Mobility Tensor of the Electron Bubble in Superfluid ^{3}He-A

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(Received 13 November 1979)

This paper reports calculation of the mobility tensor of negative ions, in ^{3}He-A for temperatures close to T_c in terms of normal-state properties from an exact solution for the anisotropic quasiparticle-rigid-ion elastic scattering amplitude in the Anderson-Brinkman-Morel state.

PACS numbers: 67.50.Fi

Ionic motion provides a sensitive probe of the elementary excitation spectra of quantum liquids at low temperatures. Recently we showed[1] that the rapid mobility increase of negative ions in ^{3}He-B just below T_c is due to a novel scattering phenomenon.[2] Here we evaluate the mobility of negative ions in ^{3}He-A where the important new feature is the anisotropy of the spectrum of elementary excitations, which leads to the observed tensor structure for the mobility.[3]

The striking constancy of the normal-state mobility throughout the Fermi-liquid regime[1] is explained by the large size of the ion, a cavity of radius $R = 10-20$ Å enclosing an electron. Since an ion constantly interacts with many quasiparticles, its motion is diffusive and essentially recoilless.[3,4] Positive ions, which are smaller, cannot at low temperatures be treated as classical nonrecoiling particles, and consequently the theory of their mobility is not so simple.

Just below T_c the scattering of superfluid quasiparticles from the ion remains practically elastic. Hence at low drift velocities collision processes which change the number of superfluid excitations cannot occur and the mobility is only limited by the ion's scattering from ^{3}He quasiparticles, whose spectrum in ^{3}He-A is given by $E_{\vec{p}}^2 = \xi_p^2 + |\Delta(\hat{p})|^2$, where $\xi_p \approx v_F(p - p_F)$ is the normal-state energy measured from the ^{3}He chemical potential, v_F and p_F are the ^{3}He Fermi velocity and momtneum, and $\underline{\Delta}(\hat{p}) = \Delta(T)\sin\theta\, e^{i\varphi}\underline{1}$ is the gap matrix for quasiparticles in the direction $\hat{p} = \vec{p}/p$ relative to the $\hat{l}$ vector. Since the gap is equal for both spin projections, spin can be suppressed in what follows for brevity.

Because of the anisotropy of ^{3}He-A, the ion mobility is a tensor. Its principal components are easily shown to be given (in units where $\hbar = 1$; $\hat{l}$ is along z axis) by[5]

$$\frac{e}{\mu_{\parallel}} = \sum_{\vec{p}'\vec{p}} (p_z - p_z')^2 \langle |t_{\vec{p}'\vec{p}}(E_{\vec{p}})|^2\rangle \left(-\frac{\partial n}{\partial E_{\vec{p}}}\right) 2\pi\delta(E_{\vec{p}} - E_{\vec{p}'}), \tag{1a}$$

$$\frac{e}{\mu_{\perp}} = \sum_{\vec{p}'\vec{p}} \tfrac{1}{2}[(p_x - p_x')^2 + (p_y - p_y')^2] \langle |t_{\vec{p}'\vec{p}}(E_{\vec{p}})|^2\rangle \left(-\frac{\partial n}{\partial E_{\vec{p}}}\right) 2\pi\delta(E_{\vec{p}} - E_{\vec{p}'}). \tag{1b}$$

Here $\mu_{\parallel}$ ($\mu_{\perp}$) is the mobility parallel (perpendicular) to $\hat{l}$, e is the charge of an electron, $n(E) = (\exp\beta E + 1)^{-1}$ with $\beta = (k_B T)^{-1}$ is the Fermi function, and $t_{\vec{p}'\vec{p}}(E_{\vec{p}})$ is the amplitude for scattering a superfluid quasiparticle at energy $E_{\vec{p}}$ from momentum $\vec{p}$ to $\vec{p}'$.

It is useful to express the mobilities in the form

$$\frac{e}{\mu_{\parallel,\perp}} = n_3 p_F \int dE \left(-\frac{\partial n}{\partial E}\right) \langle \sigma_{\parallel,\perp}^{\mathrm{tr}}(E)\rangle_{\mathrm{av}}, \tag{2}$$

where n_3 is the number density of ^{3}He. Angular brackets here denote averages of the parallel and perpendicular momentum transfer cross sections over the Fermi surface.[5] These may be expressed with help of the differential cross section

$$\frac{d\sigma}{d\Omega'}(\hat{p}',\hat{p},E)=\left(\frac{m^*}{2\pi}\right)^2\frac{E}{[E^2-|\Delta(\hat{p}')|^2]^{1/2}}\langle|t_{\vec{p}'\vec{p}}(E)|^2\rangle\frac{E}{[E^2-|\Delta(\hat{p})|^2]^{1/2}}, \quad (3)$$

where m^* denotes the ^{3}He effective mass. The total cross section equals

$$\sigma^{\rm tot}(\hat{p},E)=\int d\Omega(\hat{p}')\,d\sigma(\hat{p}',\hat{p},E)/d\Omega'.$$

If one ignored the coherent nature of the superfluid excitations by replacing the differential cross section by a constant, one would obtain the formulas derived by Bowley.[6] Bromley[7] takes energy-independent *Ansätze* for the differential cross section, while Soda[8] suggests calculating it by performing a Bogoliubov transformation on the normal-state interaction.

