67B, 84 (1977).

[6]A similar picture was independently suggested by N. N. Nikolaev and S. Pokorski, Phys. Lett. 80B, 290 (1979); N. N. Nikolaev and A. Ya. Ostapchuck, CERN Report No. TH-2575, 1978 (to be published). However, their model rests heavily on details of a specific quark recombination model, it is not applicable to lepton production, and their predictions involve a free choice of functions and adjustable parameters.

[7]We assume that quarks can propagate long distances through the nucleus as free quarks. We have not addressed ourselves to the problem of what is the mechanism that makes this possible, whereas confinement distances are believed to be of the order of nucleon dimensions. The answer to this problem has to wait first for the solution of the confinement problem.

[8]It is well known both theoretically (see, for instance, Ref. 6 and references therein) and experimentally (lack of significant cascading within nuclei) that formation times of large x particles are long compared with nuclear dimensions.

[9]G. Berlad *et al.*, Phys. Rev. D 13, 161 (1976), and references therein.

[10]E. M. Levin and L. L. Frankfurt, Pis'ma Zh. Eksp. Teor. Fiz. 2, 3105 (1965) [JETP Lett. 2, 108 (1965)]; H. J. Lipkin and F. Sheck, Phys. Rev. Lett. 16, 71 (1966).

[11]G. Belletini *et al.*, Nucl. Phys. 79, 609 (1966); S. P. Denisov *et al.*, Nucl. Phys. B61, 62 (1973); A. S. Carroll *et al.*, Phys. Lett. 80B, 319 (1979).

[12]S. Barshay *et al.*, Phys. Rev. C 11, 360 (1975).

[13]We used only the $\theta \neq 0°$ data of Walley *et al.*, Ref. 2d, to avoid the contribution from coherent production of resonances by the nuclear Coulomb field.

Dynamical Supersymmetries in Nuclei

F. Iachello
Department of Physics, Yale University, New Haven, Connecticut 06520, and
Kernfysisch Versneller Instituut, 9700av Groningen, The Netherlands
(Received 16 January 1980)

It is suggested that dynamical (Bose-Fermi) supersymmetries may be present in the spectra of complex nuclei. A concrete example in which a supersymmetric structure is experimentally observed is shown. In this example the supersymmetry is generated by $L = 0$ and $L = 2$ bosons together with $j = \frac{3}{2}$ fermions.

In recent years, it has been pointed out by several authors that there may exist in nature examples of supersymmetric structures in which bosonic and fermionic degrees of freedom are linked together in a single theoretical framework.[1] However, no concrete example in which a supersymmetric structure is experimentally observed has been reported so far. In this Letter, I suggest that dynamical supersymmetries may be present in complex nuclear spectra and I show one concrete example where the supersymmetric structure is experimentally observed.

By analogy with an ordinary dynamical symmetry (which applies to a system either of bosons *or* of fermions separately), one can define a dynamical supersymmetry as that situation in which (i) the states of the combined system of bosons and fermions can be simultaneously classified with a complete set of group-theoretical labels; (ii) these states are split but not mixed by the Hamiltonian, H, and (iii) the *same* expression for the eigenvalues of the system in terms of the complete set of group-theoretical labels described *both* bosonic and fermionic spectra.

The investigation of this problem was stimulated in part by the fact that similar attempts have been made in elementary-particle physics and in part by the known occurrence of several ordinary dynamical symmetries in the spectra of complex nuclei.[2] In particular, the spectra of the platinum nuclei with an even number of protons and neutrons have been shown to display a dynamical symmetry associated with the group O(6).[3] The states of these nuclei have then been classified by the set of labels related to the group chain[4] $U(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2)$, denoted by N, σ, τ, ν_Δ, L, and M. The group U(6) and its subgroups are generated by $L = 0$ and $L = 2$ bosons. In terms of these labels, the expectation value of the most general Hamiltonian which splits but does not mix the states can be written as[4]

$$E(N, \sigma, \tau, \nu_\Delta, L, M) = -\tfrac{1}{4}A\sigma(\sigma+4) + \tfrac{1}{6}B\tau(\tau+3) + CL(L+1), \quad (1)$$

where, in comparison with Eq. (12) of Ref. 4, the constant $\frac{1}{4}AN(N+4)$ has been removed, since it does not contribute to the excitation energies. An ordinary symmetry associated with the group chain $O(6) \supset O(5) \supset O(3)$ is then present whenever the spectrum of an even-even nucleus can be described by (1). I note here that, in this case, in which only bosonic spectra are considered, the group-theoretical labels appearing in the classification scheme are only those belonging to *tensor* representations of $O(6) \supset O(5) \supset O(3)$.

