Microtearing Modes and Anomalous Transport in Tokamaks

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Microtearing (high-m) modes driven by the electron temperature gradient are found to be unstable for present tokamak parameters. A self-consistent calculation of the nonlinear saturation of this instability yields magnetic fluctuations $|\tilde{B}|/B \simeq \rho_e/L_T$. The associated crossfield electron thermal conductivity is shown to be inversely proportional to density, consistent with Alcator scaling, and comparable in magnitude with that inferred from experiments.

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A major mystery in present tokamak experiments is the anomaly in the electron energy confinement. The crossfield electron thermal conductivity scales as $\chi_{et} \simeq (10^4\ \mathrm{cm^2/sec})[(10^{14}\ \mathrm{cm^{-3}})/n]$, two orders of magnitude higher than the neoclassical value.[1] The importance of magnetic fluctuations in producing thermal transport has been recognized.[2-5] Radial magnetic fluctuations allow electrons to move along $\vec{B}$ in the radial direction, thereby coupling the parallel thermal conductivity $\chi_{e\parallel}$ to $\chi_{e\perp}$. Previous work has largely been concerned with transport in an assumed spectrum of magnetic fluctuations.[2] More recently, the magnetic fluctuations associated with drift waves in a finite-β plasma have been invoked as a source of the magnetic fluctuations.[4,5]

Microtearing modes (high-m drift-tearing) are an alternate source of magnetic fluctuations which could enhance thermal transport in tokamaks. While these modes are stable in the collisionless regime $\omega_{*e} \gg \nu_{et}$,[6-8] destabilization by the electron temperature gradient T_e' was predicted in the collisional regime $\omega_{*e} < \nu_{et}$.[7] The expansion of thermal energy along the magnetic fluctuation, which destabilizes the microtearing mode, also produces a net outward flux of thermal energy. Thus, thermal transport is inherently the cause and the effect of this instability. The previous theory of this instability was based on a number of questionable assumptions. In particular, Coppi *et al.*[9] have shown that the potential $\tilde{\varphi}$, previously neglected, has a strong stabilizing influence on collisionless tearing modes, casting doubt on the existence of this collisional instability.

Here, we investigate the stability of microtearing modes and the associated electron thermal transport. We first show that these modes are driven unstable by T_e' in present tokamaks, even in the absence of toroidal effects. The unstable spectrum compares well with recent observations of magnetic fluctuations on the Macrotor tokamak.[10] Nonlinearly, these modes produce magnetic islands which saturate when $|\tilde{B}|/B \simeq \rho_e(T_e'/T_e)$ by transferring energy to stable long-wavelength modes. The resulting thermal conductivity $\chi_{e\perp}$ scales inversely with density and is consistent with experimental measurements of energy confinement time.

We consider a slab plasma with density and temperature gradients (scale lengths L_n and L_T) in a sheared magnetic field $\vec{B} = B_0\hat{z} + (x/L_s)\hat{y}$. In a low-$\beta$ system the electromagnetic perturbations can be represented by the parallel vector potential $\tilde{A}_\parallel$ and the scalar potential $\tilde{\varphi}$. For $\tilde{A}_\parallel$, $\tilde{\varphi} \sim \exp(-i\omega t + ik_y y)$, the linearized drift-kinetic equation for electrons becomes[6]

$$[i(\omega - k_\parallel v_\parallel) + (\nu/2)(\partial/\partial\xi)(1-\xi^2)(\partial/\partial\xi)]\tilde{f}_e = e\tilde{E}_\parallel v_\parallel f_0/T_e - (ik_y c/B)(\tilde{\varphi} - \tilde{A}_\parallel v_\parallel/c)(\partial f_0/\partial x), \tag{1}$$

