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Theory of Current Generation by Electrostatic Traveling Waves in Collisionless Magnetized Plasmas

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The general formula which gives the current density generated by an electrostatic traveling wave in a collisionless magnetized plasma is derived within the framework of quasilinear theory. This formula shows that the ion response is indispensable to generating the current via wave-particle interaction. The current which is obtained by use of this formula in the case of lower hybrid waves is in agreement with the experimental values.

There is a great deal of interest in the generation[1-11] of toroidal currents by traveling waves in order to allow tokamaks to run in the steady state. Recently, Wong[12] observed current generation by unidirectional electrostatic plama waves. In this Letter, the general formula which gives the current density generated by an electrostatic traveling wave in a collisionless magnetized plasma is derived within the framework of quasilinear theory (when the wave amplitude is so small that nonlinear Landau dampling, mode-mode coupling, and trapped particle effects can be neglected, the quasilinear theory is a good approximation). This formula is expressed in terms of the electric susceptibilities and shows that the ion response to the wave is indispensable to generating the current. The current which is obtained by use of this formula in the case of lower hybrid waves is in agreement with the experimental values observed by Wong.[12]

First, let us consider a high-frequency electrostatic wave in which the ion motion can be neglected. I define the wave momentum as the momentum of the nonresonant electrons. When the wave is attenuating or growing, momentum is exchanged between the wave and the resonant electrons. If the momentum of the electron distribution function which is the sum of the wave momentum and the momentum of the resonant electrons is initially zero, it must still vanish after the wave has damped out. Not only the resonant electrons but also the nonresonant electrons contribute to the current so that the currents generated by them cancel each other. Therefore, in this case the wave-particle interaction generates no net plasma current.[13]

Next, let us consider a low-frequency electrostatic wave in which the ion motion cannot be neglected. In this case current generation by the wave-particle interaction can be interpreted physically as follows. Suppose that an electrostatic wave in a plasma has been excited by an external system and that then the interaction between the plasma and the external system is switched off at a time $t=0$. The total momentum of the plasma remains constant after $t=0$:

$$P_e^W + P_e^R + P_i^W = P_0, \qquad (1)$$

where P_e^W is the nonresonant electron momentum, P_e^R is the resonant momentum, P_i^W is the nonresonant ion momentum, P_0 is the initial momentum at $t=0$, and where we have neglected the resonant ions. Here, the initial momentum P_0 is determined by the interaction between the plasma and the external system. As will be shown later, P_e^W and P_i^W are proportional to the intensity of the electric field. Then, one obtains the currect density j,

$$j = \frac{e}{m_i} P_i^W - \frac{e}{m_e}(P_e^W + P_e^R) = \frac{e}{m_i} P_i^W + \frac{e}{m_e}(P_i^W - P_0) \simeq \frac{e}{m_e}(P_i^W - P_0), \qquad (2)$$

where the notation is standard. Therefore, supposing that the wave has damped out at $t=\infty$, the current density J generated by the wave-particle interaction is given by

$$J \equiv j(t=\infty) - j(t=0) = -\frac{e}{m_e} P_i^W(t=0). \qquad (3)$$

The resonant electrons have taken up the momentum which the ions had at $t=0$ and the current generated in the plasma is proportional to this momentum. Equations (1)–(3) show that the plasma current is not generated by momentum exchange between electrons themselves, but momentum exchange between ions and electrons. Note the fact that the current flows in the direction opposite to the initial ion momentum $P_i^W(t=0)$ and is carried by the resonant electrons. This is in agreement with the experiment by Wong.[12]

I start from the Vlasov equation for the α component of the distribution function $f_\alpha(\vec{r},\vec{v},t)$,

$$\frac{\partial}{\partial t}f_\alpha+\vec{v}\frac{\partial}{\partial \vec{r}}f_\alpha+\frac{e_\alpha}{m_\alpha}\left(\vec{E}+\frac{\vec{v}}{c}\times\vec{B}_0\right)\frac{\partial}{\partial\vec{v}}f_\alpha=0, \tag{4}$$

where E is an electrostatic field which lies in the x-z plane, B_0 is an external constant magnetic field in the z direction, α denotes electrons or ions, and the other symbols are self-evident. Suppose that the distribution function can be divided into a rapidly varying part $\tilde{f}_\alpha=\tilde{f}_\alpha(t,\vec{v})\times\exp(ik\vec{r}-i\omega t)$ and a slowly varying part

