Where Have All the Resonances Gone? An Analysis of Baryon Couplings in a Quark Model with Chromodynamics

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This paper reports on the results of an extensive analysis of baryon couplings in a quark model with chromodynamics. The amplitudes which emerge from the analysis resolve the problem of "missing" baryon resonances by showing that a very large number of states essentially decouple from the partial-wave analyses; those resonances which remain are in remarkable correspondence to the observed states in both their masses and decay amplitudes.

The nonrelativistic three-quark model for baryons has been an extraordinarily successful one. The early work of Greenberg, Dalitz, and their collaborators showed that the low-lying baryon resonances could be interpreted as simple orbital excitations in the symmetric quark model, and their subsequent SU(6)-based analyses showed that this interpretation could be made quantitative.[1,2] This foundational work suffered, however, from several possibly related problems: (1) In the absence of a specific dynamical picture, many [SU(6) tensor] parameters were required in the fitting of a limited amount of spectroscopic data; (2) decay analyses indicated that the baryon compositions expected from the spectroscopic fitting were inadequate; and (3) many of the states expected by the model were not seen.

More recent work has built upon these foundations a dynamical model of baryons which incorporates some of the features one might expect to emerge from quantum chromodynamics. Details may be found in several articles[3] and in recent reviews,[4] but the physics of the model may be summarized as consisting principally of flavor)independent "confinement" perturbed by color hyperfine interactions:

$$H_{\rm hyp}=\sum_{i<j}\frac{2\alpha_s}{3m_im_j}\left[\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(r_{ij})+\frac{1}{r_{ij}^{\,3}}\left(\frac{3\vec{S}_i\cdot\vec{r}_{ij}\vec{S}_j\cdot\vec{r}_{ij}}{r_{ij}^{\,2}}-\vec{S}_i\cdot\vec{S}_j\right)\right]. \quad (1)$$

The result has been a model which with only a few natural parameters (masses, potential strengths, etc.) has led to a good description of baryon spectroscopy. In this Letter we report on the results of an extensive analysis of baryon couplings in this model.[5]

Since it was our purpose to study the couplings of the baryons generated by a nonrelativistic model, it was natural and consistent to describe these couplings also in the explicit nonrelativistic quark-model approach. We found, in any event, that the alternatives—like l-broken $SU(6)_w$ or relativistic quark models—are very similar in their predictions. Thus we assumed that the decay of a baryon proceeds via single-quark transition operators of the type

$$e_i(\tfrac{1}{2}\vec{\sigma}_i\cdot\vec{k}\times\vec{\epsilon}^*+i\vec{p}_i'\cdot\vec{\epsilon}^*)\exp(-i\vec{k}\cdot\vec{r}_i)$$

for photons and

$$(g\vec{k}\cdot\vec{\sigma}_i+h\vec{\sigma}_i\cdot\vec{p}_i')\exp(-i\vec{k}\cdot\vec{r}_i)$$

for mesons where $\vec{\epsilon}$ is the photon polarization vector, e_i, $\frac{1}{2}\vec{\sigma}_i$, $\vec{r}_i$, and $\vec{p}_i'$ are the charge, spin, position, and final momentum of the ith quark, and $\vec{k}$ is the momentum of the emitted boson. An important feature of our analysis was based on the fact that apart from an overall strength factor, all of the states in a given SU(6)×O(3) supermultiplet share common partial-wave amplitudes for pseudoscalor-meson emission independent of their flavor or total angular momentum; the sharing actually goes much further in that the amplitudes which appear depend only on the total excitation quantum number N of the harmonic oscillator and the value of L^P so that, e.g., all D_S and D_M (i.e., all $[56,2^+]$ and $[70,2^+]$) decays are governed by the same amplitudes. These "universal" nonrelativistic amplitudes are displayed in Table I from which it can further be seen that the amplitudes fall into two classes: (1) "structure-independent" amplitudes like P_0, D, and F which have only the k dependence dictated by angular momentum considerations multiplying the gentle "elastic form factor" $\exp(-\frac{1}{6}k^2/\alpha^2)$, and (2) "structure-dependent" amplitudes like S, P_0', and P which are highly sensitive to the structure of the states and the nonrelativistic interpretation of the momentum transfer. We responded to this observation by taking an approach that is different from the usual one adopted in explicit quark models and specifically eschewed attempting to compute

TABLE I. The universal partial-wave amplitudes for pseudoscalar-meson emission to unmixed ground states.