where a velocity-dependent, Lorentz collision operator describes electron-ion collisions with $\nu(v) = \nu_{et}(v_e/v)^3$, $\nu_{et} = 4\pi n_0 Z^2 e^4 \ln\Lambda/m^2 v_e^3$, $\xi = v_\parallel/v = \vec{v}\cdot\vec{B}/vB$, and $k_\parallel = k_y x/L_s$. This equation can be solved by expanding $\tilde{f}_e$ in a Legendre series. From Ampere's law and the quasineutrality condition, we obtain

$$(\partial^2/\partial x^2 - k_y^2)\tilde{A}_\parallel = -(4\pi/c)\sigma_{\parallel e}\tilde{E}_\parallel = -(4\pi/c)\sigma_{\parallel e}(i\omega\tilde{A}_\parallel/c - ik_\parallel\tilde{\varphi}), \tag{2}$$

$$(c^2/c_A^2)(\omega + \omega_{*i})[\partial^2/\partial x^2 - k_y^2]\tilde{\varphi} = -4\pi k_\parallel \tilde{E}_\parallel \sigma_{\parallel e}, \tag{3}$$

$$\sigma_{\parallel e} = (2i/3\pi^{3/2})\omega_{pe}^2\int_0^\infty ds\, s^4 \exp(-s^2)[\omega - \omega_*(s)][i\omega(\nu - i\omega)\alpha_1 - k_\parallel^2 v_e^2 s^2/3]^{-1}, \tag{4}$$

where $c_A^2 = B^2/4\pi Mn$, $s = v/v_e$, $\omega_* = \omega_{*n}[1 + \eta_e(s^2 - \frac{3}{2})]$, $\omega_{*n} = k_y cT_e/eBL_n$, $\eta_e = L_n/L_T$, $\omega_{*i} = (T_i/T_e)\omega_{*n}$, and α_1 is given by the continued fraction

$$\alpha_n = 1 + k_\parallel{}^2 v^2 (n+1)^2/(2n+1)(2n+3)[i\omega - (\nu/2)(n+1)(n+2)][i\omega - (\nu/2)n(n+1)]\alpha_{n+1}. \tag{5}$$

For $|k_\parallel v/(\nu - i\omega)| \ll 1$, $\alpha_1 \approx 1$ and $\sigma_{\parallel e}$ reduces to that derived in Ref. 6.

The tearing branch of Eqs. (2) and (3) corresponds to modes with even $\tilde{A}_\parallel(x)$ and odd $\tilde{\varphi}(x)$, i.e., $\tilde{B}_x(0) = ik_y\tilde{A}_\parallel \neq 0$ with $\tilde{\varphi}(0) = 0$, and $\tilde{E}_\parallel \to 0$ at large $|x|$. The usual (macro) tearing modes, with $k_y L_n \lesssim 1$, are global modes driven by the magnetic field energy. Microtearing modes are short-wavelength modes, with $k_y L_n \gg 1$, localized near the rational surface ($x = 0$). At large $|x|$, $\tilde{A}_\parallel \sim \exp(-k_y|x|)$, corresponding to a negative Δ' in the usual tearing-mode terminology. The magnetic perturbations increase the magnetic energy by stretching the field lines and thus are stabilizing. The magnetic perturbations, however, also tip the magnetic field in the direction of the temperature gradient T_e' and the resulting expansion of the thermal energy along $\vec{B}$ can overcome the stabilizing magnetic forces to produce instability.[6,11]

In previous analytic theories, $\tilde{\varphi}$ was neglected in Eq. (2) and $\tilde{A}_\parallel(x)$ was approximated by $\tilde{A}_\parallel(0)$ (constant ψ approximation) under the assumption that $\sigma_{\parallel e}$ was strongly localized around $k_\parallel = 0$. Neither of these approximations can be justified for present tokamaks. Because of the complicated spatial dependence of $\sigma_{\parallel e}(x)$, we numerically solve Eqs. (2) and (3) using invariant imbedding methods.[7] The frequency is approximately $\omega_r = \omega_{*n}(1 + 1.5\eta_e)$. The growth rate γ/ν_{ei} is shown as a function of $k_y \propto \omega_{*n}/\nu_{ei}$ in Fig. 1(a). A positive temperature gradient $\eta_e > 0$ is destabilizing at intermediate wavelengths $\omega_{*n}/\nu_{ei} \sim 10$ and is stabilizing for both short and long wavelengths: This differs from previous theory where $\eta_e > 0$ was always destabilizing. The mode structure is illustrated in Fig. 2(a) for $\eta_e = 1$, $k_y\rho_s = 0.1$, and $\nu_{ei}/\omega_{*n} = 10$.

