which in the limit $|\mu| \to 0$ corresponds to Eq. (13) and in the limit $\varphi \to 0$ to Eq. (16).

These results can also be extended to include the effects of symmetry-breaking fields. One finds logarithmic corrections to power-law scaling for the conjugate order parameter at $q = q_c$, and an essential singularity in $q - q_c$ for quantities like the discontinuity in the zero-field magnetization at $T = T_c$, in analogy to Eqs. (13) and (9), respectively.[16]

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[(a)]Permanent address: Board of Studies in Physics, Division of Natural Sciences, University of California, Santa Cruz, Cal. 95060.

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[14]An alternative approach which leads to the same result, Eq. (13), is to parametrize the flows in terms of $\psi(x) = C$ fixing $\varphi(x)$ outside the critical region, say $\varphi(x) = 1$. The corresponding regularity condition on the scaling function $f_s(1, C; \mu)$ now becomes $\lim_{C \to 0} f_s(1, C; \mu) = \text{const}$.
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Two-Spin Correlation Functions of an Ising Model with Continuous Exponents

Barry M. McCoy and Jacques H. H. Perk
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
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An Ising model on a square lattice is studied, where one row of horizontal bonds has an energy E_1' different from all other horizontal bonds. The correlation of two spins is calculated in this row, resulting in exponents β and η which depend on E_1'. The long-distance behavior of the correlation for fixed $T \neq T_c$ is found to have different forms depending upon the value of E_1'.

Since the discovery by Baxter[1] of a two-dimensional (2D) statistical mechanical model whose specific heat exponent depends on the parameters of the Hamiltonian there has been widespread recognition that many other 2D models such as the massive Thirring model,[2] the Ashkin-Teller model,[3] and the planar rotator[4] [O(2)] model will have correlation functions with continuous critical indices. For the region where the mass gap vanishes these models all bear some relation in leading order to the Gaussian model.[5] However, when there is a mass gap the only exact information known is the correlation length of the Baxter model.[6] It is thus extremely interesting to investigate the phenomenon of continuous critical exponents in a simple modification of the Ising model. In this Letter we discuss the principal properties of the two-point function of this model.

Let E_1 (E_2) be the horizontal (vertical) interaction energies of the homogeneous Ising model on the square lattice,[7,8] but let one row (called zero) of horizontal bonds have an altered interaction energy E_1'.[9] Consider the two-spin correlation function $\langle\sigma_{00}\,\sigma_{0N}\rangle$ in the row of the altered horizontal bonds E_1'. Because the lattice is reflection symmetric about this row, the correlation function can be calculated as the $N\times N$ Toeplitz determinant, so that

$$\langle\sigma_{00}\,\sigma_{0N}\rangle = \det|a_{i-j}|,\quad i,j=1,\dots,N, \tag{1}$$

where the elements a_n may be calculated either by the operator technique[7,10] or by Pfaffian methods[8,11] as

$$a_n=(2\pi)^{-1}\int_0^{2\pi} d\theta\, e^{-in\theta}K(e^{i\theta}), \tag{2}$$

$$K(e^{i\theta})=[C(e^{i\theta})+\kappa]/[1+\kappa C(e^{i\theta})], \tag{3}$$

$$C(e^{i\theta})=\left[\frac{(1-\alpha_1e^{i\theta})(1-\alpha_2e^{-i\theta})}{(1-\alpha_1e^{-i\theta})(1-\alpha_2e^{i\theta})}\right]^{1/2}. \tag{4}$$

$C(e^{i\theta})$ is well-known generating function for the pure $E_1'=E_1$ case,

$$\kappa=\tanh[(E_1'-E_1)/kT],\quad \alpha_1=Z_1(1-Z_2)/(1+Z_2),$$

$$\alpha_2=Z_1^{-1}(1-Z_2)/(1+Z_2),\quad Z_i=\tanh[E_i/kT],$$

the square root being defined to be +1 at $\theta=\pi$.

The following are some of the principal properties of the two-point function.

