the already known $10^+$ states. The single-particle wave functions have a positive slope at $R=0$. The transition densities were calculated following Lee (Ref. 8), with an additional phase of $(-1)^j p^{-j} h$.

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# Autoionization Accompanying Emission of Internal Bremsstrahlung in Beta Decay

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$K$-shell autoionization accompanying internal bremsstrahlung in the decay of $^{204}$Tl has been observed. Its probability was determined to be $P_{IB,K}=(8\pm3)\times10^{-8}$ per $\beta$ decay for internal bremsstrahlung in the energy range from 88 to 394 keV. A comparison with theoretical expectations is presented.

In our recently reported measurement[1] of the internal bremsstrahlung (IB) in the $K$-capture decay of $^{204}$Tl a continuous photon spectrum was observed in coincidence with lead x rays. In the present paper we show that the origin of this observation is simultaneous $K$-shell autoionization and emission of IB in the $\beta^-$ decay of $^{204}$Tl. Autoionization accompanied by IB in $\beta$ decay has not been reported before, nor have theoretical predictions of this effect been published, to our knowledge.

The experimental setup and procedures were described before.[1] Coincidences between $K$ x rays recorded by a Ge(Li) detector and $\gamma$ rays detected by a NaI(Tl) crystal were analyzed and stored in a two-parameter mode. The x-ray spectrum summed over the whole energy range of the IB is shown in Fig. 1(a). The prominent Hg $K$–x-ray peaks are due to coincidences with IB accompanying the $K$-capture decay of $^{204}$Tl.

The presence of the Pb x rays may be due to several processes which had to be carefully examined. The contributions due to scattering of the Hg $K\beta$ x rays from the collimator located between the source and the Ge(Li) detector, contributions of Pb x rays excited within the source and in the lead bricks of the housing as well as the contributions due to source impurities were found negligible. The only other contribution could arise from autoionization followed by external bremsstrahlung (EB) produced by the $\beta$ particles interacting with matter surrounding the source. The experimental setup could not distinguish between EB and IB which occur in coincidence with the Pb $K$ x rays following autoionization.

In order to prevent the $\beta$ particles from reaching the detectors it was necessary to sandwich

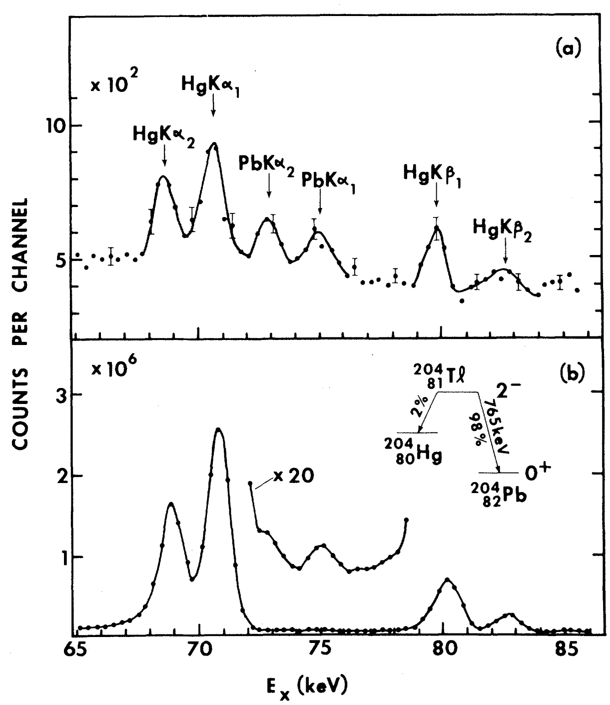


FIG. 1. (a) The x-ray spectrum in coincidence with IB in the energy interval 88–600 keV obtained in 329.5 h. (b) The x-ray singles spectrum. The inset shows the decay scheme of $^{204}$Tl.

the source between 320-mg/cm$^2$-thick beryllium filters. Since the source was very thin, the EB was produced by electrons interacting with Be. To account for contributions to the spectra due to EB, the Be filters were replaced by 320-mg/cm$^2$-thick filters of Al and Cu. Since the intensity of EB is proportional to the atomic number of the material in which the $\beta$ particles are stopped, the contribution of the EB to the data should be proportional to the atomic number of the filter. Thus by extrapolating the data obtained with filters of different atomic numbers to a filter of atomic number 0, one obtains a result free of contributions from coincidences of EB with Pb $K$ x rays.