The difference between the present numerical results and previous theory arises from the retention of φ and the spatial variation of $\tilde{A}_\parallel(x)$ in Eq. (2). To separately illustrate the importance of these two effects, we solved Eq. (2) arbitrarily neglecting $\tilde{\varphi}$ and found that the modes are damped and that increasing η_e typically increases stability. The enhanced stability with increasing η_e results from an increase in the distortion of the magnetic islands (thus further increasing the magnetic energy), as illustrated in Fig. 1(b). For $\eta_e > 0$, the "local" eigenfrequency of tearing mode [obtained by solving the local version of Eq. (2)]

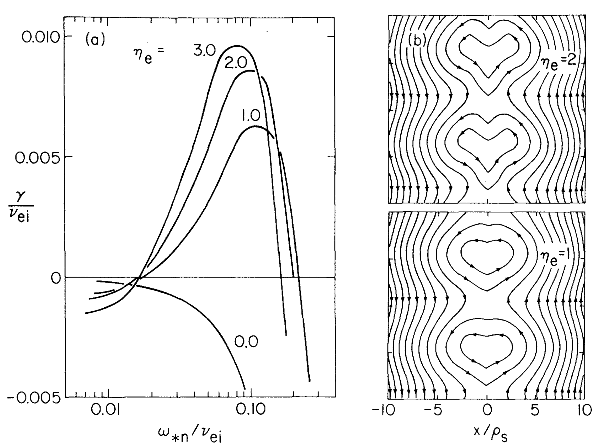

FIG. 1. (a) The growth rate γ/ν_{ei} is shown as a function of $k_y \propto \omega_{*n}/\nu_{ei}$ for $\eta_e = 0$, 1, 2, and 3, with $L_n/L_s = 0.05$, $\beta = 0.01$, and $T_e = T_i$. (b) The magnetic island structure is shown for $\eta_e = 1$ and 2 with $\omega_{*n}/\nu_{ei} = 0.1$ and other parameters as in (a).

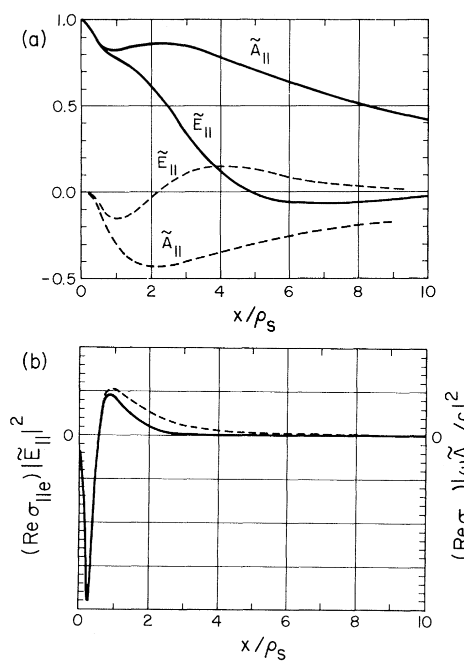

FIG. 2. The real (solid) and imaginary parts of $\tilde{A}_\parallel$ and $\tilde{E}_\parallel$ are shown in (a) for $\eta_e = 1$, $k_y\rho_s = 0.1$, and $\omega_{*n}/\nu_{ei} = 0.1$ with other parameters as in Fig. 1. In (b) the local particle heating rate $\tilde{J}_\parallel\tilde{E}_\parallel = (\mathrm{Re}\,\sigma_{\parallel e})|\tilde{E}_\parallel|^2$ (dashed curve) and that of the inductive field alone (solid curve) are shown.