In addition to the spectra of even nuclei, there are experimentally available in this region also spectra of odd-A nuclei. These spectra are obviously fermionic and require for their description both bosonic and fermionic degrees of freedom.[5] One may then inquire whether or not it is possible to describe both even (bosonic) spectra and odd (fermionic) spectra within the same group-theoretical framework. Because of the stringent conditions (i)–(iii), it is clear that this is not possible in general. In fact a supersymmetric situation will only occur whenever the angular momentum of the fermion, j, has some specific value and, moreover, the boson-boson, fermion-fermion, and boson-fermion interactions of Ref. 5 are in some particularly simple relationship with each other. In the particular case discussed here, where the boson symmetry is $O(6) \supset O(5)$, there is only one particular value of j for which a simple situation will occur, $j=\frac{3}{2}$, since $j=\frac{3}{2}$ is the angular momentum of the fundamental *spinor* representation of O(6) and O(5). As a consequence, when $j=\frac{3}{2}$, one can construct a classification scheme which includes both bosonic and fermionic states. In this classification scheme, bosonic states will belong to tensor representations of O(6) and O(5), while fermionic states will belong to their spinor representations.[6] The enlargement of the representation space to include also spinor representations, corresponds to the introduction of the universal covering groups of O(6), O(5), and O(3), denoted by $\mathrm{Spin}(6) \supset \mathrm{Spin}(5) \supset \mathrm{Spin}(3)$, respectively.[7] The quantum numbers which are needed to classify uniquely the states are as follows:

(a) Three quantum numbers $(\sigma_1, \sigma_2, \sigma_3)$ which characterize the irreducible representations of Spin(6). For systems with N bosons and either zero or one fermion (even or odd representations), which are those considered in this Letter, the values of σ_1, σ_2, and σ_3 are given by

$$\sigma_2 = \sigma_3 = 0, \ \sigma_1 = N, N-2, \ldots, 0 \text{ or } 1 \tag{2}$$

for even representations, and by

$$\sigma_2 = \sigma_3 = \tfrac{1}{2}, \ \sigma_1 = N+\tfrac{1}{2}, \ N-\tfrac{1}{2}, \ N-\tfrac{3}{2}, \ldots, \tfrac{1}{2} \tag{3}$$

for odd representations.

(b) Two quantum numbers (τ_1, τ_2) which characterize the irreducible representations of Spin(5). The values of (τ_1, τ_2) contained in each Spin(6) representation $(\sigma_1, \sigma_2 = \sigma_3 = \frac{1}{2})$ are given by

$$\tau_2 = 0, \ \tau_1 = \sigma_1, \sigma_1 - 1, \ldots, 0 \tag{4}$$

for even representations, and by

$$\tau_2 = \tfrac{1}{2}, \ \tau_1 = \sigma_1, \sigma_1 - 1, \ldots, \tfrac{1}{2} \tag{5}$$

for odd representations.

(c) A quantum number ν_Δ which takes into account the fact that the step from Spin(5) to Spin(3) is not fully reducible ($\nu_\Delta = 0, 1, 2, \ldots$ for even representations and $\nu_\Delta = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$ for odd representations).

