when $r \to \infty$]. If S_ν decreases faster than any exponential, $G_{\mu\nu}$ will do the same.

In a former work[3] we dealt with the same problem but we made different assumptions. In particular, the gauge potential A_μ was assumed to have the representation

$$A_\mu(\vec{x},t) = \int_{m_0}^{\infty} dm \exp[-mf(r)]A_\mu(m,\theta,\varphi,t) \quad (12)$$

and $f(r)$ could be, for example, r^η, $\eta > 0$. Here, we made the assumption that the *field* $G_{\mu\nu}$ is decreasing like $\exp(-mr)$. The possibility that A_μ is not *simultaneously* decreasing in all directions when $G_{\mu\nu}$ does is left open. No representation like Eq. (12) is assumed; we only deal with tempered distributions.

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Vibrational States in the Υ Spectroscopy

W. Buchmüller and S.-H. H. Tye
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
(Received 13 December 1979)

Vibrational states, which are expected (in the string picture) in addition to ordinary $(Q\overline{Q})$ bound states, are investigated in the Υ spectroscopy by use of a phenomenological approach. The lowest 1^{--} vibrational state is predicted to be below the continuum threshold, between the radial excitations Υ'' and Υ'''. Its mass is estimated to be 990 ($\pm$ 90) MeV above Υ with a leptonic width of 0.20 ($\pm$ 0.15) keV.

Current knowledge of the ψ family[1] clearly indicates that, to a good approximation, quarkonium systems can be described as nonrelativistic quantum-mechanical bound states of a heavy quark-antiquark pair. Many potential models have been suggested to describe quarkonium systems, and their agreement with experimental data is good.[2,3]

It was pointed out earlier[4] that we should expect energy levels in any quarkonium system in addition to those given by potential models. The expectation of their presence in quarkonium spectroscopy is suggested by rather general considerations:

(1) In theories of quark confinement, a confining potential between a quark-antiquark pair arises from a string (or tube) of color-electric flux linking the quark and the antiquark. Such a string presumably emerges as a consequence of the strong self-interaction of the colored gluons in quantum chromodynamics. Relativistic invariance requires the embedding of such a string in Minkowski space to be covariant. This is possible only if the energy-momentum localized on the string is associated with dynamical degrees of freedom. The extra states we mentioned earlier are simply the quantized excitations of such modes. They correspond to the vibrations of the string.

(2) Typical masses of light mesons are large compared to the masses of the bound quarks. The level degeneracy of the mesonic mass spectrum increases exponentially,[5] i.e., $\sim \exp(cE_{\text{had}})$, where c is a constant and E_{had} is the hadronic mass under consideration. This suggests that the level degeneracy of a quarkonium system may also increase much faster than one expects on the basis of a potential model. The string model clearly demonstrates this.

To illustrate this point, we present the Υ spectroscopy,[6] as it results from our calculation, in the form of a Chew-Frautschi plot shown in Fig. 1 (fine and hyperfine structures have been ignored; details of the calculation will be described below). The quantization of the vibrational modes yields extra states in addition to those given by ordinary potential models. All states carry a "vibrational" quantum number $v = 0, 1, 2, \ldots$ in addition to the principal quantum number n (corresponding to radial excitations) and the orbital angular momentum L. We observe that at energies below 10.3 GeV, the spectrum contains only nonvibrational states and hence a potential model is

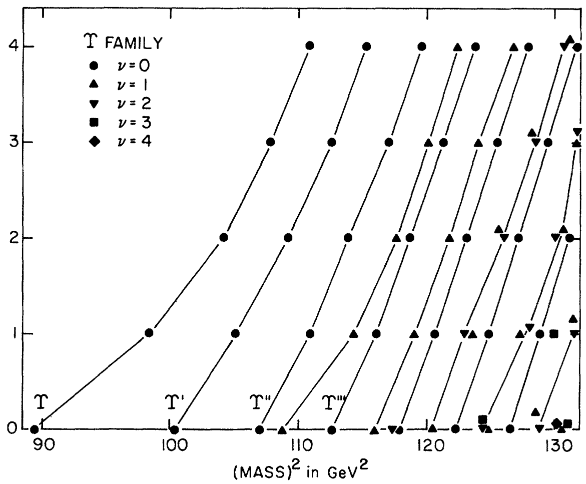

FIG. 1. The Chew-Frautschi plot of the γ family where spin effects have been neglected. L is the orbital angular momentum and v the vibrational quantum number. The lines linking states with different L are drawn to guide the reader's eyes.

sufficient. As we go to higher energies, two phenomena occur:

(a) The Regge slopes start increasing. For $L \geqslant 2$, the Regge slopes for both the leading trajectory and the daughter trajectories are roughly parallel. The smallness of the Regge slopes at low energy is due to heavy-quark-mass effects and the deviation of the potential from a pure linear one at small distances. The asymptotic Regge slopes approach that of light mesons (i.e., $\alpha' \sim 1$ GeV^{-2}) since both of the above effects become negligible for large values of L.