Multiplet	Amplitude	Nonrelativistic model[a]
$[56,0^+]$	P_0	$[g-\frac{1}{3}h](k/\alpha)$
$[70,1^-]$	S	$[(g-\frac{1}{3}h)(k/\alpha)^2+3h]$
	D	$[g-\frac{1}{3}h](k/\alpha)^2$
$[56',0^+]$ and $[70,0^+]$	P_0'	$[(g-\frac{1}{3}h)(k/\alpha)^2+2h](k/\alpha)$
	F_0'	0
$[56,2^+]$ and $[70,2^+]$	P	$[(g-\frac{1}{3}h)(k/\alpha)^2+5h](k/\alpha)$
	F	$[g-\frac{1}{3}h](k/\alpha)^3$
$[20,1^+]$	P'	0
	F'	0

[a]The full amplitudes denoted by the symbols in column 2 are obtained by multiplying column 3 by $\alpha(kE'/\pi M_R)^{1/2}\exp(-\frac{1}{6}k^2/\alpha^2)$.

the structure-dependent amplitudes in terms of g and h. In practice this meant that we parametrized our decay amplitudes by four reduced partial-wave amplitudes (the factors in square brackets in Table I[6]) instead of the two "elementary" amplitudes g and h. Recalling that our main objective was to test the model for baryon structure, this relaxation of the decay model seems to us both sensible and prudent. On the other hand, we treated our photon amplitudes in totally standard fashion.[7]

To compare to experiment we combined our calculated decay amplitudes for all of the states associated with up to two orbital or one radial excitation in the nonrelativistic quark model with the compositions of these states from the "QCD-improved" baryon model of Refs. 3 and 4. The result was to generate a set of amplitudes with remarkable properties:

(1) The model automatically explains the apparent absence of many predicted states from the partial-wave analyses. This is illustrated in the case of the positive-parity excited baryons in Figs. 1 and 2 (similar effects occur for the negative-parity states). These figures show that the problem of missing resonances is not really a problem: *Over half of all of the predicted resonances are too inelastic to be easily seen*! One of the nicest examples of this is in the $\Lambda_{(3/2)^+}$ sector where only one of the seven predicted resonances couples significantly to $N\bar{K}$. This result has a very simple interpretation as an SU(3) breaking effect in the "*uds* basis" first discussed in Ref. 3. In addition to confirming the decouplings that could be seen easily in the *uds* basis, the calculation revealed further very extensive decouplings many of which are the result of strong intermultiplet mixings by the hyperfine interaction (1) which will be discussed below. Our conclusion is that the complete baryon spectrum anticipated by the naive nonrelativistic quark model is present and that there is no need for schemes to eliminate, for example, the 70-even SU(6) supermultiplets.

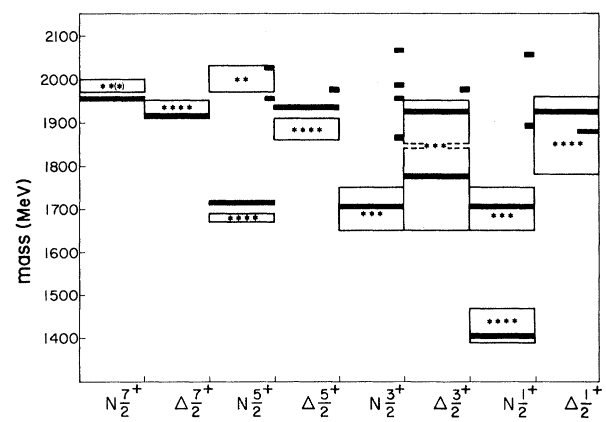

FIG. 1. The pattern of decouplings in the $S=0$ positive-parity excited baryons. The regions in which the masses of the observed resonances probably lie are denoted by open boxes, in which are given the resonances' ratings in stars. The predicted resonances are denoted by bars whose lengths indicate their predicted visibility relative to the strongest resonance in the partial wave, as given by the magnitude of their peak elastic amplitudes. Further details are given in Ref. 5.

