \tfrac{1}{6}\rho_{ij}] + \text{Yukawa terms}. \tag{6}$$

For small fermion masses, the Yukawa terms in (6) will generally be negligible. If the F_i are positive, then for $T^2 \gtrsim 2\mu_1^2/F_1$ the coefficient of $\varphi_1^\dagger \varphi_1$ in $V(T)$ will be positive and the system will undergo a transition to phase in which $SU(2) \otimes U(1)$ is unbroken $[\langle \varphi_i(T) \rangle = 0]$. However, Weinberg[8] and more recently Mohapatra and Senjanović[10] and Zee[11] have emphasized in analogous models that some of the F_i can be negative; in this case, the symmetry need not be restored at high T.

It is even possible to have a transition to a state of lower symmetry.[8] We will choose parameters so that $F_{1,2} < 0$. This turns out to require $F_3 > 0$ so that for sufficiently large T we may have a transition to the phase with $SU(2) \otimes U(1)$ completely broken. As an existence proof that all of these conditions can be satisfied, choose

$$\lambda_1 = \lambda_2 \equiv \lambda \gg g^4, |\rho_{ij}|, \qquad \sigma_{12} > -\lambda, \qquad -\sigma_{13} = -\sigma_{23} \equiv \sigma > 3\lambda + \sigma_{12} + 3X, \qquad \lambda_3 > \sigma^2/\lambda, \tag{7}$$

where $X = \frac{1}{8}(3g^2 + g'^2) \simeq 0.16$. The condition $\lambda \gg g^4$ allows us to neglect radiative corrections to V. For a typical set of numbers, choose $\lambda \simeq -\sigma_{12} \simeq g^2 \simeq 0.4$, $\sigma \gtrsim 1.3$, $\lambda_3 \gtrsim 4.1$. The only purpose of introducing φ_3 was to lower the energy of φ_1 and φ_2 at high temperatures because of their coupling to φ_3. We see that there is a range of parameters which satisfy the above conditions, but a rather large value for λ_3 is required. This value is not so large as to violate tree-level unitarity, which would occur[12] for $\lambda_3 \gtrsim 8\pi/3$, but it may lead to serious difficulties with the renormalization-group equations[13] for running quartic couplings.[14]

For large T, the effective-mass quantities $M_1^2(T)$ and $M_2^2(T)$ defined by $M_i^2(T) \equiv \mu_i^2 - \frac{1}{2}F_i T^2$ will be positive. $V(T)$ will have an extremum, $\langle \varphi_1(T)\rangle = (0\; v_1(T))^T/\sqrt{2}$ and $\langle \varphi_2(T)\rangle = (v_2(T)\; 0)^T/\sqrt{2}$, with

$$\begin{pmatrix} v_1^2(T) \\ v_2^2(T) \end{pmatrix} = \frac{1}{(\lambda_1\lambda_2 - \frac{1}{4}\sigma_{12}^2)} \begin{pmatrix} \lambda_2 & -\frac{1}{2}\sigma_{12} \\ -\frac{1}{2}\sigma_{12} & \lambda_1 \end{pmatrix} \begin{pmatrix} M_1^2(T) \\ M_2^2(T) \end{pmatrix}, \tag{8}$$

and $\langle \varphi_3(T)\rangle = 0$. This will be (at least) a local minimum if

$$v_{1,2}^2 > 0,$$
$$2 > \sigma_{12}/(\lambda_1\lambda_2)^{1/2} > -1, \tag{9}$$
$$|M_3^2| + \tfrac{1}{2}\sigma_{13}v_1^2 + \tfrac{1}{2}\sigma_{23}v_2^2 > 0.$$

The second-order transition between these phases occurs at T_c such that $v_2(T_c) = 0$. For the special case (6) these conditions are fulfilled if $2 > \sigma_{12}/\lambda > -1$ and $|\mu_2^2| > \mu_1^2$. In this case $|F_1| = |F_2| \leqslant O(\lambda \simeq g^2)$, $F_3 \simeq \lambda_3$, and the phase transition occurs for

$$T_c = A\mu_1/\sqrt{\lambda} = (246\ \text{GeV})A, \tag{10}$$

where

$$A = \left[\frac{\lambda|\mu_2|^2/\mu_1^2 + \sigma_{12}/2}{\frac{1}{2}|F_2|(1-\sigma_{12}/2\lambda)}\right]^{1/2}. \tag{11}$$

A is typically of order unity, but can be made much larger or smaller by adjusting parameters. We will assume $T_c \gtrsim 1$ TeV.