The intensities of the Pb $K\alpha_1$ x rays obtained by summing the coincidence spectra over three 102-keV intervals of the IB spectrum are plotted versus the atomic number of the filter in Fig. 2. The straight lines were obtained by weighted least-squares fits to the experimental points. The spectra were corrected for random coincidences, which constituted less than 5% of the intensity of the Pb $K\alpha$ peaks, absorption of the x rays and

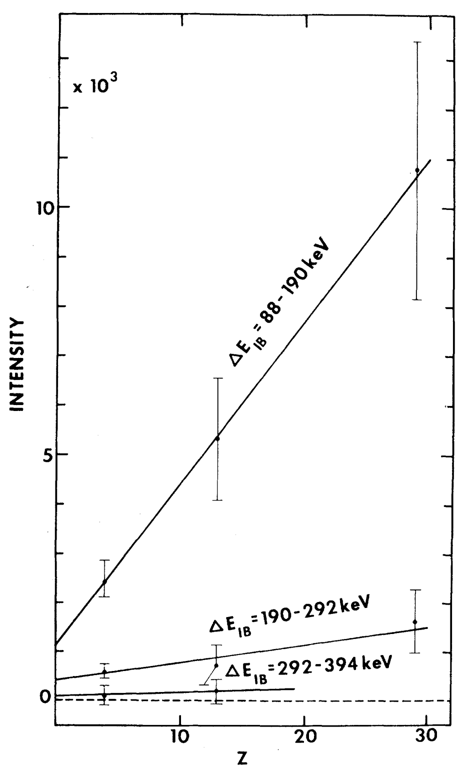


FIG. 2. The intensity in relative units of Pb $K\alpha_1$ x rays integrated over three energy intervals of the IB vs the atomic number of the filter, for filters of beryllium, aluminum, and copper.

bremsstrahlung in the filters, and the efficiency of the NaI(Tl) detector. The continuous background on which the x rays are superimposed was substracted as described in Ref. 1. The rapid increase of this background with the atomic number of the filter made it impossible to determine the intensity of the weak x-ray peaks in coincidence with the 292–394-keV bremsstrahlung in the run with the copper filter.

The Pb $K\alpha_2$–x-ray peak was not included in the calculation because it may contain a contribution from $K\alpha_1$ x rays of Tl excited within the source. The magnitude of this contribution could not be reliably estimated. The Tl x rays would be in coincidence with external as well as internal bremsstrahlung.

It is convenient to express the probability of IB accompanying $K$-shell autoionization in $\beta$ decay since this fraction does not depend on the efficiency of the Ge(Li) detector. The latter probability was determined from the intensity of the Pb $K\alpha_1$ x rays in the singles spectrum shown in Fig. 1(b) to be $P_K = (1.0 \pm 0.8)\times 10^{-4}$. This value is based on the $K$-capture to $\beta^-$ branching ratio of 0.01525 $\pm$ 0.002 of Leutz and Ziegler[2] and is in very good agreement with the results of other measurements.[3,4] The ratio $P_{IB,K}/P_K$ is given in Table I for three energy intervals of the IB. Using the value of $P_K$ determined in the present experiment, one obtains, for the total probability of the effect per $\beta$ decay in the energy interval studied, $P_{IB,K} = (8 \pm 3)\times 10^{-8}$. The measured probability[5,6] of IB is also given in Table I. For the whole energy range covered in the present experiment $P_{IB} = (9.4 \pm 0.8)\times 10^{-4}$. Thus the magnitude of $P_{IB,K}$ is very close to the product $P_K P_{IB}$.