(d) The angular momentum J and its z component M. The values of J contained in each (τ_1, τ_2) representation are given by the rules of Ref. 4 for even representations, and by

$$J = [2(\tau_1 - \tau_2) - 6\nu_\Delta + \tfrac{3}{2}], \ [2(\tau_1 - \tau_2) - 6\nu_\Delta + \tfrac{1}{2}], \ldots, \ \{(\tau_1 - \tau_2) - 3\nu_\Delta - \tfrac{1}{4}[1 - (-1)^{2\nu_\Delta}] + \tfrac{3}{2}\} \tag{6}$$

for odd representations. Rule (6) gives the following angular momentum content in the lowest representations:

(τ_1, τ_2)	$\nu_\Delta = 0$	$\nu_\Delta = \frac{1}{2}$	$\nu_\Delta = 1$
$(\frac{1}{2}, \frac{1}{2})$	$J = \frac{3}{2}$		
$(\frac{3}{2}, \frac{1}{2})$	$J = \frac{7}{2}, \frac{5}{2}$;	$J = \frac{1}{2}$	
$(\frac{5}{2}, \frac{1}{2})$	$J = \frac{11}{2}, \frac{9}{2}, \frac{7}{2}$;	$J = \frac{5}{2}, \frac{3}{2}$	
$(\frac{7}{2}, \frac{1}{2})$	$J = \frac{15}{2}, \frac{13}{2}, \frac{11}{2}, \frac{9}{2}$;	$J = \frac{9}{2}, \frac{7}{2}, \frac{5}{2}$;	$J = \frac{3}{2}$
...	...	...	...

Once the classification scheme has been constructed, one can write down immediately the extension of the energy formula (1) to include both fermionic and bosonic spectra, so that

$$E(N, (\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2), \nu_\Delta, J, M) = -\tfrac{1}{4}A[\sigma_1(\sigma_1+4) + \sigma_2(\sigma_2+2) + \sigma_3^2] + \tfrac{1}{6}B[\tau_1(\tau_1+3) + \tau_2(\tau_2+1)] + CJ(J+1). \tag{7}$$

Here $\sigma_1(\sigma_1+4) + \sigma_2(\sigma_2+2) + \sigma_3^2$ is the eigenvalue of the quadratic Casimir operator of Spin(6), $\tau_1(\tau_1+3) + \tau_2(\tau_2+1)$ is that of Spin(5), and $J(J+1)$ is that of $\mathrm{Spin}(3) \approx \mathrm{SU}(2)$. This formula reduces to (1) when $\sigma_2 = \sigma_3 = 0$, $\tau_2 = 0$, and $J = L$. The corresponding odd and even spectra are shown in Figs. 1 and 2.