(1) *Spontaneous magnetization in the row.*—The critical temperature of this model is still given by $\alpha_2=1$ (as it is for $\kappa=0$). When $T<T_c$ ($\alpha_2<1$) the magnetization in the zeroth row, $\mathfrak{M}_0$, may be calculated by Szegö's theorem[12] in the form

$$\ln\mathfrak{M}_0^2=\ln\{\lim_{N\to\infty}\langle\sigma_{00}\,\sigma_{0N}\rangle\}=\tfrac{1}{8}\int_0^{2\pi}\frac{d\theta_1}{2\pi}\int_0^{2\pi}\frac{d\theta_2}{2\pi}\left[\frac{\ln K(\exp(i\theta_1))-\ln K(\exp(i\theta_2))}{\sin[\frac{1}{2}(\theta_1-\theta_2)]}\right]^2. \tag{5}$$

We thus find that, as $T\to T_c^-$ ($\alpha_2\to 1^-$),

$$\mathfrak{M}_0\sim f(\kappa_c)(1-\alpha_2^2)^{\beta(\kappa_c)}, \tag{6}$$

$$\beta(\kappa_c)=\tfrac{1}{2}\{\pi^{-1}\arccos[2\kappa_c/(1+\kappa_c^2)]\}^2=\tfrac{1}{2}\{\pi^{-1}\arccos\tanh[2(E_1'-E_1)/kT_c]\}^2, \tag{7}$$

$$\ln f(\kappa)=\tfrac{1}{8}\int_0^{2\pi}\frac{d\theta_1}{2\pi}\int_0^{2\pi}\frac{d\theta_2}{2\pi}\{\sin[\tfrac{1}{2}(\theta_1-\theta_2)]\}^{-2}\{[\ln K_0(\exp(i\theta_1))-\ln K_0(\exp(i\theta_2))]^2+2\beta(\theta_1-\theta_2)^2\}, \tag{8}$$

where $K_0(e^{i\theta})$ is given by (3) with $C(e^{i\theta})$ replaced by $ie^{-i\theta/2}[(1-\alpha_1e^{i\theta})/(1-\alpha_1e^{-i\theta})]^{1/2}$. The exponent $\beta(\kappa_c)$ depends continuously on κ_c and agrees with the related calculation of Bariev.[9] Note that $\beta(+1)=0$, $\beta(0)=\frac{1}{8}$, $\beta(-1)=\frac{1}{2}$, and $\ln f(\kappa)$ vanishes when $\kappa=+1$ and diverges when $\kappa\to-1$. These two limits reflect the fact that when $E_1'\to+\infty$ ($-\infty$) the spins in the row are all parallel (antiparallel).

(2) *Correlation at $T=T_c$.*—When $T=T_c$ the leading term as $N\to\infty$ may be determined by calculating the ratio of $\langle\sigma_{00}\sigma_{0N}\rangle$ to the $N\times N$ Cauchy determinant generated by $\tilde{C}(e^{i\theta})=\exp\{-i(\theta-\pi)[2\beta]^{1/2}\}$ following the method of Wu.[8,13] We find

$$\langle\sigma_{00}\sigma_{0N}\rangle\sim A(\kappa_c)f(\kappa_c)N^{-2\beta(\kappa_c)}, \tag{9}$$

where $f(\kappa)$ is given by (8) and

$$A=(1-2\beta)\exp\{-[2\beta\gamma+\sum_{n=1}^{\infty}(2\beta)^n(\zeta(2n-1)-1)/n]\} \tag{10}$$

with γ denoting Euler's constant and $\zeta(s)$ the ζ function. Thus the critical index $\eta(\kappa)$ obeys the usual hyperscaling relation $(d-2+\eta)\nu=2\beta$ with $\nu=1$ and $d=2$.