We have therefore demonstrated the existence of a model for which $SU(3)^c \otimes SU(2) \otimes U(1)$ is broken to $SU(3)^c$ for $T > T_c$.

For $M_X \gg T \gg T_c$, we have

$$v_1(T) \simeq v_2(T) \sim (-F_i T)^{1/2}/\sqrt{\lambda} \leqslant T,$$
$$m_1, m_2 \simeq (-F_1)^{1/2} T < T,$$
$$m_3 \simeq (F_3)^{1/2} T \gtrsim T, \tag{12}$$

where the four massive gauge bosons (mixtures of $W^{\pm}, Z, \gamma$) have masses $\simeq g v_i \leqslant gT$. $m_{1,2}$ are the masses of the Higgs-particle eigenstates which are mixtures of φ_1 and φ_2, and m_3 are the masses of the bosons in φ_3 (which do not mix with $\varphi_{1,2}$). Fermion masses are of order

$$m_F(T) \sim [m_F(0)/v_i(0)]v_i(T)$$
$$\sim m_F(0) G_F^{1/2} T \ll T. \tag{13}$$

For $T \lesssim M_X$ the superheavy scalar and vector particles can no longer be neglected and additional terms will be added to (6). A phase transition to an unbroken SU(5) phase is probable. There may also be intermediate phases [e.g., with $SU(3)^c \otimes SU(2) \otimes U(1)$ unbroken] for $T \lesssim M_X$, either due to the onset of superheavy thresholds or possibly from the effects of T-dependent effective coupling constants.[10]

There should be essentially no magnetic monopoles in our model. Any monopoles produced during intermediate phases at $T \lesssim M_X$ will become unstable once the $SU(3)^c$ phase is entered. They would presumably either decay or be confined in pairs which could subsequently annihilate. Stable monopoles of mass $m_m \simeq 10^{16}$ GeV could, in principle, exist for $T < T_c$, but the number $r \simeq \exp(-m_m/T_c)$ expected from thermal fluctuations when $T \simeq T_c$ is utterly negligible.

Fermion masses are always $\ll T$, so that the usual scenarios for producing a baryon asymmetry will be unchanged. Also, for $T < T_c$, $U(1)^{em}$ is restored so nucleosynthesis at $T \simeq 1$ MeV is not affected.

The most interesting feature is that electric charge is not conserved and the gauge bosons (including the photon) and the fermions and Higgs particles are massive for $T_c < T \lesssim M_X$. In fact, the gauge-boson masses $M \simeq gT$ are negligible compared to the electron plasma frequency

$$\omega_p(T) \sim [4\pi n_e(T)e^2/m_e(T)]^{1/2} \simeq 400T, \tag{14}$$

and can therefore be ignored. The fermion and (hopefully) the Higgs masses are small enough not to be problematic.

The reaction rate for charge-nonconserving reactions is[15]

$$\Gamma(T) = \langle \sigma v\rangle_T n(T) \simeq 10^{23} g\alpha^2 T(\text{GeV})\ \text{sec}^{-1}, \tag{15}$$

where we have assumed

$$\langle \sigma v\rangle_T \simeq c\langle\sigma\rangle_T \approx \frac{\alpha^2 v_2(T)^2}{T^4}c \approx \frac{\alpha^2 c}{T^2} \tag{16}$$

and a number density $n(T) \sim g n_\gamma(T)/2$, with[15] $g = g_B + 7g_F/8 \gtrsim 100$. $g_{B,F}$ are the number of boson and fermion light degrees of freedom at T. This is large compared to t^{-1}, where $t(\text{sec}) = 2.4 \times 10^{-6} g^{-1/2} T^{-2}$ (GeV) is the age of the universe $[\Gamma(T)t(T) \sim 10^{14}/T \text{ (GeV)}]$, so that the charge-nonconserving reactions are in equilibrium for $T \geqslant T_c$. There will be a small net charge density n_Q in the present universe left over from fluctuations from equilibrium at $T > T_c$. Only charge fluctuations on the scale of the observable universe are distinguishable from the standard scenario, and so we will assume a total charge $N_Q \leqslant N_\gamma^{1/2}$ in the observable universe. (Actually, the net charge will probably be much smaller because charged Higgs bosons become massless for $T \sim T_c$. They could be produced prolifically out of the vacuum to neutralize any excess charge produced earlier.[16] With $N_\gamma \simeq 10^{86}$, this implies $n_Q < 10^{-43} n_\gamma \sim 10^{-34} n_B$ in the present universe, where n_B is the baryon density. This is far smaller than the observational limit[17,18] $n_Q/n_B \simeq 10^{-18}$ from galaxies and cosmology.

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