It is possible that final-state interactions are responsible for some of this discrepancy, but again preliminary results[18] indicate only a (5–10)% reduction. There may exist significant strength in S_L at high ω that we are missing, but that is considered unlikely. It is clear that the data should be extended to higher q approaching $2K_F$ so that we have greater confidence in our independent-particle model and avoid any complication from Pauli correlations.

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Widths of Σ-Hypernuclear States

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The widths of states of Σ-hypernuclei have been estimated in two models. Widths as small as one to a few megaelectronvolts are found, a result which should hold for some isolated simple Σ-hypernuclear states. The standard procedure by use of the two-body absorptive cross section is shown to overestimate the width by one or more orders of magnitude.

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Recently (K,π) experiments with nuclear targets leading to Σ-hypernuclei have been carried out,[1,2] and one anticipates that there might soon be considerable new data with improved energy resolution. The crucial question governing the future of this field of study is the width of Σ-hypernuclear states. The conventional wisdom is that the $\Sigma\to\Lambda$ conversion in nuclear matter will lead to a typical strong interaction width of 50–150 MeV for these states. If that is true the study of Σ-hypernuclei will be very limited in comparison with Λ-hypernuclei. The present data,[2] however, show that there are states with widths less than 10 MeV. It is the purpose of the present note to demonstrate that such small widths can be expected for isolated Σ-hypernuclear states, and that the study of Σ-hypernuclei should be most rewarding.

The main calculations are estimates of the width of a Σ-N state produced in a (K^-,π^-) reaction on a N-N cluster in a nucleus. Two models are used. First, the width is estimated using plane-wave final Λ-N states. Since the final center-of-mass momentum is approximately $2m_\pi c$, this should be adequate for the purposes of the present work. In the second calculation the Σ-$N\to$ Λ-N width is estimated by means of the model of broken SU(3) symmetry which has been proposed[3,4] for the treatment of analog hypernuclear states. In this model one considers Λ analog states, $|\Lambda\mathrm{AS}\rangle$, and Σ analog states, $|\Sigma\mathrm{AS}\rangle$, degenerate in the SU(3) limit and split mainly by mass differences. Although microscopic calculations indicate that the $|\Lambda\mathrm{AS}\rangle$ might not be detectable,[4,5] the basic two-baryon force seems more favorable for the $|\Sigma\mathrm{AS}\rangle$. In any case, we only apply the model to the two-baryon problem here.

In using this broken SU(3) model, it is the width of possible Σ analog states which is being estimated; however, the results will be approximately valid for other shell-model–type Σ-hypernuclear states. It is simply a model to include final-state interactions. Although the final numerical results are somewhat model dependent, it will be shown that the widths obtained are an order of magnitude smaller than previous conjectures.

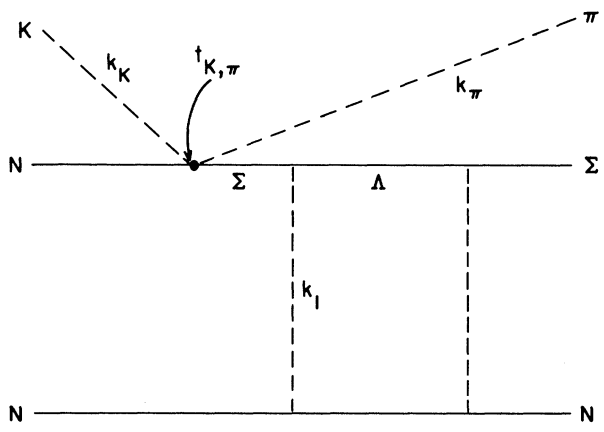

FIG. 1. The Σ-Λ conversion process leading to the width of Σ-hypernuclear states being considered in this work.

