Development of Atmospheric Cosmic-Ray Showers

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In a recent paper Thornton and Clay strongly suggest a very early development in the atmosphere for small cosmic-ray showers and consequently require a change in primary mass composition in the energy range $10^{15}-10^{17}$ eV. It is suggested here that when allowance is made for inconsistencies in the data the necessity for the early development of small showers—and hence the inferred rapid change in primary composition—is removed.

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Thornton and Clay[1] have claimed evidence for a change of the primary cosmic-ray chemical composition between 10^{15} and 10^{17} eV per nucleus. This claim is very important because the nature of the primary particles above about 10^{14} eV is one of the big unsolved problems in cosmic-ray physics. The basis for their claim was the depths of shower maximum inferred from the measured width (in time) of the atmospheric Cherenkov radiation signal in individual cosmic ray showers using a single detector. Specifically, the depths of maximum increase from 440 g cm^{-2} at $\sim 10^{15}$ eV to 680 g cm^{-2} at $\sim 10^{17}$ eV.

The fundamental assumptions of their interpretation are in (i) the form of the variation of the width of the Cherenkov signal with the distance from the core at which the measurement was made (this relationship is important because it allows measurements made with a single detector at varying core distances to be normalized to a prescribed core distance) and (ii) the form of the relation between the full width half maximum (FWHM) of the light pulse at such a fixed core distance (300 m has been chosen) and the depth of the electron cascade maximum in the shower.

This second assumption can be made only on the basis of computer simulations of shower development. The authors have chosen to use the relation subsequently published by Kalmykov *et al.*[2] [The expression quoted is incorrect and should read the following: Height of maximum $= 17.05 - 9.17 \log_{10} \tau(300 \text{ m})$ km, where $\tau(300 \text{ m})$ is the FWHM in nanoseconds.] Implicit in the derivation of this expression was a power-law relationship of the form FWHM $\propto r^n$, where n is between 1.7 and 2.4, for core distances in the range $300 < r < 600$ m (Kalmykov *et al.*[2]). For the depths of maximum appropriate to the small showers in question, the appropriate index quoted by Kalmykov *et al.* is close to 2.0. Other calculations (Protheroe and Turver,[3] McComb and Turver[4]) are in agreement with this value and, further, indicate its continued validity to smaller distances.

Clay and Thornton, however, used a power-law relation for the FWHM with an exponent $n = 1.4$. This choice was made on the basis of a multiple regression analysis of measurements with a single detector in individual showers of varying electron size, differing zenith angle, and over a range of core distances 150–350 m (J. R. Prescott, private communication). Cosmic-ray shower studies with use of a single detector are known to be difficult because of the inability to remove unambiguously the strong r dependence of the measured quantity within an individual shower and we regard the choice of $n = 1.4$ to be incorrect. Thornton and Clay, as co-authors of a contemporaneous paper discussing the same data sample, have assumed an alternative relation of the form $a + br^2$ (see Thornton *et al.*[5]). Such a strong r dependence has been measured in individual showers with an array of eight detectors by our own group (Hammond *et al.*[6]) in showers of energy $10^{17}-10^{18}$ eV over the core-distance range 150–500 m and was confirmed by our simulation predictions (see, e.g., Protheroe and Turver[3]).

The consequence of adopting too small a value for the exponent n in the power-law relationship would be to systematically underestimate the depth of cascade maximum when the recorded core distance is less than 300 m. The underestimation would be greatest for measurements nearest the core which in most air shower arrays are in the smaller showers of the sample [this was clearly so for the Adelaide data reported in 1978 (see Thornton and Clay[7])] and the effect is in the direction of removing the need to postulate a change in primary mass.

We wish also to point out the differing sensitivity of the FWHM at various core distance to the depth of cascade maximum. The variation of FWHM with cascade maximum depth for core distances in the range 100–350 m has been calculated.[4] There is a strong sensitivity at core distances of 300 m ($\simeq 10$ ns per 100 g cm^{-2}) where previous measurements have purposely been centered (see Hammond *et al.*,[6] Kalmykov *et al.*[2]). At 200 m this sensitivity decreases to ~ 3.5 ns per 100 g cm^{-2} and measurements made at 100 m are predicted to be practically invariant with depth of maximum. Measurements made at distances as small as 150 m (as is almost inevitable in the smaller showers) are, regardless of any systematic effects due to normalizing to a distance of 300 m, clearly much less sensitive to changes in depth of maximum. Increasingly large statistical uncertainties would thus be expected for the derived depths of maximum in smaller showers.