Here we calculate the quasiparticle-ion cross section in ^{3}He-A, which we use to evaluate the ion mobility tensor. To obtain the cross section we work out the branch-averaged squared transition amplitude

$$\langle|t_{\vec{p}'\vec{p}}(E)|^2\rangle=\frac{1}{2}\sum_{\substack{\text{final}\\ \text{branches}}}|\langle\Psi_{\vec{p}'}(E)|\underline{T}_{\vec{p}'\vec{p}}(E)|\Psi_{\vec{p}}(E)\rangle|^2. \quad (4)$$

The scattering states, $\Psi_{\vec{p}}(E)$, depend on energy and momentum and are spinors in particle-hole space. In the presence of the divergent density of states for superfluid excitations it is vital to account for the repeated scattering of ^{3}He quasiparticles from an ion. This is described by the full scattering $\underline{T}$ matrix, which we write as the Nambu matrix

$$\underline{T}_{\vec{p}'\vec{p}}(E)=\begin{pmatrix} t_{\vec{p}'\vec{p}}{}^1(E) & t_{\vec{p}',-\vec{p}}{}^2(E)\\ t_{-\vec{p}',\vec{p}}{}^3(E) & t_{-\vec{p}',-\vec{p}}{}^4(E)\end{pmatrix}. \quad (5)$$

$\underline{T}$ is calculated from the Lippmann-Schwinger equation $\underline{T}(E)=\underline{V}+\underline{V}\underline{G}(E)\underline{T}(E)$, where $\underline{V}$ is the ion potential and the matrix $\underline{G}(E)$ is the superfluid propagator. Since modifications of $\underline{T}$ occur for $E\simeq\Delta$, we need it only for initial and final quasiparticle momenta equal to the Fermi momentum. Expressing $\underline{V}$ in terms of the normal-state T matrix, $\underline{T}^N$, and propagator, $\underline{G}^N$, we thus find

$$\underline{T}(E)=\underline{T}^N+\underline{T}^N[\underline{G}(E)-\underline{G}^N]\,\underline{T}(E). \quad (6)$$

This equation determines the T matrix in the superfluid in terms of the normal-state interaction, for which we have employed a partial-wave expansion in Legendre polynomials. The ion is assumed rigid, with a potential that of a hard sphere of radius R. Fermi-liquid effects are essentially subsumed in the choice of potential. Order-parameter distortions close to the ion may be neglected.[2]

The matrix equation (6) amounts to coupled equations which connect $t_{\hat{p}'\hat{p}}{}^1$ with $t_{-\hat{p}'\hat{p}}{}^3$ and $t_{\hat{p}',-\hat{p}}{}^2$ with $t_{-\hat{p}',-\hat{p}}{}^4$. These are amplitudes for an initial normal-state quasiparticle $\hat{p}$ and quasihole $-\hat{p}$ to scatter to final quasiparticle $\hat{p}'$ and quasihole $-\hat{p}'$ states. We have solved these equations using expansions in spherical harmonics. In contrast with ^{3}He-B, where full rotational symmetry could be exploited, the present case is only axially symmetric and thus only m, the projection of angular momentum onto the anisotropy axis, is a good quantum number. On the other hand, all orbital momenta l' and l are possible. The solution may be represented as[5]

$$\underline{T}_{\hat{p}'\hat{p}}(E)=-\frac{4}{N(0)}\sum_m\sum_{l'l}\begin{pmatrix} Y_{l'm}{}^*(\hat{p}')t_{l'l,m}{}^1(E)Y_{lm}(\hat{p}) & Y_{l',m-1}{}^*(\hat{p}')t_{l'l,m}{}^2(E)Y_{l,-m}{}^*(-\hat{p})\\ Y_{l',-m-1}(-\hat{p}')t_{l'l,m}{}^3(E)Y_{lm}(\hat{p}) & Y_{l',-m}(-\hat{p}')t_{l'l,m}{}^4(E)Y_{l,-m}{}^*(-\hat{p})\end{pmatrix}, \quad (7)$$

where $N(0)$ is the density of states in normal ^{3}He. Because of the large radius of the scattering potential many partial waves must be included above. In numerical calculations we kept sixteen partial waves and thus for $m=0$ the coefficients $t_{l'l,m}{}^{(i)}$ are 16× 16 matrices. We therefore find it advantageous to employ parity properties of the scattering process by introducing symmetrized scattering amplitudes.[5] We thus achieve block diagonalization in angular momentum space and need half as large matrices.