(3) *Large-N behavior for $T<T_c$.*—When $T<T_c$ the index of $K(e^{i\theta})$ is zero and we may make the Wiener-Hopf factorization $K(e^{i\theta})=P_<^{-1}(e^{i\theta})Q_<^{-1}(e^{-i\theta})$, where $P_<(\xi)$ and $Q_<(\xi)$ are both analytic for $|\xi|<1$ and continuous and nonzero for $|\xi|\le 1$. Since $K(e^{i\theta})=K^{-1}(e^{-i\theta})$ we have $P_<(e^{i\theta})Q_<(e^{i\theta})=1$ and the calculation[14] of the case $\kappa=0$ may be generalized to give

$$\langle\sigma_{00}\sigma_{0N}\rangle=\mathfrak{M}_0^2\exp[-\sum_{n=1}^{\infty}F_<^{(2n)}], \tag{11}$$

where

$$F_<^{(2n)}=n^{-1}(2\pi)^{-2n}\prod_{j=1}^{n}\oint d\xi_{2j-1}\xi_{2j-1}{}^{N}P_<(\xi_{2j-1})P_<(\xi_{2j-1}{}^{-1})(\xi_{2j-1}-\xi_{2j})^{-1}\times\oint d\xi_{2j}\xi_{2j}{}^{-N}P_<{}^{-1}(\xi_{2j})P_<{}^{-1}(\xi_{2j}{}^{-1})(\xi_{2j}-\xi_{2j+1})^{-1}, \quad (12)$$

with $\xi_{2n+1}\equiv\xi_1$, $|\xi_{2j+1}|<1$, and $|\xi_{2j}|>1$.

The splitting function $P_<(e^{i\theta})$ can be explicitly calculated using Jacobi's Θ functions. However, the analytic properties of $P_<(e^{i\theta})$ may be read off from $K(e^{i\theta})$ and this will suffice for the calculation of the major features of the $N\to\infty$ behavior.

The kernel $K(e^{i\theta})$ has square-root branch points at α_1, α_2, $\alpha_2{}^{-1}$, and $\alpha_1{}^{-1}$. In addition it will have zeros for these values of ξ, where $C(\xi)+\kappa=0$ and poles where $1+\kappa C(\xi)=0$. On the sheet of the square root defined by $C(-1)=+1$ the function $C(\xi)$ is never negative real. Therefore if $\kappa>0$ there are neither zeros nor poles. However, for $\kappa<0$ there are solutions and we find zeros of $K(\xi)$ at $\beta_1{}^{-1}$ and β_2, where

$$\left.\begin{matrix}\beta_1{}^{-1}\\ \beta_2\end{matrix}\right\}=\tfrac{1}{2}(\alpha_1-\kappa^2\alpha_2)^{-1}\{(1-\kappa^2)(1+\alpha_1\alpha_2)\pm[(1-\kappa^2)^2(1-\alpha_1\alpha_2)^2+4\kappa^2(\alpha_1-\alpha_2)^2]^{1/2}\}, \quad (13)$$

and we find poles at β_1 and $\beta_2{}^{-1}$. The β_i have the following properties: (i) If $\kappa=0$, then $\beta_1=\alpha_1$ and $\beta_2=\alpha_2$; (ii) if $-1<\kappa<0$, then $-1<\beta_1<\alpha_1<\alpha_2<\beta_2<1$. When $-1<\kappa<1$, function $P_<(e^{-i\theta})$ has square-root branch points at α_2 and α_1 [at which $P_<(\xi)$ is finite and nonzero if $\kappa\neq 0$] and when $-1<\kappa<0$, $P_<(e^{-i\theta})$ has a zero at β_2 and a pole at β_1. Thus we find from (12) that, for $0<\kappa<1$,

$$\langle\sigma_{00}\sigma_{0N}\rangle\sim\mathfrak{M}_0{}^2\{1+a_1(\kappa)N^{-3}\alpha_2{}^{2N}\}, \quad (14)$$

where only $a_1(\kappa)$ depends on κ, and diverges for $\kappa\to 0$. For $-1<\kappa<0$, the situation is more complicated. In asymptotically evaluating the ξ_2 integral [of (12) with $n=1$] the contour is deformed outward and the leading singularity is now the pole of $P_<{}^{-1}(\xi)$ at $\beta_2{}^{-1}$. The ξ_1 contour is deformed inward and the singularity closest to the unit circle will be either the square root at α_2 or the pole at β_1. If

$$-[(1+\alpha_2{}^2)(\alpha_1+\alpha_2)/2\alpha_2(1+\alpha_1\alpha_2)]^{1/2}<\kappa<0, \quad (15)$$

then $-\beta_1<\alpha_2$ and the leading behavior is

$$\langle\sigma_{00}\sigma_{0N}\rangle\sim\mathfrak{M}_0{}^2\{1+a_2(\kappa)N^{-3/2}(\alpha_2\beta_2)^N\}. \quad (16)$$