To facilitate the theoretical evaluation of $P_{IB,K}$ we assume that the observed effect is entirely due to $K$-electron shakeoff and simultaneous emission of internal bremsstrahlung in the $\beta$-decay process. The probability of emission of IB in $\beta$ decay was originally calculated by Knipp and Uhlenbeck[7] and by Bloch[8] (KUB) using perturbation

TABLE I. Comparison of the ratio $P_{IB,K}/P_K$ with $P_{IB}$ in the $\beta^-$ decay of $^{204}$Tl.

| $E_{IB}$ (keV) | $10^4 P_{IB,K}/P_K$ | $10^4 P_{IB}$[a] |
|---|---|---|
| 88–190 | 5.6 ± 2.7 | 6.6 ± 0.7 |
| 190–292 | 2.1 ± 0.9 | 2.0 ± 0.2 |
| 292–394 | 0.3 ± 0.9 | 0.7 ± 0.1 |

[a]Average of values from Refs. 5 and 6.

theory. To account for autoionization accompanying emission of IB it is necessary to modify the KUB calculation and include the wave function of the atomic 1$s$ electron in the initial state of the system undergoing decay and the wave function of the ejected atomic electron in the intermediate and final states. Ejection of an electron from the atom occurs as a result of the sudden change of the Coulomb potential during the $\beta$ decay[9,10] to an intermediate state. The bremsstrahlung photon is radiated by the $\beta$ particle in the transition from the intermediate to the final state. The matrix element of the process is

$$M=\sum_l \frac{\langle\psi_f|H_\gamma|\psi_l\rangle\langle\psi_l|H_\beta|\psi_i\rangle}{W-W_l}, \quad (1)$$

with

$$\psi_i=\psi_e(K,Z)\psi_\nu\psi_i(N),$$
$$\psi_l=\psi_e(W_e,Z)\psi_\beta(W_\beta{}')\psi_f(N),$$
$$\psi_f=\psi_e(W_e,Z')\psi_\beta(W_\beta)\psi_\gamma(k)\psi_f(N),$$

where $\psi_e(K,Z)$ and $\psi_e(W_e,Z')$ are the wave functions of the $K$ electron and the electron ejected with energy $W_e$, respectively; $Z$ and $Z'$ are the initial and final atomic numbers; $\psi_\gamma(k)$, $\psi_\nu$, and $\psi_\beta$ are the wave functions of the photon, the neutrino, and the $\beta$ particle; $W_\beta{}'$ and $W_\beta$ are the energies of the $\beta$ particle in the intermediate and final states, respectively; $W_0$ and $W_l$ are the total energies in the initial and intermediate states; $H_\beta$ and $H_\gamma$ are the Hamiltonians of the weak interaction and the interaction of the $\beta$ particle with the electromagnetic field. Antisymmetrization of the wave functions of the two electrons in the intermediate and final states would not change the result of the calculation appreciably. This is due to the fact that the contribution to the intensity of the IB due to radiation by the ejected atomic electron is negligible since its energy is small on the average compared to the energies of the $\beta$ particles which contribute to the bremsstrahlung spectrum above 88 keV.

The calculations are greatly simplified if $E_e+B_K\ll E_0$, where $E_e$ is the kinetic energy of the ejected atomic electron, $B_K$ is its binding energy, and $E_0$ is the maximum kinetic energy of the $\beta$ particle in ordinary $\beta$ decay. When this condition is satisfied one can neglect the dependence of the density of final states of the $\beta$ particle, the antineutrino and the IB photon upon $E_e$ and $B_K$. This makes factorization of the density of final states possible. With use of the formalism and notation of Chang and Falcoff,[11] Lewis and Ford,[12] and Stephas and Crasemann[10] the probability per $\beta$ decay that autoionization and emission of an IB photon will occur simultaneously can be written, after separation of the atomic and nuclear variables in the matrix element, as

$$P_{\text{IB},K}=P_{\text{IB}}P_K \quad (2)$$

with

$$P_K=\pi^{-2}\int|\langle\psi_e(W_e,Z')|\psi_e(K,Z)\rangle|^2(W_e{}^2-1)^{1/2}W_e\,dW_e, \quad (3)$$

$$P_{\text{IB}}=(2\psi)^{-8}N^{-1}\int k^2dk\int dW_\beta(W_0-W_\beta)^2(W_\beta-k)[(W_\beta-k)^2-1]^{1/2}\int d\Omega_\beta\int d\Omega_\nu\int d\Omega_k|M_c|^2, \quad (4)$$

where $N$ is the probability of ordinary $\beta$ decay and $M_c$ is the matrix element remaining after the overlap integral appearing in Eq. (3) is factored out from $M$ [Eq. (1)]. The integrals over $\Omega_\nu$, $\Omega_k$, and $\Omega_\beta$ indicate integration over the direction of the antineutrino, the photon, and the $\beta$ particle as well as summation over their polarizations.

