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### Longitudinal and Transverse Inelastic Electron Scattering from <sup>56</sup>Fe

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Inelastic-electron-scattering cross sections for <sup>56</sup>Fe have been measured in the continuum region. The longitudinal and transverse inelastic response functions have been determined for vector momentum transfers, q, from 210-410 MeV/c and for energy losses  $0 \le \omega \le 220$  MeV.

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Deep-inelastic electron scattering can provide significant insight into the constituents of nuclei and their dynamics.<sup>1-5</sup> In general the inelastic (e, e') spectrum results from the excitation of both discrete final states and a continuum region which dominates at higher energy loss,  $\omega$ , and vector momentum transfer, q. The electromagnetic interaction for this region can be separated into two response functions<sup>2</sup>: the longitudinal response function,  $S_{L}(q, \omega)$ , which is Coulomb scattering; and the transverse response function,  $S_{\rm T}(q,\omega)$ , which is a combination of electronic and magnetic scattering. In this Letter we present the first separation of  $S_{\rm L}$  and  $S_{\rm T}$  for both discrete states and the continuum over a significant range of the  $(q, \omega)$  plane. Until it is possible to

perform double or triple-coincidence experiments,  $S_{\rm L}$  is considered the most appropriate source for information on the elusive nucleon short-range pair-correlation function. Deviations from the simple impulse approximation through the appearance of other possible reaction channels can be delineated through the separation; for example, some meson-exchange corrections are expected to be entirely transverse in nature.<sup>6</sup> With a sufficiently large atomic number, some major effects of interest relate to the physics of nuclear matter. Therefore, Fe was chosen as a representative target nucleus.

The  $S_{\rm L}$  and  $S_{\rm T}$  components in (e, e') are formulated using plane-wave Born approximation where the cross section is given by the following equation<sup>2</sup>:

$$\frac{d^{2}\sigma}{d\Omega \, dE'} = \frac{d\sigma_{\text{Mott}}}{d\Omega} \frac{4\pi}{M_{T}} \left\{ \left(\frac{q_{\mu}}{q}\right)^{4} S_{L}\left(q,\omega\right) + \left[\frac{1}{2}\left(\frac{q_{\mu}}{q}\right)^{2} + \tan^{2}\left(\frac{1}{2}\theta\right)\right] S_{T}\left(q,\omega\right) \right\},\\ \frac{d\sigma_{\text{Mott}}}{d\Omega} = \frac{\alpha^{2}\cos^{2}\left(\frac{1}{2}\theta\right)}{4E^{2}\sin^{4}\left(\frac{1}{2}\theta\right)}.$$

The variables are the following: four-momentum,  $q_{\mu}{}^2 = q^2 - \omega^2$ ; incident energy, *E*; target mass,  $M_T$ ; laboratory scattering angle,  $\theta$ ; and finestructure constant,  $\alpha$ . The response functions  $S_L(q,\omega)$  and  $S_T(q,\omega)$  contain, in principle, all the information on the electromagnetic structure of

the nucleus.<sup>1,2</sup>  $S_{\rm L}$  and  $S_{\rm T}$  are determined from measured cross sections by choosing two appropriate angles and energies such that q and  $\omega$  are the same for both angles and energies. In practice we take a set of many incident energies at two angles. We interpolated between the measured data points to obtain the response surface of the cross sections as a function of q and  $\omega$ . Once this surface has been determined  $S_{\rm L}$  and  $S_{\rm T}$  can be extracted as a function of  $\omega$  for any q within the measured range of data.

This experiment was carried out at the Massachusetts Institute of Technology Bates Linear Accelerator. A 75-mg/cm<sup>2</sup>-thick natural-iron target was used and cross sections were measured for scattered electron energies ranging from the elastic peak down to 60-70 MeV final energy. The scattering angles and incident energies were as follows: 90° at 150.3, 200.4, 249.7, 298.7, 320.8, 348.8, and 372.6 MeV; 140° at 100.3, 130.5, 160.3, 190.0, 220.3, 249.8, 291.0, 329.9, and 368.6 MeV; 160° at 100.9, 150.7, 175.4, 200.8, 225.1, 250.3, 299.7, and 368.6 MeV. The absolute cross section was verified at each energy and angle by measuring the elastic e + p scattering using a polyethylene target.