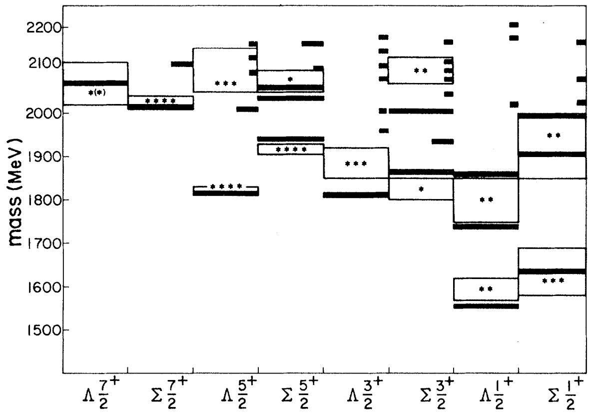

FIG. 2. The pattern of decouplings in the $S=-1$ positive-parity excited baryons. The coding here is as in Fig. 1 except that the predicted visibility is based on the sum of the magnitudes of the peak amplitudes from $N\overline{K}$ to $N\overline{K}$, $\Sigma\pi$, and $\Lambda\pi$.

(2) Those states which are predicted to remain coupled are in very good correspondence in both their masses and decay amplitudes with the observed resonances; i.e., the model correctly predicts the signs and magnitudes of almost all known baryon couplings. Among these results (hundreds of amplitudes are involved) we would like to highlight four as examples of evidence for the QCD-like components of this quark model (the spectroscopic evidence has been discussed elsewhere[3,4]):

(a) The previously mentioned decouplings due to $m_s \neq m_d$ visible in the *uds* basis are a consequence of the assumption that the confinement forces are flavor independent with flavor symmetry breaking due to the quark mass differences, as expected in quantum chromodynamics; in fact, these decouplings are perhaps the best evidence available for this assumption.

(b) In the $N_{(1/2)^+}$ sector the decay analysis succeeds because of a hyperfine-interaction–induced ^2P_M-^4P_M mixing of the correct sign and large magnitude. The necessity of such a mixing angle has been apparent from the time of the earliest decay analyses[5]; its presence in our model indicates that the tensor part of the hyperfine interaction (1) is present with the sign and magnitude expected relative to the contact term.

(c) The hyperfine interactions produce extensive D_S-D_M (i.e., $[56,2^+]$-$[70,2^+]$) mixings with several important effects. In the $N_{(5/2)^+}$, $\Delta_{(5/2)^+}$, and $N_{(3/2)^+}$ sectors, for example, D_S and D_M states would have had approximately equal couplings to $N\pi$; the strong mixing has the correct sign to enhance the coupling of the observed resonance and essentially decouple the other one. As another example of an effect of this type of mixing we mention the $N_{(5/2)^+}(1688)$ photon decays where $A_{3/2}{}^n$ arises entirely from the strong D_S-D_M mixing which at the same time leaves undisturbed the famous near zero in the $A_{1/2}{}^p$ amplitude.[7]

(d) Finally, we mention that the hyperfine interactions also mix S_M (i.e., $[70,0^+]$) components into the ground states which provide explanations for the signs and magnitudes of various observed amplitudes which are forbidden by SU(6).[8]

This concludes our brief report on the results of our analysis. While we believe that the successes of this model are quite outstanding, it should be borne in mind that it is after all only a crude model of limited numerically accuracy (typically amplitudes are predicted to $\pm 25\%$). Perhaps as a consequence, scattered among its successes are a few probable failures, e.g., the infamous $N_{(3/2)^+}(1810)$ photon amplitudes. Despite these shortcomings, we believe that the evidence is now quite strong that the model has successfully described at least the dominant features of baryon physics.