In this case the approach to the limit is still monotonic but the correlation length now depends on κ even though ν is still equal to 1. If, on the other hand,

$$-1<\kappa<-\left[\frac{(1+\alpha_2{}^2)(\alpha_1+\alpha_2)}{2\alpha_2(1+\alpha_1\alpha_2)}\right]^{1/2}, \quad (17)$$

then $-\beta_1>\alpha_2$ and the leading behavior is

$$\langle\sigma_{00}\sigma_{0N}\rangle\sim\mathfrak{M}_0{}^2\{1+(-1)^N a_3(\kappa)(\beta_2|\beta_1|)^N\}, \quad (18)$$

where we have made explicit the fact that the approach to the limit is oscillatory. This behavior is expected because a large negative value of E_1' tends to make the spins in the zeroth row antialign. Note that if $T\to T_c$ ($\alpha_2\to 1$) then the region (17) shrinks to the point -1 only and hence this local antiferromagnetic behavior is eventually washed out by the ferromagnetic behavior of the bulk.

(4) *Large-N behavior for $T>T_c$.*—When $T>T_c$ the index of $K(e^{i\theta})$ is not zero. We therefore consider the shifted kernel $K_>(e^{i\theta})=-e^{i\theta}K(e^{i\theta})$ and make the factorization $K_>(e^{i\theta})=P_>{}^{-1}(e^{i\theta})\times Q_>{}^{-1}(e^{-i\theta})$, where again $P_>(\xi)$ and $Q_>(\xi)$ are analytic for $|\xi|<1$ and continuous and nonzero for $|\xi|\leq 1$, and $P_>(e^{i\theta})Q_>(e^{i\theta})=1$. We may again generalize the studies made[8,14] for $\kappa=0$ to find

$$\langle\sigma_{00}\sigma_{0N}\rangle=\mathfrak{M}_{0>}{}^2(\kappa)\sum_{n=0}^{\infty}g^{(2n+1)}\exp\Bigl(-\sum_{n=1}^{\infty}F_>{}^{(2n)}\Bigr), \quad (19)$$

where $\mathfrak{M}_{0>}{}^2(\kappa)$ is obtained by applying Szegö's theorem to $K_>$,

$$g^{(2n+1)}=(2\pi i)^{-1}\oint d\xi_0\xi_0{}^{N-1}P_>(\xi_0)P_>(\xi_0{}^{-1})\prod_{l=1}^{2n}(2\pi i)^{-1}\oint d\xi_l\xi_{l-1}\xi_l{}^{N}(1-\xi_{l-1}\xi_l)^{-1}\times\prod_{l=1}^{n}P_>{}^{-1}(\xi_{2l-1})P_>{}^{-1}(\xi_{2l-1}{}^{-1})P_>(\xi_{2l})P_>(\xi_{2l}{}^{-1}), \quad (20)$$

where $|\xi_i| < 1$ and $F_>^{(2n)}$ is obtained from (12) with $P_<$ replaced by $P_>$ and N by $N+1$. At $T\to T_c^+$ we find that $M_{0>}^2(\kappa)$ vanishes at $(T-T_c)^{2\beta(-\kappa)}$. Furthermore in the scaling limit, $g^{(2n+1)}$ contains the temperature-dependent multiplicative factor $(T-T_c)^{2[\beta(\kappa)-\beta(-\kappa)]}$. Therefore the full correlation function (19) has the factor $(T-T_c)^{2\beta(\kappa)}$ in the scaling limit and hence the high-temperature hyperscaling relation involving η holds.

