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Search for a Nonlinear Variant of the Schrödinger Equation by Neutron Interferometry

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A slow-neutron interferometer system has been used to test a nonlinear variant of the Schrödinger equation $i\hbar\, \partial\psi(\vec{r},t)/\partial t = [(-\hbar^2/2m)\nabla^2 + U(\vec{r},t)]\psi - b\ln(a^3|\psi|^2)\psi$. If this equation were correct, then, as Shimony has suggested, repositioning an attenuating plate downstream in a neutron beam would produce a phase modification. No measurable phase shift beyond experimental uncertainty was found and an upper limit of 3.4×10^{-13} eV for the energy constant b was established.

Recently, Shimony[1] suggested that newly developed neutron interferometer systems offer sufficient sensitivity to test the physical reality of some nonlinear variants of the Schrödinger equation (SE), in which an additional energy term of the form $F(|\psi^2|)$ appears in the Hamiltonian. In an earlier theoretical investigation, Bialynicki-Birula and Mycielski[2] had pointed out that, unlike the liner SE, such nonlinear equations can have traveling wave-packet solutions that do not spread spatially with time. Nevertheless, several features of the linear theory and its interpretation were shown to be retainable: Born interpretation of the wave function, Galilean invariance, and conservation of probability. Furthermore, these authors show that, in the case of a multiple-particle system, such nonlinear equations imply physically unattractive correlations between noninteracting particles unless the function F is taken to be logarithmic. Consequently, in the case of a single-particle system, the specific modification of the SE proposed by these writers is

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = \left[\frac{-\hbar^2}{2m}\nabla^2 + U(\vec{r},t)\right]\psi - b\ln(a^3|\psi|^2)\psi . \qquad (1)$$

Here, a is a length that need not be a universal constant, since a change in its value is equivalent to adding an unobservable constant potential to the Hamiltonian. The energy constant b, however, must be a fundamental constant, the same for all systems, and, in addition, must be positive if nonspreading wave packets are to be achieved. Bialynicki-Birula and Mycielski estimate from experimental values of the Lamb splitting of the energy levels of hydrogen that

$$b < 4\times10^{-10}\ \text{eV}.$$

Such nonlinear modifications of the SE imply, as Shimony has pointed out, that a mere downstream repositioning of an intensity attenuating plate in a particle beam would cause a change of phase not predicted by the linear SE. For the general case that the nonlinear term is $F(|\psi^2|)$ it was shown that the phase shift Δ would be expected to be

$$\Delta = (d/\hbar)(m/2E)^{1/2}[F(|\psi^2|) - F(\alpha^2|\psi^2|)], \qquad (2)$$

where d is the translated distance of the attenuator (of intensity attenuation α^2) and m and E are the mass and kinetic energy of the particles in the beam. For the particular logarithmic form

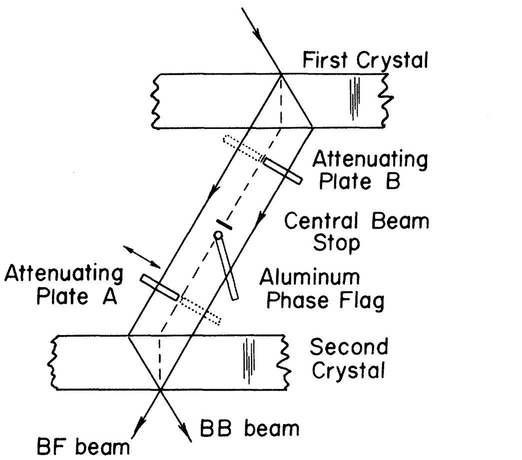

FIG. 1. Neutron interferometer used in the search for a nonlinear term in the Schrödinger equation. Coherence exists between rays on the left and right sides of the beam passing between the two perfect-crystal plates. Phase effects are exhibited in the intensity of the outgoing beams, BB and BF. Intensity attenuating plates are positioned as shown and these are shuttled left and right in the experiment. The solid and the dotted shuttle positions are called configurations I and II, respectively.