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[6]We have chosen to represent both the "structure-independent" and "structure-dependent" reduced amplitudes by constants. This is done both for simplicity and because we believe that the emission of a real pion will tend to wash out any momentum dependence of these amplitudes.

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Properties of Finite-Range Classical Solutions to Yang-Mills Theory

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We study the classical solutions of Yang-Mills equations of motion where the field tensor $G_{\mu\nu}$ is assumed to decrease exponentially with r, as r, the spatial distance, goes to infinity. Sources are introduced and they are also assumed to decrease exponentially fast at infinity. We find that $G_{\mu\nu}$ has to decrease at least as fast as the sources at infinity. If we think that within a classical approach to confinement the size of the sources should not matter, then our result is an indication that quantum effects are necessary to achieve it.

The Yang-Mills theory (YMT) has become a field of current interest nowadays, and it is a widespread conjecture that this theory will be shown to confine color degrees of freedom. In this context, we would like to study the classical solutions of YMT in order to see if they have any chance to correspond to what we know as hadrons. Hadrons have a finite size. This finite size can be expressed by an exponential decrease of the fields which constitute a hadron. We shall therefore be interested in classical solutions where $G_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+[A_\mu,A_\nu]$ is decreasing exponentially when r, the spatial distance from the center of the hadron, tends to infinity. We also introduce sources. The reason for them is that Pagels and Coleman[1] proved that classical glueballs with finite energy cannot exist. Of course, the sources also have to be of finite size. And, in fact, Sikivie and Weiss[2] have constructed short-range solutions, but where sources had the same decrease at spatial infinity as the glue field. In such solutions, the structure of the source can never be neglected because the source is always of the same order of magnitude as the field. If the source is shrunk to a point, the hadron will look pointlike. That is the reason why, in our opinion, for a classical approach one should have a faster decrease of the source at spatial infinity than for the glue field.

In this work our aim is to show that what happens for the Sikivie-Weiss solution is a general feature of all short-range classical solutions of YMT. More precisely, if the source S_ν is decreasing at spatial infinity like $\exp(-Mr)$, then $G_{\mu\nu}$ will have the same behavior, provided $G_{\mu\nu}$ decreases exponentially. Then, in any finite-range solution the structure of the source is important. This leads us to think that quantum effects are necessary to obtain confinement.

Let us proceed to the proof. We shall assume that A_μ, $[A_\mu,A_\nu]$, $[A^\mu,G_{\mu\nu}]$, and S_μ are tempered distributions in order to be able to define Fourier transforms. The Fourier transform of a distribution $V(\vec{x},t)$ will be

$$V^{\mathfrak{F}}(\vec{p},\omega)=\int_{-\infty}^{+\infty}dt\,d^3x\,\exp[-2\pi i(\vec{p}\cdot\vec{x}+\omega t)]V(\vec{x},t). \qquad (1)$$

The exponential decrease of $G_{\mu\nu}$ [$G_{\mu\nu}\sim\exp(-m\times r)$ as $r\to\infty$] will translate itself into $G_{\mu\nu}{}^{\mathfrak{F}}$ being an analytic function in the polytube T_1: $\{(\mathrm{Im}2\pi\vec{p})^2<m^2\}$. Our goal will be to show that if $M>2m$ then $G_{\mu\nu}{}^{\mathfrak{F}}$ is analytic in the larger polytube T_2: $\{(\mathrm{Im}2\pi\vec{p})^2<4m^2\}$, which implies that $G_{\mu\nu}\sim\exp(-2\times mr)$ as $r\to\infty$. This will be achieved by first taking a gauge so that, in this gauge, $A_\mu{}^{\mathfrak{F}}$ will be analytic in $T_1{}^{1+}$: $\{0<(\mathrm{Im}2\pi\vec{p})^2<m^2,\ \mathrm{Im}p_1>0\}$. This, in turn, will enable us to show that $G_{\mu\nu}{}^{\mathfrak{F}}$