decreases strongly as $|k_{\parallel}|$ increases. Thus, the mode tries to rotate faster (downward) in the vicinity of $k_{\parallel}=0$, distorting the island into the heart shapes shown in Fig. 1(b). The distortion of the magnetic islands is found to be important (from scaling arguments and computation) for $\eta_e^2\beta L_s^2/L_n^2>1$. When $\eta_e=0$, the local eigenfrequency is approximately ω_{*n}, nearly independent of $k_{\parallel}$, and the distortion is not significant.

The role of $\tilde{\varphi}$ in producing instability is illustrated in Fig. 2(b). The particle heating rate $\tilde{J}_{\parallel}\tilde{E}_{\parallel}=(\mathrm{Re}\sigma_{\parallel e})|\tilde{E}_{\parallel}|^2$ is shown in the dashed curve. The instability is driven by the electrons close to the rational surface where $\tilde{J}_{\parallel}\tilde{E}_{\parallel}<0$ (electrons give energy to the wave) while in the outer region the electrons are stabilizing since $\tilde{J}_{\parallel}\tilde{E}_{\parallel}>0$. The solid curve in Fig. 2(b) shows the electron heating from the induction field alone, $(\mathrm{Re}\sigma_{\parallel e})|\omega\tilde{A}_{\parallel}/c|^2$. The potential $\tilde{\varphi}$ is important at large $|x|$ where it shorts out the induction field (forcing $\tilde{E}_{\parallel}\to 0$), thereby strongly reducing the stabilizing effect of local electron heating in the outer region. Thus $\tilde{\varphi}$ has an important *destabilizing* influence. In the long- and short-wavelength regimes of Fig. 1, the magnetic field bending overcomes the destabilizing influence of $\tilde{\varphi}$.

In present tokamaks, $\eta_e>0$ destabilizes a spectrum of high-m modes. Recent probe measurements of magnetic fluctuations on the Macrotor tokamak revealed a broad spectrum of waves peaked at 25 kHz and extending up to 100 kHz.[10] For the parameters $T_e\simeq 100$ eV, $B=2$ kG, and $n_e\sim 4\times10^{12}$, the collision frequency is 500 kHz and our maximum linear growth rate corresponds to 50 kHz, in agreement with the observations.

The magnitude of the thermal transport depends on the saturation amplitude of the magnetic fluctuations. We study the saturation of the microtearing instability for zero shear and $k_{\parallel}=0$ by solving the drift kinetic equation,

$$[\partial/\partial t+v_{\parallel}(\tilde{\vec{B}}/B)\cdot\nabla-(\nu/2)(\partial/\partial\zeta)(1-\zeta^2)(\partial/\partial\zeta)]\tilde{f}=(ev_{\parallel}f_0/cT_e)(\partial/\partial t+v_D\partial/\partial y)\tilde{A}_z, \tag{6}$$

where $v_D=\omega_*/k_y$. These approximations lead to a considerable simplification of the nonlinear problem and are motivated by the observation that the most unstable modes in Fig. 1(a) are well described by the local dispersion relation [Eq. (2) with $\partial/\partial x=0$] at $k_{\parallel}=0$ and thus are essentially independent of the shear. The shear-free version of this mode is discussed elsewhere.[12] The dominant nonlinearity in Eq. (6) arises from the electron motion along the fluctuating magnetic field. We solve Eq. (6) iteratively to third order in $\tilde{B}$ to obtain

$$\begin{aligned}\tilde{f}_{ek}=-i(eT_e/c)v_{\parallel}f_0\tilde{A}_k\{&(\omega-\omega_*)/(\nu-i\omega)\\&+i\textstyle\sum_{k'}(|\vec{k}\cdot\tilde{\vec{B}}_{k'}|^2/B^2)(v^2/3\nu^2)[\omega-\omega_*-(\omega'-\omega_*')]/(\omega+\omega')\}\\&+i(e/3T_ec)[(\omega'-\omega_*')/\omega\nu]v^2f_0\textstyle\sum_{k'}\tilde{A}_{k'}(\vec{k}'\cdot\tilde{\vec{B}}_{k-k'}/B),\end{aligned} \tag{7}$$