Let us first consider a two-nucleon cluster in a nucleus. The main width seen in a (K^-,π^-) reaction exciting a Σ-hypernuclear state arises from the Σ-$N\to\Lambda$-N processes. The t matrix for the (K^-,π^-) reaction including this process is

$$T^{K^-,\pi^-}=\frac{t_{K^-,\pi^-}F_N(k_{K^-},k_{\pi^-})}{\langle\Sigma N|E_0-H_{\Sigma N}-V^\dagger(E-H_{\Lambda N}+i\epsilon)^{-1}V|\Sigma N\rangle}, \tag{1}$$

where V is the Σ-$N\to\Lambda$-N interaction, $H_{\Sigma N}$ is the Hamiltonian in the Σ-N space and $H_{\Lambda N}$ in the Λ-N space. The explicit form of V is given below. The numerator is the conventional multiple-scattering form with t_{K^-,π^-} being the two-body t matrix and F_N related to the nuclear form factor. The third term in the denominator is the effective Hamiltonian arising from the process illustrated in Fig. 1. From this one obtains the width of the Σ-N states as seen in the (K,π) cross section

$$\Gamma_{\Sigma N}/2=\pi\int\frac{d^3k''}{(2\pi)^3}\langle\Sigma N|V|\Lambda N,k''\rangle\langle\Lambda N,k''|V|\Sigma N\rangle\,\delta(E-E_\Lambda). \tag{2}$$

Equations (1) and (2) are quite general. For the interaction V the one-pion–exchange potential (OPEP) interaction is used:

$$V\approx V_{\Sigma\Lambda}{}^{\mathrm{OPEP}}=\frac{gg_{\Sigma\Lambda}}{4\pi}\frac{m_\pi{}^3}{4M_NM_{\Sigma\Lambda}}\left[\frac{1}{3}\vec{\sigma}\cdot\vec{\Sigma}+S_{12}\left(\frac{1}{3}+\frac{1}{mr}+\frac{1}{m^2r^2}\right)\right]\frac{e^{-mr}}{mr}\vec{\tau}\cdot\vec{\mathrm{T}}, \tag{3}$$

where Σ and T are $\Sigma\to\Lambda$ transition spin and isospin operators, $S_{12}=3(\overleftrightarrow{rr})^2(\sigma\Sigma)^2/r^2$ is a tensor force operator for the Σ-$N\to\Lambda$-N potential. The coupling constants are defined in the convention of Nagels *et al.*,[6] and $M_{\Sigma\Lambda}\equiv\frac{1}{2}(M_\Sigma+M_\Lambda)$.

Let us now determine the width of a Σ-N cluster assuming that the final Λ-N cluster consists of the Λ and N in plane-wave states. Taking the Σ-N cluster in an s state, and using the OPEP interaction of Eq. (3), one finds after averaging over spin and isospin that

$$\Gamma_{NN}=2\frac{kE_NE_\Lambda}{(E_\Lambda+E_N)m_\pi{}^3}\left(\frac{gg_{\Sigma\Lambda}m_\pi{}^3}{4M_NM_{\Sigma\Lambda}}\right)^2[m^3R_0{}^2+18m^3R_2{}^2], \tag{4}$$

where

$$R_0=\int dr\,r^2R_{\Sigma N}(r)j_0(kr)e^{-mr}/mr$$

and

$$R_2=\int dr\,r^2R_{\Sigma N}(r)j_2(kr)\left(\frac{1}{3}+\frac{1}{mr}+\frac{1}{m^2r^2}\right)e^{-mr}/mr,$$

and where $k=283$ MeV/c, $E_N=(k^2+M_N{}^2)^{1/2}$, $E_\Lambda=(k^2+M_\Lambda{}^2)^{1/2}$, and $R_{\Sigma N}(r)$ is the radial Σ-N cluster wave function. It is evident from the form of R_0 and R_2 that the answer is model dependent and that it is particularly important to take the short-range correlations into account. We use the model $R_{\Sigma N}(r) = N(e^{-\alpha r} - e^{-\beta r})$, a model which at least provides for the vanishing of the relative wave function at short distance. For a 4-MeV binding at the cluster we find

$$\Gamma_{NN}^{\text{plane wave}} = 1.23 - 1.8 \text{ MeV} \tag{5}$$

for $4 < \beta/\alpha < 7$. The model dependence of the result is an important consideration, to which we return below.