$$f_{\alpha 0}=f_{\alpha 1}(v_\perp,v_\parallel,t)+f_{\alpha 2}(v_\perp,v_\parallel,\varphi,t)$$
$$(f_{\alpha 1}>>f_{\alpha 2},\ \tan\varphi\equiv v_y/v_x),$$

and that the slow spatial dependence can be neglected. Then one obtains, correct to first order in the small parameter $(1/\omega)(\partial/\partial t)$,

$$\tilde{f}_\alpha=\sum_n i\frac{e_\alpha}{m_\alpha}\frac{\exp(i\xi_\alpha\sin\varphi-in\varphi-i\omega t)}{n\Omega_\alpha+k_\parallel v_\parallel-\omega}\left[E_\perp\frac{\partial f_{\alpha 1}}{\partial v_\perp}\frac{n}{\xi_\alpha}J_n(\xi_\alpha)+E_\parallel\frac{\partial f_{\alpha 1}}{\partial v_\parallel}\frac{\partial J_n}{\partial\xi_\alpha}\right]$$
$$-\frac{e_\alpha}{m_\alpha}\sum_{n,l,q}\frac{\exp(i\xi_\alpha\sin\varphi-in\varphi-i\omega t)}{(n\Omega_\alpha+k_\parallel v_\parallel-\omega)[(n-l+q)\Omega_\alpha+k_\parallel v_\parallel-\omega]}\frac{\partial}{\partial t}\left(E_\perp\frac{\partial f_{\alpha 1}}{\partial v_\perp}\frac{n}{\xi_\alpha}J_n+E_\parallel\frac{\partial f_{\alpha 1}}{\partial v_\parallel}\frac{\partial J_n}{\partial\xi_\alpha}\right), \tag{5}$$

where Ω_α is the Larmor frequency, J_n is a Bessel function, $\xi_\alpha\equiv k_\perp v_\perp/\Omega_\alpha$, and where I have assumed that

$$\partial\tilde{f}_\alpha(t,\vec{v})/\partial t>>(e_\alpha/m_\alpha)\vec{E}\cdot\partial f_{\alpha 2}/\partial\vec{v}$$

Note that in Eq. (5) I do not assume that $f_{\alpha 1}$ is Maxwellian. From Eqs. (4) and (5), the z component of the slowly varying momentum equation is

$$\frac{\partial}{\partial t}m_\alpha\Gamma_{\alpha\parallel}=\tfrac{1}{4}\rho_\alpha E_\parallel{}^*+\text{c.c.}=\frac{\partial}{\partial t}\left(\frac{k_\parallel}{16\pi}\frac{\partial\chi_\alpha{}^H}{\partial\omega_r}|E|^2\right)+\frac{1}{8\pi}k_\parallel\chi_\alpha{}^A|E|^2, \tag{6}$$

where

$$\Gamma_{\alpha\parallel}\equiv\int d^3v\,v_\parallel f_{\alpha 1},\quad \rho_\alpha\equiv e_\alpha\int d^3v\,f_{\alpha 1},$$

$$\frac{\partial\chi_\alpha{}^H}{\partial\omega_r}\equiv\text{Re}\sum_n\frac{4\pi e^2}{m_\alpha k^2}\int dv_\perp 2\pi v_\perp\int dv_\parallel\frac{J_n}{(n\Omega_\alpha+k_\parallel v_\parallel-\omega)^2}\left(\frac{n\Omega_\alpha}{v_\perp}\frac{\partial f_{\alpha 1}}{\partial v_\perp}J_n+k_\parallel\frac{\partial f_{\alpha 1}}{\partial v_\parallel}\frac{\partial J_n}{\partial\xi_\alpha}\right),$$

$$\chi_\alpha{}^A\equiv\text{Im}\sum_n\frac{4\pi e_\alpha{}^2}{m_\alpha k^2}\int dv_\perp 2\pi v_\perp\int dv_\parallel\frac{J_n}{n\Omega_\alpha+k_\perp v_\perp-\omega}\left(\frac{n\Omega_\alpha}{v_\perp}\frac{\partial f_{\alpha 1}}{\partial v_\perp}J_n+k_\parallel\frac{\partial f_{\alpha 1}}{\partial v_\parallel}\frac{\partial J_n}{\partial\xi_\alpha}\right),$$

ω_r is the real part of the wave frequency, and I have used $\sum_{n=-\infty}^{\infty}J_n{}^2=1$ and $\sum_{n=-\infty}^{\infty}J_nJ_{n+l}=0$ $(l\neq 0)$.[14] Note that the first and second terms of the right-hand side of Eq. (6) result from the nonresonant and the resonant particles, respectively.