In contrast with the B-phase work,[2] extensive numerical computations are required in the present calculation. Here we want to describe some interesting features of these[5]: All initial and final quasiparticle directions are allowed for $E>\Delta$, but when $E<\Delta$ these are restricted to angles where $E>\Delta(\hat{p})$. The intermediate-state propagator diverges for the threshold angle $\theta_0=\arcsin(E/\Delta)$ and in contrast to the B phase, virtual transitions, which give rise to an off-shell contribution to the propagator, have an important effect on the scattering amplitude, and also give rise to quasibound levels with energies below Δ.

Examples of the computed differential cross section are illustrated in Fig. 1 for $\hat{p}$ and $\hat{p}'$ in the same plane as $\hat{l}$. The cross section displays strong energy dependence and anisotropy. For E approaching Δ from above, the forward-scattering peak is broadened, while less scattering occurs into the directions of the maximum gap. Note that for $E<\Delta$ the differential cross section vanishes for $\theta_0<\theta<\pi-\theta_0$.

The angular-averaged total cross section is remarkably constant for energies $E>\Delta$. Below Δ it displays several resonances. The averaged transport cross sections, however, are severely reduced for energies approaching the gap from above. When $E<\Delta$ we find pronounced peaks in the parallel-momentum-transfer cross section at the quasiparticle resonances, while the perpendicular one rapidly tends to zero.[5]

Inserting the transport cross sections into Eqs. (2) we find the results which have been compared with those of Ref. 6 in Fig. 2. The latter yields for $(1-\mu_N/\mu_{\parallel,\perp})$ the slopes 0.42 and 0.47, respectively, as functions of Δ/T for $T_c-T\ll T_c$. To within numerical accuracy we find $(1-\mu_N/\mu)$ $\simeq 0.56\Delta/T$ for *both* $\mu_\parallel$ and $\mu_\perp$. The small anisotropy we predict near T_c is in agreement with the data of Roach, Ketterson, and Roach.[3] At lower temperatures we find a much larger difference between $\mu_\parallel$ and $\mu_\perp$ than in Ref. 6, again in agreement with experiment.[3,5]

In Fig. 3 we compare our calculation for $(\mu_\parallel+\mu_\perp)/2$ at 28.4 bars with that of Ref. 6 and experiments[1,3] around this pressure. These were carried out in an external magnetic field applied perpendicular to the electric field, such that the average $(\mu_\parallel+\mu_\perp)/2$ was measured.

To take into account strong-coupling effects, we assume that Δ^2 scales as the specific heat discontinuity, and employ

$$\Delta(T)=[(\Delta C/C_N)_{\text{meas}}/(\Delta C/C_N)_{\text{ABM}}]^{1/2}(\tfrac{5}{4})^{1/2}\Delta_{\text{BCS}}(T),$$

where $(\Delta C/C_N)_{\text{ABM}}=1.29$,[9] and take $(\Delta C/C_N)_{\text{meas}}$ as 1.82.[10,11] Our results agree well with the most extensive set of available data, that of

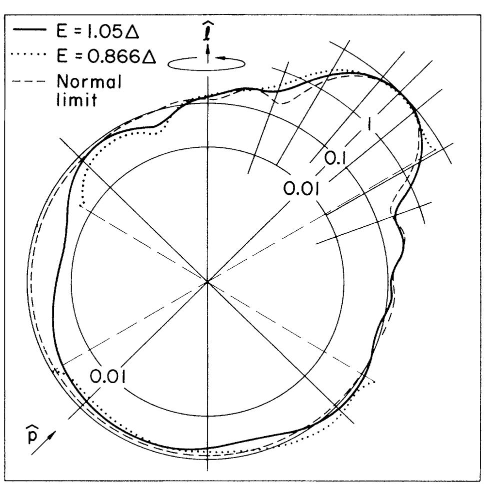

FIG. 1. Polar plot of the differential ^{3}He-ion cross section in the Anderson-Brinkman-Morel (ABM) state as a function of $\hat{p}'$, with fixed $\hat{p}$ at 45° to $\hat{l}$ and $\hat{p}$, $\hat{p}'$, and $\hat{l}$ coplanar. The radial scale is logarithmic in units of πR^2 and an ion radius $R=8.45/p_F$ was used.