Next, consider the coupled-channels model of the (K^-, π^-) reaction leading to $|\Sigma N\rangle$ and $|\Lambda N\rangle$ two-baryon clusters which are coupled by the interaction V. Because of the large energy involved, the final Λ-N particles will most likely leave the nucleus, but here we assume that the direct coupling to the Σ-N cluster proceeds through the $|\Lambda\text{AS}\rangle$, which in this case is just a Λ-N cluster. As typical kinematics, for 600-MeV/c K^- momentum, for the reaction leading to the Σ-hypernuclear state the π^- momentum is about 512 MeV/c and about 586 MeV/c for the Λ-hypernuclear state. A lower K^- momentum would be more suitable for the recoilless reaction, but the present estimate gives typical results. For the present calculation we consider that an $L = 0$, Σ-N state has been produced.

Using this model of saturation by the ΛAS, one obtains from Eq. (2)

$$\frac{\Gamma}{2} = \frac{1}{2\pi}\frac{E_0}{k_0}\frac{M_\Sigma - M_\Lambda}{k_0} k_0^3 \left|\psi_\Lambda\left(\frac{k_\Lambda}{2}\right)\right|^2 \Delta E, \tag{6}$$

with

$$\Delta E = \langle \Sigma N | V^\dagger V | \Sigma N\rangle / (M_\Sigma - M_\Lambda). \tag{7}$$

In Eq. (6) k_0 is the pion momentum for the intermediate Λ-N state, and k_Λ is the momentum of the Λ corresponding to the δ function in Eq. (2). The quantity ΔE defined in Eq. (7) is the real energy shift produced by the Σ-Λ coupling of Eq. (1) in the analog-state model. Note that this expression for ΔE is also the closure result if the average energy difference between Σ- and Λ-hypernuclear states is $M_\Sigma - M_\Lambda$. Also note that equations analogous to Eqs. (6) and (7) can be obtained by use of the dispersion relation for the Σ-N scattering amplitude corresponding to the operator V. Recognizing that ΔE and $\Gamma/2$ are the real and imaginary parts of that amplitude, one has the dispersion relation

$$\Gamma/2 = \pi^{-1} \mathrm{P}\int_{E_{\text{th}}} dE'\, \Delta E(E')/(E' - E). \tag{8}$$

Using the extension of Eq. (7) for $\Delta E(E')$ and assuming that the threshold for the Λ-N state is $M_\Sigma - M_\Lambda$ below the Σ-N state, one sees that the width is expected to be of the same order as the energy shift ΔE. Let us proceed by estimating ΔE.

Assuming that the Σ-N cluster is in a $L = 0$ state, and averaging over spin and isospin, one finds from Eqs. (7) and (3) that

$$\Delta E = \frac{1}{M_\Sigma - M_\Lambda}\frac{g^2}{4\pi}\frac{g_{\Sigma\Lambda}^2}{4\pi}\left(\frac{m_\pi^3}{4M_N M_{\Sigma\Lambda}}\right)^2 (R_S^2 + 15R_T^2), \tag{9}$$

where

$$R_S^2 = \langle \Sigma N | (e^{-mr}/mr)^2 | \Sigma N\rangle, \quad R_T^2 = \langle \Sigma N | \left[\left(\frac{1}{3} + \frac{1}{mr} + \frac{1}{m^2 r^2}\right)\frac{e^{-mr}}{mr}\right]^2 | \Sigma N\rangle.$$

It is quite clear that the short-range repulsive nature of the force, which causes the wave function to tend to 0 faster than r^{-4} at the origin, must be used, and that the result for the tensor force is sensitive to the model of this short-range behavior. We evaluate expression (9) by choosing an average radius r which gives the correct value of $\langle (e^{-mr}/mr)^2\rangle$ for a wave function of the form $R_{\Sigma N}(r) \sim e^{-\alpha r} - e^{-\beta r}$, with α chosen to give the cluster a binding energy of ≈ 4 MeV and various choices for β. Typically, for $\beta = 4\alpha$, one finds

$$\Delta E \approx 0.8 \text{ MeV}$$

for the energy shift in the two-nucleon system. This gives $\Gamma_{NN}^{\text{analog}} \approx$ 1–2 MeV, consistent with the plane-wave result of Eq. (5).