The probability of $K$-electron shakeoff, $P_K$, is in the form given by the early theoretical calculations of the effect.[9,13] In their simplest form, with the overlap integral evaluated by use of nonrelativistic hydrogenic wave functions, these calculations give[13] for $^{204}$Tl, $P_K=0.97\times10^{-4}$.

Expressions for $P_{\text{IB}}$ based on Eq. (4) have been derived by KUB for allowed $\beta$ transitions, and by Chang and Falcoff,[11] and Madansky *et al.*[14] for first-forbidden ones. In the case of $^{204}$Tl the calculated probability of emission of IB in the energy range 88 to 394 keV with Coulomb corrections included[15] is $P_{\text{IB}}=7.6\times10^{-4}$.

The condition $E_e+B_K\ll E_0$, which made possible the factorization of $P_{\text{IB},K}$ is not well fulfilled for $^{204}$Tl, in which case, on the average, $E_e+B_K=0.23E_0$. When this condition is not assumed to hold and $E_0$ is replaced by $E_0-E_e-B_K$ in the unfactorized expression for $P_{\text{IB},K}$, one obtains by a numerical calculation a value for $^{204}$Tl smaller by about a factor of 3 than the value given by the factorized expression. This result is primarily a consequence of the very strong dependence of the intensity of the IB upon $E_0$. Thus the good agreement which was obtained with the experimental result using the factorized expression is fortuitous.

The small theoretical value may be because we neglected contributions to the internal ionization due to direct collisions of the $\beta$ particle with the $K$ electrons and contributions due to internal Compton scattering of the IB on the $K$ electrons.

The contribution of the direct-collision mechansim to autoionization in $\beta$ decay has been estimated by Feinberg[9] as $B_K P_K/E_\beta$, which gives a lower limit in the case of $^{204}$Tl of $\sim 0.1P_K$. Although this is a lower limit only, a comparison of the experimental values of $P_K$ with existing theoretical calculations[11] which neglect the collision effect indicates that this contribution is unlikely to be appreciably higher.

An order-of-magnitude estimate of the contribution of the internal Compton scattering of the IB can be obtained by use of Compton-scattering cross sections on the $K$ electrons. Assuming hydrogenic wave functions for the $K$ electrons one gets for the contribution of this effect a value of $\sim 0.1P_K$.

It should also be noted that the KUB calculations underestimate the intensity of the IB in the case of $^{204}$Tl by about 20%. The agreement between the measured intensity of the IB in $^{204}$Tl and the predictions of the theory has been shown[16] to be significantly improved when detour transitions are taken into account.

Thus the total contribution of mechanisms other than shakeoff and emission of IB in transitions via intermediate electron states, neglecting interference effects, may amount to $\sim 40\%$ of the value of $P_{\mathrm{IB},K}$ estimated on the assumption that this is the only effect present.

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# Sequential-Scattering Model for Relativistic Heavy-Ion Collisions

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A microscopic model, constructed within the framework of the Boltzmann equation, is formulated and used to calculate particle spectra from high-energy nucleus-nucleus collisions. Formation of composite particles is treated according to Hagedorn's statistical-thermodynamical approach of strong interactions. Inclusive proton, deuteron, triton, and $^3$He cross sections from 400-MeV/nucleon $^{20}$Ne + $^{208}$Pb are calculated and found in good agreement with experiment.

Recent experiments at the Bevelac on medium- and high-energy heavy-ion collisions have produced impressive inclusive measurements of double-differential cross sections.[1] They were obtained for protons, light ions, and also pions over a large momentum and angular range and supplement the first results[2] for this type of heavy-ion reaction. Attempts for a theoretical description[3] of the measured proton spectra have been based mainly on two different assumptions.