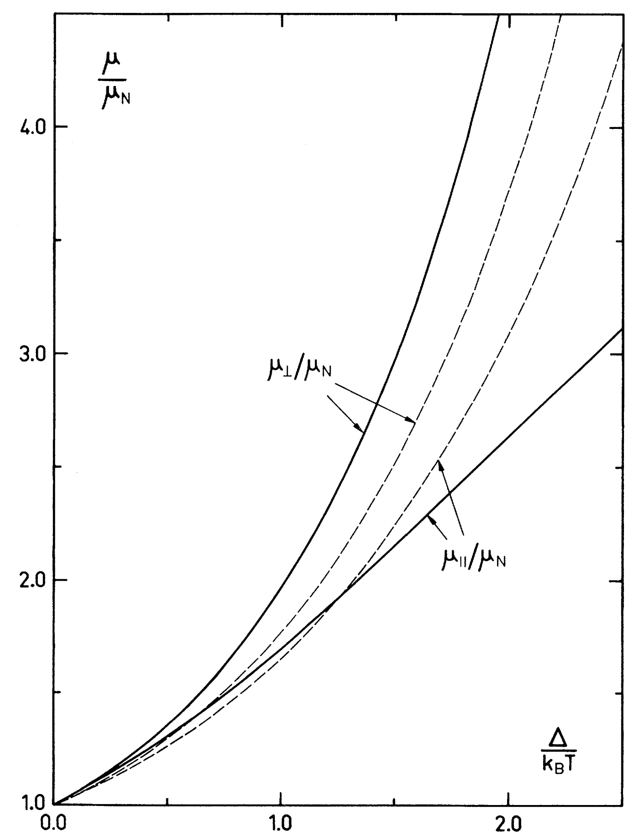

FIG. 2. The caluclated ion mobility tensor in ^{3}He-A relative to the constant normal-state mobility, μ_N, as a function of $\Delta(T)/T$. Dashed curves are the approximation of Ref. 6.

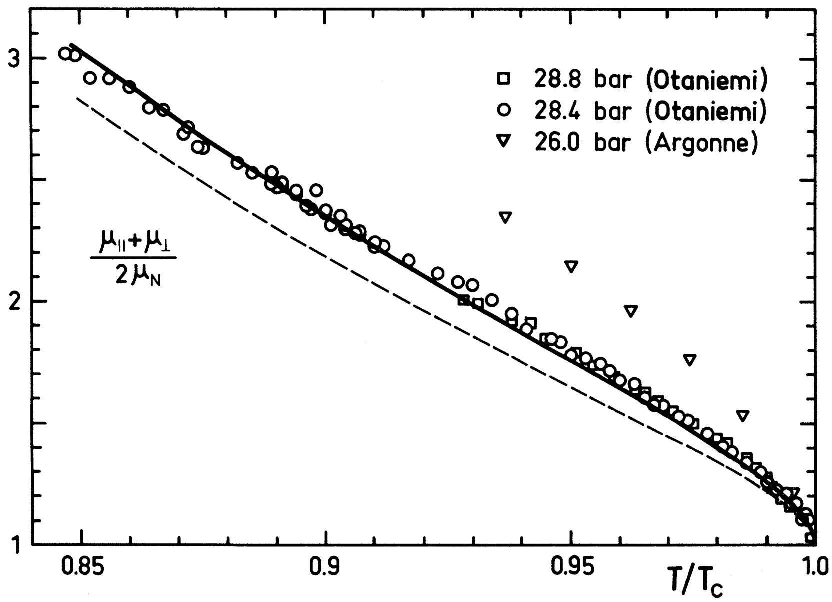

FIG. 3. The computed mobility (thick line) of negative ions in ^{3}He-A as a function of T/T_c, compared with data from Refs. 1 (squares and circles) and Ref. 3 (triangles). Dashed line is Bowley's approximation (Ref. 6).

Ahonen *et al.*[1] While the agreement with experiment is better than that for the B phase, we do not regard the difference as significant. The reason for the large discrepancy between the data of Refs. 1 and 3 at similar pressures is unclear, but may be due to differences in the temperature scale. The result of Bowley's approximation[6] does not differ much from ours in this comparison of averages. These calculations provide further stimulus to perform experiments to measure the anisotropy of the A-phase mobility, and also to use ions as a probe of the parameters of liquid ^{3}He.

We thank A. I. Ahonen, J. B. Ketterson, and O. V. Lounasmaa for useful communications. One of us (M.S.) is grateful to the Emil Aaltonen Foundation and to the Finnish Cultural Foundation for grants and to NORDITA for support during a stay in Copenhagen. This research was also supported by the U. S. National Science Foundation under Grants No. DMR78-21068 and No. DMR78-21069.

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