Furthermore, the precise pressure-altitude relationship for the atmosphere could be a cause of further uncertainty in the ascribed depth of maximum. For example, the depths appropriate to an altitude of 6 km above sea level according to the atmosphere used by Clay and Thornton and that quoted in Handbook of Geophysics[8] are 430 and 492 g cm^{-2}, respectively. This difference, 62 g cm^{-2}, alone represents a substantial part of the claimed effect at $\sim 10^{15}$ eV.

The difficulty in making and interpreting useful measurements in small showers is illustrated by the following example. Consider that showers have a maximum at a depth of 680 g cm^{-2} for a sea-level size of 2×10^7 electrons and that the depth of maximum changes by ~ 85 g cm^{-2}/decade of primary energy (or sea-level size) for unchanging mass composition. For a size of 2×10^5 particles such as a conservative model predicts a depth of maximum of ~ 500 g cm^{-2} (5.8 km altitude according to a 30° latitude winter atmosphere[8]) for which the Kalmykov conversion relation gives a $\tau(300\text{ m})$ of 16.8 ns. If such measurements are made at 150–200 m, the analysis procedures used by Thornton and Clay (i.e., $n = 1.4$) would suggest a true pulse width of 7.9 ns, measured as 9.5 ns with an Adelaide-like 5.3 ns system. Thornton and Clay claim a depth of 430 g cm^{-2} (6.0 km using their exponential atmosphere) based on a FWHM at 300 m of 16.0 ns, in turn derived from a measurement (similarly assumed to be at 150–200 m) of 9.2 ns with a 5.3 ns system. Thus an alternative explanation *may* exist not requiring an unconventional mass composition (i.e., a rapid change in composition with energy).

There is apparently a further inconsistency. In an earlier paper, Thornton and Clay[7] have stated "We can say that our best estimate of the FWHM (300) for showers of mean size 5.5×10^5 is 20 ± 1 ns." Such a statement, combined with the relation of Kalmykov *et al.*[2] and their exponential atmosphere suggests a depth of maximum of 486 ± 13 g cm^{-2}. This conflicts with the lower value of about 440 g cm^{-2} given by Thornton and Clay[1] and is, in fact, close to what we would expect for a conventional composition.

Clay and Thornton claim support for their interpretation from the only other estimate of shower maximum at small sizes—that by Antonov and Ivanenko.[9] However, Antonov, Ivanenko, and Kuzmin[10] make clear the large statistical and interpretive uncertainties in reducing the measurements. The errors in assigned depths of maximum are at least 100 g cm^{-2} and this value is bigger than the difference between the conventional and unconventional mass compositions. Therefore such data cannot be taken as supporting evidence for Clay and Thornton's interpretation of their data.

To summarize, our criticisms of the conclusion of Thornton and Clay are fourfold. Firstly, we note an apparent inconsistency in the derivation of depths of maximum resulting from incomplete use of model predictions. Secondly, we draw attention to the relative insensitivity to depth of maximum of measurements closer to the core than 300 m in the smaller showers (which is where most of the measurements on small showers are made). Thirdly, there exists an alternative model of the atmosphere which may be more reliable. Finally, we note an apparent conflict in the basic data quoted in an earlier paper and those inferred from the recent publication. The effect of employing the alternatives would in each case increase the ascribed depths of maximum for showers and give better accord with the conventional mass composition.

We conclude that there is no firm evidence at present for the suggestion of a rapid change in the depth of maximum of cosmic-ray showers of the size range 10^5–10^7 particles implying a rapidly changing mass composition. Further, we emphasise the difficulties in applying Cherenkov-light techniques, appropriate to large showers, to the (less-sensitive) smaller showers.

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