(b) Vibrational states show up in the spectroscopy. As energy increases, the number of vibrational states increases much faster than the number of nonvibrational states. At high energies, the level degeneracy of vibrational states clearly dominates. This pattern of the Υ family resembles that of light mesons when their masses are much bigger than the quark masses involved. Of course, the final exponential degeneracy is not accessible to a nonrelativistic potential model since the validity of the latter breaks down before we reach such energies. However, even at low energies where its application (at least qualitatively) is supposed to be valid, a potential model may have too few states to be compatible with experiments. We note that the qualitative features of the Chew-Frautschi plot remain unchanged if we use another potential as long as it has the following two features: (i) It fits the ψ spectroscopy and the Υ'-Υ splitting, and (ii) it approaches a linear potential at large distances.[3,7]

The continuum threshold in the ψ spectroscopy occurs at 3.7 GeV. Hence the first vibrational level ψ_v in the ψ family appears as a broad resonance. For the Υ spectroscopy, the first 1^{--} vibrational level occurs around the Υ'' and is expected to be below the continuum threshold.[8] Unhampered by background threshold effects and Okubo-Zweig-Iizuka-allowed decays, its identification should be less ambiguous than that in the ψ spectroscopy. In this Letter, we attempt to estimate, in a phenomenological approach, more precisely its position and leptonic width, and to obtain a better understanding of the types of uncertainties involved. This would be useful in differentiating a vibrational level from any other extra level arising, for example, as a consequence of S-D mixing.

The earlier estimate of the lowest 1^{--} vibrational state (at 4.0 GeV) in the ψ spectroscopy[4] was carried out in the quark-confining string model.[9] In its simplest version, the nonrelativistic approximation leads to a pure linear potential, which is clearly not sufficient to describe the ψ and Υ spectroscopies. A more detailed application[10] of the quark-confining string model is needed to give a realistic comparison with experiments.[11]

Here we adopt a phenomenological approach: We choose a potential, interpolating between a Coulomb and a linear potential at small and large distances respectively, which describes the ordinary ($Q\bar{Q}$) bound states[12]; the effective potential, determining the vibrational states, is then obtained by changing the linear part of the potential as calculated by Giles and Tye.[4] There the vibrational potential $V_v(r)$ behaves like r for large distances and approaches a constant for $r \to 0$. It has been derived by quantizing vibrational modes in the simplest version of the quark-confining string model, with use of a variational approximation. Here we need a quantization of vibrational modes in the more complicated situation where the potential for nonvibrational states involves a Coulomb term for small distances. This may lead to a more singular behavior of $V_v(r)$ for $r \to 0$. In quantum chromodynamics (QCD), one may visualize a vibrational state as a coherent color excitation. Because of the strong self-interaction of the gluons, the quark-antiquark potential may be changed even at short distances as compared to the nonvibrational state. This and dimensional arguments support the possibility of different Coulomb potentials in vibrational and nonvibrational states. We will make use of this possibility in our phenomenological analysis. The picture of a coherent color excitation also gives a hint for the dominant decay of a vibrational state. In quantum electrodynamics, any possible vibration will be damped immediately by the radiation of soft photons. In QCD, confinement prohibits the emission of soft gluons; we may, however, expect a decay via the emission of soft pions, e.g., $\Upsilon_v \to \Upsilon\pi\pi$.

In order to determine how sensitive our calculation of vibrational energy levels is with respect to the chosen potential, we have carried out our phenomenological approach for the Cornell potential[3] $V_{\rm CL}$ and the Bhanot-Rudaz potential[7] $V_{\rm BR}$. Note that the Coulomb part of $V_{\rm CL}$ has to be modified in the Υ range in order to obtain the observed splitting $M(\Upsilon')-M(\Upsilon)=560$ MeV. If we start from the Cornell potential $V_{\rm CL}$ the effective potential for vibrational states is given by

$$V^{\rm vib}(r)=V_{\rm CL}(r)+[V_v(r)-r/a^2]+(A_v/r),$$

where $V_v(r)$ is given in Ref. 4 and r/a^2 is the linear potential piece in $V_{\rm CL}$. A_v is a constant, small compared to the Coulomb part of $V_{\rm CL}$; it is adjusted to fit the ψ_v data.