Summing Eq. (6) over the plasma component and noting that $\sum_\alpha\rho_\alpha E_\parallel{}^*+\text{c.c.}=0$, the wave momentum equation is

$$\frac{\partial}{\partial t}\left(\frac{1}{16\pi}k_\parallel\frac{\partial\epsilon^H}{\partial\omega_r}|E|^2\right)+\frac{1}{8\pi}k_\parallel\epsilon^A|E|^2=0, \tag{7}$$

where

$$\partial\epsilon^H/\partial\omega_r\equiv\partial\chi_i{}^H/\partial\omega_r+\partial\chi_e{}^H/\partial\omega_r,\quad \epsilon^A\equiv\chi_i{}^A+\chi_e{}^A.$$

Equation (7) shows that the total momentum is conserved.[15]

From Eqs. (6) and (7), the current density generated by an arbitrary electrostatic wave is expressed as

$$j_{\parallel}=\frac{k_{\parallel}}{16\pi}\frac{e}{m_e}|E|^2\left[\frac{\partial\chi_i^H}{\partial\omega_r}\left(\frac{m_e}{m_i}+\frac{1-m_e\chi_i^A/m_i\chi_e^A}{1+\chi_i^A/\chi_e^A}\right)-\frac{\partial\chi_e^H}{\partial\omega_r}\left(1-\frac{1-m_e\chi_i^A/m_i\chi_e^A}{1+\chi_1^A/\chi_e^A}\right)\right]+\theta, \qquad (8)$$

where θ is a constant of integration determined by the initial condition. Here I have assumed that the electric field is so weak that the damping rate is well approximated by the initial linear one. Equation (8) shows that the current generated by the wave-particle interaction is proportional to the intensity of the electric field.

If the ion motion can be neglected ($\partial\chi_i^H/\partial\omega_r=\chi_i^A=0$) as in case of Langmuir waves, it is seen from Eq. (8) that the wave-particle interaction generates no plasma current. In the case of a wave for which χ_i^A is comparable with χ_e^A and $\partial\chi_e^H/\partial\omega_r=0$, e.g., the ion-acoustic wave, Eq. (8) becomes

$$j_{\parallel}\simeq\frac{k_{\parallel}}{16\pi}\frac{e}{m_e}|E|^2\frac{\partial\chi_i^H}{\partial\omega_r}\left(1+\frac{\chi_i^A}{\chi_e^A}\right)^{-1}+\theta. \qquad (9)$$

Let us for instance consider a lower hybrid wave.[12,16] The susceptibilities for this wave are

$$\chi_e^H=\frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\Omega_e^2}-\frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega_r^2},\quad \chi_i^H=-\frac{k_\perp^2\omega_{pi}^2}{k^2\omega_r^2},$$
$$\chi_e^A=\left(\frac{\pi}{2}\right)^{1/2}\frac{k_e^2}{k^2}\frac{\omega_r}{k_\parallel v_e},\quad \chi_i^A=0, \qquad (10)$$

where ω_{pe}, ω_{pi} are the electron and ion plasma frequencies, respectively, and k_e is the electron Debye wave number. Then, the generated current density is

$$j_\parallel\simeq\frac{k_\parallel}{16\pi}\frac{e}{m_e}\frac{\partial\chi_i^H}{\partial\omega_r}|E|^2+\theta$$
$$=\frac{k_\parallel}{8\pi}\frac{e}{m_e}\frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_r^3}|\mathbf{E}|^2+\theta. \qquad (11)$$

Note that Eq. (11) does not contain the electronic susceptibilities. If we put $k_\parallel/2\pi=\frac{1}{7}$ cm^{-1}, $k_\perp/2\pi=\frac{1}{4}\times10^{-1}$, $\omega_r/2\pi=50$ MHz, plasma density $N=2\times10^{10}$ cm^{-3} and $|\mathbf{E}|=40$ V/cm into Eq. (11), the current generated by the wave-particle interaction is 10^{-2} A. This is in agreement with the experimental value observed by Wong.[12]

Finally, let me discuss the current generation by an electromagnetic wave which has field momentum as well as particle momentum. As the wave decays, the field momentum is converted to particle momentum. Therefore, even if the ions are assumed to be immobile, the electrons gain momentum so that a plasma current is generated.

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