assuming $\omega',\omega\ll\nu$. The first-order term produces a current perturbation but no density perturbation so that the potential $\tilde{\varphi}$ is zero in first order and justifies the neglect of $\tilde{\varphi}$ in Eq. (6). In second order, only a density perturbation is produced. Computing $\tilde{J}_{ek}$ and using Ampere's law yield

$$\begin{aligned}\omega=\omega_k&+\tfrac{3}{4}i\Gamma(\tfrac{9}{2})\eta_e\omega_k\omega_{*n}/\nu_{ei}-i(3\sqrt{\pi}/8)k_y^2\nu_{ei}c^2/\omega_{pe}^2\\&-\tfrac{5}{12}i\Gamma(\tfrac{11}{2})\sum_{k'}\frac{v_e^2}{\nu_e}\frac{|\vec{k}\cdot\tilde{\vec{B}}_{k'}|^2}{B^2}\left(\frac{\omega_{*n}-\omega_{*n}'}{\omega_k+\omega_k'}\right)\eta_e,\end{aligned} \tag{8}$$

where $\omega_k=\omega_{*n}(1+5\eta_e/2)$. The first imaginary term represents the destabilizing influence of η_e and the second arises from field-line bending. The nonlinear term scales as $k^2D_{\perp}$ where $D_{\perp}=(v_e^2/\nu_{ei})|\tilde{B}|^2/B^2$ is the perpendicular diffusion coefficient. The nonlinear term is stabilizing for $k'<k$ and destabilizing for $k'>k$ and thus leads to an energy flow from short- to long-wavelength modes. Since the long-wavelength modes are stable [see Fig. 1(a)], this flow saturates the instability when the growth and damping rates balance at $|\tilde{B}|/B\approx\rho_e/L_T$.

For typical tokamaks, $|\tilde{B}|/B\sim10^{-4}$–10^{-5}, in good agreement with the magnitude of the fluctuations measured in the Macrotor tokamak.[10] Moreover, the fluctuations in this machine were found to be essentially independent of the density. With $|\tilde{B}|/B=\rho_e/L_T$, the local crossfield transport coefficient becomes

$$D_{\perp}\simeq v_e^2\rho_e^2/\nu_eL_T^2. \tag{9}$$

This result depends sensitively on L_T, which is known to be substantially smaller than both L_n and the minor radius a. To estimate the global average diffusion rate of a discharge, we estimate L_T from the heat balance equation $\eta J^2 = (\partial/\partial x)D_\perp(\partial/\partial x)nT_e \simeq D_\perp nT_e/L_T^2$ and eliminate L_T from Eq. (9) to obtain $D_\perp \simeq (c^2/\omega_{pe}^2)(T_e/m)^{1/2}/qR$, where $qR = Bc/4\pi J$. Remarkably, this is the same as the Ohkawa scaling.[13] The magnitude of the diffusion coefficient is 10^4 cm^2/sec for $n = 10^{14}$/cm^3 and $T_e = 1$ keV which is comparable to that inferred from experiments.

In conclusion, we expect the η_e-driven microtearing modes to cause substantial transport in the more collisional tokamaks such as Alcator and and Macrotor, where a large number of modes with $\omega_{*e} < \nu_{ei}$ are driven unstable. In higher-temperature machines such as PLT (Princeton Large Torus) which have fewer unstable modes, the diffusion coefficient in Eq. (9) is not expected to be valid. On the basis of linear theory, increasing T_e in this machine could actually improve confinement by further reducing the number of unstable microtearing modes. Other mechanisms may, of course, destabilize these modes in the higher-temperature regimes of toroidal plasmas.

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