The first two (1^{--}) vibrational states in the ψ spectroscopy are estimated to be at 4.0 and 4.4 GeV.[4] We may therefore identify the first vibrational state with the well-established resonance at 4.03 GeV ($\Gamma_{ee}\sim 0.7$ keV).[1] There are also some indications for a small resonance at 3.96 GeV ($\Gamma_{ee}\sim 0.3$ keV) in the e^+e^- annihilation channel.[1] Hence we can adjust A_v to fit either state. The mass and the leptonic width of the vibrational states in the Υ family are then predicted. We have carried out the same analysis for the Bhanot-Rudaz potential $V_{\rm BR}$, which we also used for the calculation of the Chew-Frautschi plot (with ψ_v at 4.03 GeV as an input for the 1^{--} vibrational state). Our results for the lowest 1^{--} vibrational state Υ_v are summarized in Table I. We note that Υ_v may mix with the 3^3S_1 and the 2^3D_1 states to produce Υ'' and Υ_v and possibly a third state with a very small leptonic width. If this is the case, the position and leptonic width of Υ_v are even harder to estimate. Taking these uncertainties[13] into account, we obtain

$$M(\Upsilon_v)-M(\Upsilon)=990\pm 90 \text{ MeV},$$

$$\Gamma_{ee}(\Upsilon_v)=0.20\pm 0.15 \text{ keV}.$$

In this energy region (below the continuum threshold), S-D mixing is expected to be small (this can be verified by a coupled-channels analysis).

TABLE I. The predictions of the mass and leptonic width of the lowest 1^{--} vibrational state Υ_v in the Υ spectroscopy. The Υ, Υ' masses and the leptonic width of Υ are inputs for the two potentials, with two choices for $m(\psi_v)$ and Γ_{ee}. The masses are given in gigaelectronvolts, and the leptonic widths under the masses in brackets are given in kiloelectronvolts. $m(\Upsilon)=9.460$ GeV.

Input \ Model	$V_{\rm BR}$	$V_{\rm CL}$
$m(\psi_v)=3.96$ GeV,	10.36	10.38
$\Gamma_{ee}=0.3$ keV	(0.10)	(0.07)
$m(\psi_v)=4.03$ GeV,	10.42	10.49
$\Gamma_{ee}=0.7$ keV	(0.27)	(0.12)

The next 1^{--} resonance is the 4^3S_1 state which is expected to be close to the continuum threshold (~ 10.60 GeV). If it is below the continuum threshold, then the first 1^{--} state above threshold would be the second vibrational excitation, Υ_v'. We estimate its mass to be $M(\Upsilon_v')=10.80$ (± 0.10) GeV. This resonance may provide a convenient source for B^* and B mesons.

The rapid increase of the density of vibrational states as energy increases will lead to a smooth approach to the scaling region which we expect to set in at about 1 GeV above the continuum threshold.

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Search for Narrow $\bar{N}N$ States near Threshold

C. Amsler,[a] B. D. Dieterle, J. Donahue,[b] C. C. Herrmann, C. P. Leavitt, and D. M. Wolfe
Department of Physics, University of New Mexico, Albuquerque, New Mexico 87131

and

L. B. Auerbach, N. Haik, V. L. Highland, E. Jastrzembski, and W. K. McFarlane
Department of Physics, Temple University, Philadelphia, Pennsylvania 19122

and

M. A. Mandelkern, D. C. Schultz, and J. Schultz
Department of Physics, University of California, Irvine, California 92717
(Received 14 November 1979)

A missing-mass experiment in $\bar{p}d \to NX$, where X is the system whose mass M_X is measured, was performed for interactions of antiprotons at rest and in flight below 500 MeV/c. The 4-standard-deviation upper limit for the production of narrow states, in the mass interval $M_X = 1650-1930$ MeV/c^2, is 2 mb for annihilations in flight and 0.3% of the annihilations at rest.

We have performed a missing-mass experiment in $\bar{p}d \to NX$ using a new method at the Brookhaven National Laboratory alternating-gradient synchrotron on the new low-energy separated beam, LESB II. The missing mass recoiling against a neutron (or a proton) was measured for antiproton annihilations in flight and at rest, in a long liquid deuterium target. The experiment permitted us to investigate the formation of $\bar{p}p$ (and $\bar{p}n$) bound states and resonances. The direct formation of $\bar{p}p$ bound states is not possible in hydrogen, but can be observed in deuterium, the recoil neutron removing the excess energy. This method has not been used before because of the difficulty of detecting neutrons.

Considerable interest has been shown recently in $\bar{N}N$ resonances and bound states. Potential models predict ten to twenty states in the interval 1700 to 2000 MeV/c^2, with widths between 1 and 100 MeV/c^2.[1] Similar predictions have been obtained with quark models[2] and from study of the topological structure of the scattering amplitudes.[3] Candidates for $\bar{p}p$ bound states at 1684, 1646, and 1395 MeV/c^2 have been observed in radiative tran-