In contrast to $T<T_c$ the kernel $K_>(e^{i\theta})$ [and therefore $P_>(e^{i\theta})$] now contains poles and zeros for both signs of κ. If $0<\kappa<1$, then $P_>(\xi)$ has a pole at β_2 in addition to square roots at α_2 and α_1^{-1}, where $1<\beta_2<\alpha_2<\alpha_1^{-1}$ and β_2 is still given by (13). We thus find from (20) that, for $0<\kappa<1$,

$$\langle\sigma_{00}\sigma_{0N}\rangle \sim \mathfrak{M}_0^2(\kappa)a_{1>}(\kappa)\beta_2^{-N}, \tag{21}$$

where $a_{1>}(\kappa)\to 0$. This form is not the same as that of the $\kappa=0$ special case and the correlation length depends on κ.

When $-1<\kappa<0$, then $P_>(\xi)$ has a pole at β_1^{-1}. If

$$-[2\alpha_2(1+\alpha_1\alpha_2)/(\alpha_1+\alpha_2)(1+\alpha_2^2)]^{1/2}<\kappa<0, \tag{22}$$

then $\alpha_2^{-1}>-\beta_1^{-1}$, the branch cut at α_2^{-1} dominates the integral, and we find

$$\langle\sigma_{00}\sigma_{0N}\rangle \sim \mathfrak{M}_{0>}^2(\kappa)a_{2>}(\kappa)N^{-3/2}\alpha_2^{-N}, \tag{23}$$

where $a_{2>}(\kappa)$ diverges as $\kappa\to 0$. Finally if

$$-1<\kappa<-[2\alpha_2(1+\alpha_1\alpha_2)/(\alpha_1+\alpha_2)(1+\alpha_2^2)]^{1/2}, \tag{24}$$

then $\alpha_2^{-1}<-\beta_1^{-1}$, the pole at β_1^{-1} dominates the integral, and we find

$$\langle\sigma_{00}\sigma_{0N}\rangle \sim \mathfrak{M}_{0>}^2 a_{3>}(\kappa)(-1)^N|\beta_1|^N. \tag{25}$$

This is the oscillatory behavior analogous to the regime (17) of $T<T_c$. This region also disappears as $\alpha_2\to 1$.

We conclude with a few remarks.

There are many modifications of the Ising model which will lead to continuous exponents. For example, in a square Ising lattice we may introduce one diagonal line of bonds of strength E_3. The correlation function $\langle\sigma_{00}\sigma_{NN}\rangle$ in this special diagonal may be shown to be given by an $N\times N$ Toeplitz determinant of the form (1) where the generating function is given by (3) with $\alpha_1\to 0$, $\alpha_2\to[\sinh(2E_1/kT)\sinh(2E_2/kT)]^{-1}$ and $\kappa\to\tanh(E_3/kT)$.

We may also change the energies of two successive rows of vertical bonds from E_2 to E_2'. The correlation in the row between these two changed rows of bonds is still given by (1) where the generating function is now

$$K(e^{i\theta})=[A(e^{i\theta})C(e^{i\theta})+B(e^{i\theta})]/[A(e^{-i\theta})+B(e^{-i\theta})C(e^{i\theta})], \tag{26}$$

with

$$A(e^{i\theta})=-\coth[2(E_2'^*-E_2^*)/kT]+i\sin\theta\sinh(2E_1/kT),\quad B(e^{i\theta})=\cos\theta-i\sin\theta\cosh(2E_1/kT), \tag{27}$$

where E^* is defined by $\sinh(2E^*/kT)\sinh(2E/kT)=1$.

Finally, it is also interesting to study the model[9,15] with one row of vertical bonds E_2 replaced by E_2'. For all rows the correlation is now given by a 2×2 block determinant (when $E_2'\neq E_2$). Bariev[9] has calculated the local magnetization in a row far from the modified row and finds a continuous exponent. We therefore expect that on the row adjacent to E_2' the magnetization for $T\sim T_c$ will be of the form (6) with $\beta(\kappa_c)$ as calculated by Bariev. However, the amplitude function $f(\kappa)$ must be completely different from (8) because when $E_2'\to 0$, then $\beta(\kappa)\to\frac{1}{2}$ and the model reduces to the previously studied half-plane problem.[8,16] This is in contrast to the case of the present paper where if $E_1'\to-\infty$ then β is formally $\frac{1}{2}$ but $f(\kappa)\to\infty$ and, in fact, the zeroth row becomes completely antialigned.

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