In order to extract inelastic-electron-scattering cross sections from the raw data, radiative corrections must be carried out. The elastic scattering produces a radiative tail that is significant both near the elastic peak and also for final electron energies below 100 MeV, where the tail rises exponentially with decreasing energy. This elastic tail increases as the scattering angle is decreased. To subtract this contribution, the exact radiative correction formalism of Mo and Tsai<sup>7</sup> and Stein *et al.*<sup>8</sup> was employed, giving a maximum of 21% subtraction at the largest  $\omega$  in a  $90^{\circ}$  spectrum. The form factors needed for the elastic tail subtraction were calculated separately for each laboratory scattering angle using a phaseshift solution to the Dirac equation for the Fe ground-state charge density.<sup>9,10</sup> The discrete states were treated as part of the inelastic continuum for this experiment; this is consistent with the peaking approximation formalism used for radiative corrections to the continuum. The peaking approximation is adequate in the continuum as there is no exponentially rising tail at low scattered-electron energies. The choice of equivalent radiator in the peaking-approximation formalism is important and its form was determined by a comparison to the proton elastic tail employing the exact formalism discussed above. The radiative corrections due to the finite-target-thickness effects were always less than 6% from the combined effect of the elastic tail and continuum. The accuracy of our target-thickness corrections were verified by taking measurements on a 161 $mg/cm^2$  target where the radiative effects reached the 30% level.

The measured (e, e') spectra contained background contributions from pair-produced electrons and pions which can be detected in the plastic Cherenkov counter. In order to determine the background the  $(e^-, e^+)$  and  $(e^-, \pi^+)$  yields were measured by reversing the spectrometer field. At the energies and angles of this experiment the  $e^+$  background was observed to be very low above 110-MeV final energy. An  $e^-$  background that was assumed to be equal to the  $e^+$  background was subtracted from the data. The Cherenkov detector eliminated pions below 140 MeV/c. At favorable spectrometer settings the  $\pi^-$  spectrum could be separated from the  $e^-$  on the basis of Cherenkov pulse-height information allowing determination of the  $\pi^-$  to  $\pi^+$  ratio. The momentum dependence of the  $\pi^-$  background was assumed to be the same as that of the  $\pi^+$  which is consistent with pion-photoproduction studies.<sup>11</sup> Thus by using the  $\pi^-$  to  $\pi^+$  ratio and the measured  $\pi^+$  spectra, the  $\pi^-$  background subtraction was either directly determined or computed from extrapola-



FIG. 1. (a),(b)  $S_{\rm L}$  (circles) and  $S_{\rm T}$  (solid triangles) for Fe. The errors are smaller than the points unless otherwise shown and include statistics only. The overall data has an uncertainty of  $\pm 3\%$  coming from the normalization to the proton. All q values are in units of MeV/c; the transverse and longitudinal response functions are dimensionless. For the data at q = 370 MeV/c the results for the Fermi-gas calculations are shown; the dashed line represents  $S_{\rm T}$ , the solid line  $S_{\rm L}$ .

tions using the theory of  $Tsai^{12}$  and normalizing to the measured spectra.

Shown in Figs. 1(a) and 1(b) are the results on  $S_{\rm L}$  and  $S_{\rm T}$  for Fe at q = 210, 250, 290, 330, 370,and 410 MeV/c determined from the data set generated by the  $\theta = 90^{\circ}$  and  $140^{\circ}$  spectra. For the data at 370 MeV/c, where an independent particle model should be more valid, a relativistic Fermigas calculation is shown.<sup>13</sup> This particular Fermi-gas calculation agrees with the previous results<sup>14</sup> on nuclei from Li to Pb.<sup>3</sup> A Fermi momentum,  $K_{\rm F}$ , of 260 MeV/c and an average interaction energy,  $\overline{\epsilon}$ , of 35 MeV were used. These values are consistent with the earlier measurements on unseparated data where the q was higher (450-500 MeV/c). The Fermi-gas model agrees reasonably well with the magnitude and shape of the transverse component  $S_{T}$ , needing only an adjustment in threshold behavior to compensate for the simple averaging of the interaction energy. The large magnitude of  $S_{T}$  at energy losses above the peak is consistent with the unseparated previous results<sup>3,15</sup> and the recent work on <sup>12</sup>C.<sup>16</sup> The prediction that meson-exchange corrections will produce this excess cross section is further confirmed by the present data; the strength at high  $\omega$  is in the transverse component. However, the predictions based on Refs. 6 and 13 for mesonexchange corrections in this region underestimate the magnitude of this cross section by a factor of 2.