A remarkable feature of the observed spectra

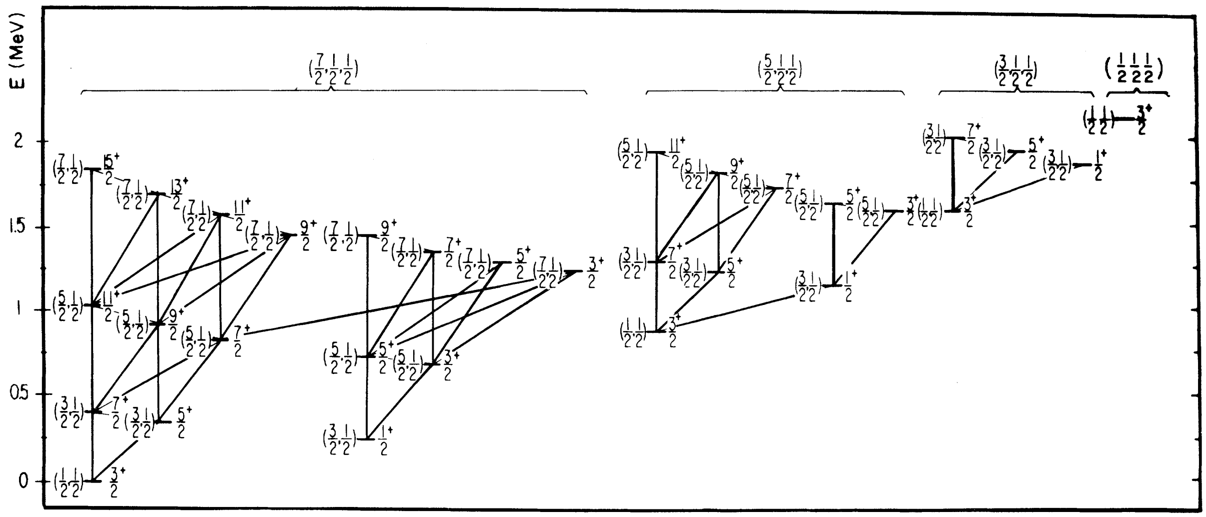

FIG. 1. Typical odd spectrum in the supersymmetric situation described by Eq. (7) ($N = 3$). The energy levels are given by Eq. (7) with $\frac{1}{4}A = 80$ keV, $\frac{1}{6}B = 60$ keV, and $C = 10$ keV. The ground state is taken as zero of the energy. The numbers in parentheses next to each level denote the Spin(5) quantum numbers (τ_1, τ_2). The numbers on the top of the figure denote the Spin(6) quantum numbers (σ_1, $\sigma_2 = \sigma_3$). The lines connecting the levels denote large electromagnetic ($E2$) transitions.

in the region of the platinum nuclei is that they display the supersymmetric structure of Figs. 1 and 2. In this region the even nuclei are well described by an O(6) symmetry.[3] The odd-proton nuclei have, among other states, a well developed structure built on the $d_{3/2}$ level. This structure appears to be intimately related to that of the adjacent even nucleus, as shown in Fig. 3 for the pair of nuclei, ${}^{192}_{78}\mathrm{Pt}_{114}$ (even states) and ${}^{191}_{77}\mathrm{Ir}_{114}$

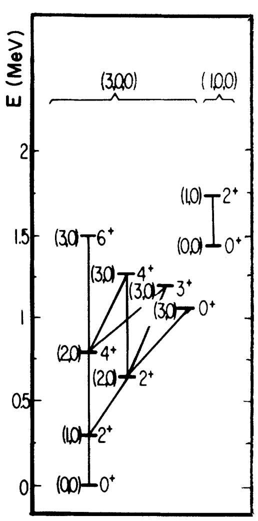

FIG. 2. Same as for Fig. 1: typical even spectrum.

(odd states). Not only the observed states can be classified in terms of the group chain Spin(6) $\supset$ Spin(5) $\supset$ Spin(3), but also their energy can be reasonably well reproduced by Eq. (7). In the figure, only the observed states belonging to the lowest representations ($\sigma_1 = N$, $\sigma_2 = \sigma_3 = 0$) and ($\sigma_1 = N + \frac{1}{2}$, $\sigma_2 = \sigma_3 = \frac{1}{2}$) are shown. Both in the case of the even and the odd nucleus, there is some additional evidence for the occurrence of higher representations ($\sigma_1 = N - 2$, and $\sigma_2 = \sigma_3 = 0$ in the even spectra and $\sigma_1 = N - \frac{1}{2}$, and $\sigma_2 = \sigma_3 = \frac{1}{2}$ in the odd spectra). However, this evidence is not conclusive and further experimental studies are needed in order to establish firmly the occurrence of these states.

In conclusion, I have suggested that the spectra of complex nuclei may provide examples of supersymmetric structures and I have shown one of these examples. This suggestion may have several implications. From the formal point of view, one may investigate the relationship between the supersymmetric structures described here and graded Lie algebras.[1] In particular, one could study the imbedding of Spin(6) into a graded group. An obvious candidate for the graded group is U(6|4) with Bose sector U(6) $\otimes$ U(4).[10] From the practical point of view, the presence of the supersymmetry will give rise to closed expressions for electromagnetic transition rates, transfer reaction intensities, etc., much in the same