Thus, the width arising from a N-N cluster is seen to be one to a few megaelectronvolts. This estimate for the two-nucleon system itself is not very useful for there does not seem to be a bound Σ-N state.[2,6] However, it can be used to estimate the widths of shell-model–type states (such as the $|\Sigma\text{AS}\rangle$). For complex nuclei the result can be stated in terms of the number of nucleon pairs, N_{eff}, which contribute to the width. Thus, the width of a typical Σ-hypernuclear state is

$$\Gamma_{\text{complex}} \approx \Gamma_{NN} N_{\text{eff}}. \tag{10}$$

From Eqs. (5) and (10) we see that this result is one to two orders of magnitude less than the values currently accepted. If one used the standard expression[7] $\Gamma \approx v\sigma_{\rm abs}\rho(0)$, then one finds values of the width of about 50 MeV[8] or larger. Note from Eq. (2), with a δ-function interaction one finds that the width arising from the Σ-N $\to \Lambda$-N two-particle cluster satisfies the relationship

$$\Gamma \propto |\psi(0)|^2, \tag{11}$$

which leads to the standard expression if one interprets $|\psi(0)|^2$ as $\rho(0)$. However, $\psi(0)$ is the *relative* wave function for the two-nucleon cluster, while the density $\rho(0)$ used in simple estimates is the average nuclear density. One can argue that the relative wave function is built into the absorptive cross section, $\sigma_{\rm abs}$, but we now show that this is not satisfactory for the type of process being considered.

I have used the model of Eq. (3) to calculate the total $\Sigma \to \Lambda$ cross section, $\sigma_{\Sigma\Lambda}$, at low energies. In the Born approximation $\sigma_{\Sigma\Lambda}$ is about a factor of 7 too large compared with experiment and detailed calculations.[9] By including an initial-state interaction to fit the Σ-N s-wave phase shift, the theoretical result can be reduced to give a satisfactory fit, in view of the crudity of the model. The results are still somewhat too large as can be expected from the absence of vector-meson contributions. This latter, by the way, will also reduce the value of the Σ-hypernuclear width given above.

Next note that for the two-body bound-state cluster the relative wave function $\psi(0)$ actually vanishes for a hard core, and is very small for realistic two-body forces. To estimate the error in using the average density $\rho(0)$ compared to a microscopic calculation, let us calculate the matrix element, $\langle \Lambda N | V_{\Sigma\Lambda}{}^{(1)} | \Sigma N\rangle$ using a Yukawa interaction, $V_{\Sigma\Lambda}{}^{(1)}$ with unit volume integral, to replace the δ function which leads to $\psi(0)$. For the parameters used in the paper and described above one finds that

$$|\langle | V_{\Sigma\Lambda}{}^{(1)} | \rangle|^2/\rho(0) \approx 2\times 10^{-4},$$

with $\rho(0)$ the typical nuclear density. The most important point is that the effect of correlations on the bound-state width is seen to be one to two orders of magnitude greater than the analogous effect for the calculation of the scattering cross section $\sigma_{\Sigma\Lambda}$. For this reason, which is mainly a consequence of the large momentum transfer, one must explicitly include the short-range repulsive correlations in the calculation of the hypernuclear width.

The magnitude of the quantity $N_{\rm eff}$ of Eq. (10) depends upon both the distortions of the K and π projectiles and the structure of the hypernuclear states. However, since both the K^- and π^- mesons are strongly absorbed at medium energies the reactions are mainly at the nuclear surface. Therefore, one can expect that only a few nucleon pairs are effective in contributing to the width. For the (K^-, π^-) reaction a related effective nucleon number for forward scattering was found to be quite small.[10,11] Thus, there should be isolated Σ-hypernuclear states, and only states for which such widths are small are being considered here. The question of the existence of approximate Σ analog states involves two-baryon and many-body symmetry and dynamics, and is the subject of future study.

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