The longitudinal component is dominated by competing mechanisms at low values of q; in particular, for Fe the excitation of resonant states appears to be almost entirely longitudinal in nature. An attempt to use an independent-particle model, such as a Fermi gas, is clearly inadequate at q much less than twice the Fermi momentum of the nucleus. A theoretical comparison with this region requires a microscopic description of the Fe nucleus. The observed falloff of overall strength of  $S_{\rm T}$  as compared with  $S_{\rm L}$  as q decreases is expected from electrodynamic differences in the nature of the longitudinal and transverse interactions.<sup>1,2</sup> The large disagreement between the magnitude of the Fermi-gas prediction for  $S_{\rm L}$  and the experimental data is observed for all  $q \ge 300 \text{ MeV}/c$ .

Shown in Fig. 2 are the results<sup>17</sup> for the Coulomb sum rule on  $S_L$ :

$$C(q) = \frac{4\pi}{M_{\rm T}} \int_0^{\omega_{\rm max}} S_{\rm L}(q,\omega) d\omega.$$

The assumption is made that the sum rule is ex-



FIG. 2. The Coulomb sum rule C(q) for Fe compared with prediction for a Fermi gas over the range of experimentally available q.

hausted in the region of quasifree scattering (the physical region of electron scattering requires  $q \ge \omega$ ). The validity of the plane-wave Born approximation was checked by forming response surfaces generated by two different data sets:  $90^{\circ}$ - $140^{\circ}$  and  $90^{\circ}$ - $160^{\circ}$ . The data presented in Fig. 2 required small extrapolation to higher  $\omega$  in a few cases; the error contains all systematic uncertainties in any interpolation or extrapolation. The close agreement between the two sets is also observed in the individual function  $S_{\rm L}$  and  $S_{\rm T}$ , indicating that Coulomb distortions in the continuum region are not significant at the present level of experimental accuracy.

The Fermi-gas prediction for the Coulomb sum rule is shown in Fig. 2 in the region of q where the independent-particle model should be valid. The magnitude of C(q) is different from the number of protons (26) for the following simple reasons: The maximum separation q = 410 MeV/cis still lower than  $2K_{\rm F}$  and Pauli correlations cause a reduction; there is a large additional reduction form the internal nucleon motion, its recoil and form factor. Note that our definition of C(q) does not divide out the nucleon form factor. The interest is not in these effects but in the deviation from our present understanding of the nucleus, in particular, the short-range pair-correlation effect on the sum rule. Previous theoretical estimates<sup>17</sup> for the reduction of strength for C(q) due only to pair correlations have indicated very small effects, less than 5%. At the high q, we now observe almost a factor-of-2 difference between experiment and the Fermi-gas model.

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It is possible that final-state interactions are responsible for some of this discrepancy, but again preliminary results<sup>18</sup> indicate only a (5-10)% reduction. There may exist significant strength in  $S_L$  at high  $\omega$  that we are missing, but that is considered unlikely. It is clear that the data should be extended to higher q approaching  $2K_F$  so that we have greater confidence in our independentparticle model and avoid any complication from Pauli correlations.

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#### Widths of $\Sigma$ -Hypernuclear States

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The widths of states of  $\Sigma$ -hypernuclei have been estimated in two models. Widths as small as one to a few megaelectronvolts are found, a result which should hold for some isolated simple  $\Sigma$ -hypernuclear states. The standard procedure by use of the two-body absorptive cross section is shown to overestimate the width by one or more orders of magnitude.

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Recently  $(K,\pi)$  experiments with nuclear targets leading to  $\Sigma$ -hypernuclei have been carried out,<sup>1, 2</sup> and one anticipates that there might soon be considerable new data with improved energy resolution. The crucial question governing the future of this field of study is the width of  $\Sigma$ -hypernuclear states. The conventional wisdom is that the  $\Sigma \rightarrow \Lambda$ conversion in nuclear matter will lead to a typical strong interaction width of 50–150 MeV for these states. If that is true the study of  $\Sigma$ -hypernuclei will be very limited in comparison with  $\Lambda$ hypernuclei. The present data,<sup>2</sup> however, show that there are states with widths less than 10 MeV. It is the purpose of the present note to demonstrate that such small widths can be expected for isolated  $\Sigma$ -hypernuclear states, and that the study of  $\Sigma$ -hypernuclei should be most rewarding.

The main calculations are estimates of the width of a  $\Sigma$ -N state produced in a (K<sup>-</sup>,  $\pi^-$ ) reaction on a N-N cluster in a nucleus. Two models are used. First, the width is estimated using plane-wave final  $\Lambda$ -N states. Since the final center-of-mass momentum is approximately  $2m_{\pi}c$ , this should be adequate for the purposes of the present work. In the second calculation the  $\Sigma$ -N  $\rightarrow$   $\Lambda$ -N width is estimated by means of the model of broken SU(3) sym-