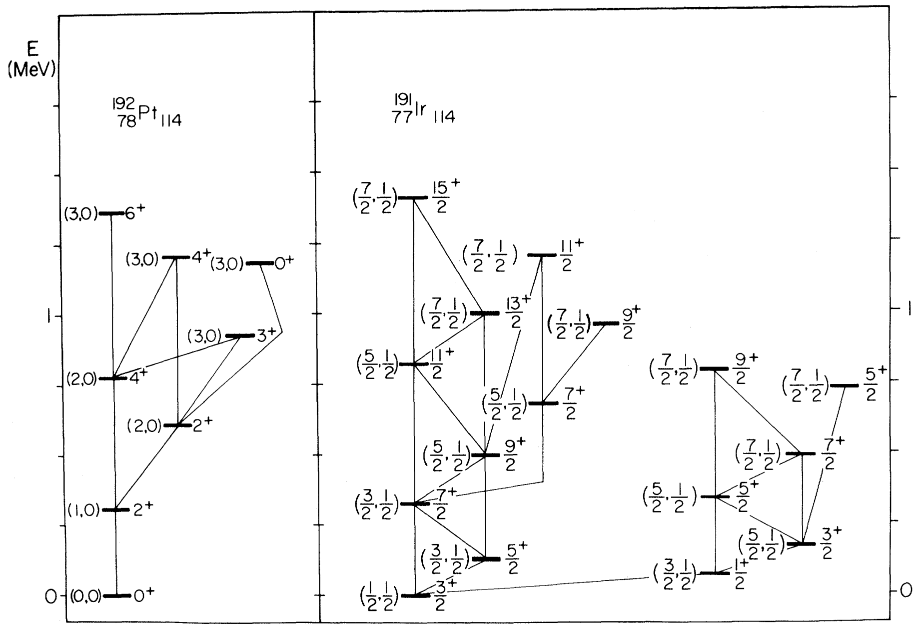

FIG. 3. An example of supersymmetric structures in heavy nuclei: the experimental spectra of $^{192}_{78}Pt_{114}$ (even spectrum, Ref. 8) and $^{191}_{77}Ir_{114}$ (odd spectrum, Ref. 9). The lines connecting the levels denote observed electromagnetic transitions ($E2$ and $M1$).

way as for an ordinary symmetry. However, these will involve now both even and odd spectra. Finally, one may note that the supersymmetry situation discussed here may be a consequence of the composite structure of the bosonic variables, which, at the microscopic level, are described by nucleon pairs.[11,12]

I wish to thank J. Wood and Ch. Vieu for bringing my attention to the spectra of odd-A nuclei in the platinum region, O. Scholten for performing the numerical calculations which illuminated the coupling of a $j=\frac{3}{2}$ spinor to the O(6) even core, and F. Gürsey and I. Bars for stimulating my interest in supersymmetries and graded Lie algebra. This work was in part supported by the Department of Energy, Contract No. EY-76-C-02-3074, and in part by the Stichting voor Fundamenteel Onderzoek der Materie (FOM).

[1]L. Corwin, Y. Ne'eman, and S. Sternberg, Rev. Mod. Phys. 47, 573 (1975), and references therein.
[2]F. Iachello, Comments Nucl. Part. Phys. 8, 59 (1978).
[3]J. A. Cizewski, R. F. Casten, G. J. Smith, M. L. Stelts, W. R. Kane, H. G. Börner, and W. F. Davidson, Phys. Rev. Lett. 40, 167 (1978).
[4]A. Arima and F. Iachello, Phys. Rev. Lett. 40, 385 (1978).
[5]F. Iachello and O. Scholten, Phys. Rev. Lett. 43, 679 (1979).
[6]The notation used here for spinor representations is that given by F. D. Murnagham, *The Theory of Group Representations* (John Hopkins, Baltimore, 1938).
[7]R. Gilmore, *Lie Groups, Lie Algebras, and Some of Their Applications* (Wiley, New York, 1974), p. 111. Note that Spin(6) ≈ SU(4), Spin(5) ≈ USp(4), and Spin(3) ≈ SU(2).
[8]M. Finger, R. Foucher, J. P. Husson, J. Jastrzebski, A. Johnson, G. Astzer, B. R. Erdal, A. Kjelberg, P. Patzelt, Å. Hoglund, S. G. Malmskog, and R. Henck, Nucl. Phys. A188, 369 (1972).
[9]J. Lukasiak, R. Kaczarowski, J. Jastrzebski, S. André, and J. Treherne, Nucl. Phys. A313, 191 (1979).
[10]P. G. O. Freund and I. Kaplansky, J. Math. Phys. 17, 228 (1976).
[11]A. Arima, T. Otsuka, F. Iachello, and I. Talmi, Phys. Lett. 66B, 205 (1977).
[12]J. Ginocchio, Phys. Lett. 85B, 9 (1979).