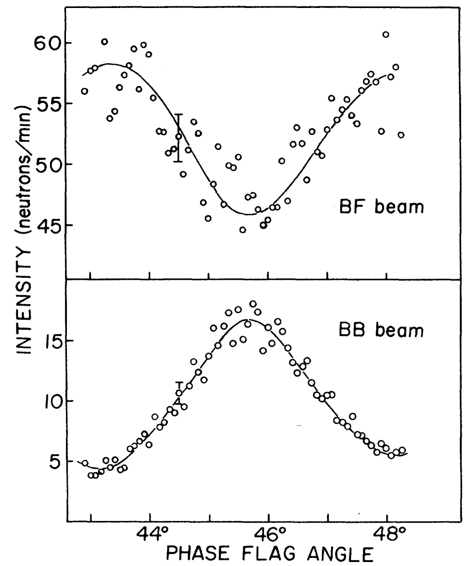

FIG. 2. Neutron-interferometer-fringe display as relative phase between left and right rays is continuously varied by rotating the Al phase flag.

of the nonlinear term in Eq. (1), the phase shift reduces to the simple form

$$\Delta = (\tau b/\hbar)\ln|\alpha^2|, \tag{3}$$

where τ is the particle transit time over the distance d.

We have searched for the existence of this Shimony phase shift in two experiments performed in a slow-neutron interferometer system with attenuating plates of LiF and Cd. No measurable effect was found within experimental uncertainty. This places an experimental upper limit on the value of the fundamental constant b of 3.4×10^{-13} eV which is over three orders of magnitude lower than the limit previously inferred from the Lamb splitting.

The experiments have been performed in a two-crystal neutron interferometer[3] in which two spatially separated, parallel, and coherent beams travel between the two perfect silicon crystals as shown in Fig. 1. Neutron wave packets (of De Broglie wavelength 1.564 Å, energy 0.0335 eV) are split coherently by the first crystal into ray components forming a broadened beam of width 10.6 mm which are then collected at the focal position of the second crystal. Radiation from this focal point is released in two directions, BB and BF, and intensity is measured in both directions by conventional neutron detectors. Complementary intensity changes are measured in the two directions when phase-changing agents are introduced in either side of the coherently split internal beam. In order to clearly delineate two propagating coherent beams, we utilize a central beam stop (width 4.2 mm) of absorbing cadmium. Convenient phase adjustment is provided by a phase flag (a transparent aluminum plate of thickness 1.99 mm which can be rotated about the central axis) in one of the beams. A sinusoidal variation of intensity (a fringe display), complementary in the two exit directions, is found as the angular position of the phase flag is changed. This is illustrated in Fig. 2, where attenuating plates have been placed in both coherent beams as in the later experiment. The higher base-line intensity in the BF beam over that of the BB beam is expected from the theory of interferometer operation. Careful positioning of elements within the interferometer is provided by micrometer control with no physical contact with the interferometer crystals, which in our case are air-bag supported to isolate them from laboratory vibration. A temperature environment constant to within a few millidegrees is also provided.

In search for the Shimony phase shift we have utilized two attenuating plates, one in each beam, which can be shuttled in automatic operation from left to right for the upstream plate and vice versa for the downstream plate. This mode of observation serves to double the sensitivity of the experiment at the expense of introducing an extraneous effect if the two plates differ in their intrinsic refractive index or thickness. This extraneous effect can be eliminated, however, by accumulating data taken with the plates interchanged. We define θ_n as the measured phase angle shift in changing from configuration (I) to configuration (II) of Fig. 1, when the plates are positioned in the normal sense of plate A being downstream and plate B being upstream in the beam. Likewise we define θ_i as the equivalent quantity when the plates are interchanges so that plate A is now upstream in the beam. It can be shown that

$$\theta_n = 2\Delta + 2(\varphi_A - \varphi_B), \quad \theta_i = 2\Delta - 2(\varphi_A - \varphi_B), \qquad (4)$$

where Δ is the Shimony phase shift from Eq. (3) and φ_A is the intrinsic phase angle introduced to the neutron beam by plate A. Thus the desired quantity Δ can be obtained from data collected with the plates in both normal and interchanged sense.

Attenuating plates for the first experiment were prepared from a single crystal of LiF. These were of dimensions $12.6\times9.4\times2.41$ mm^3 and were cut and polished in a manner to insure equivalent thickness and face parallelism. Initial transmission measurements in the neutron beam showed their transmission to be the same ($\alpha^2 = 0.4335$) within the statistical accuracy of 0.2% of the measurement. Using the accepted values for the Li and F coherent neutron scattering amplitudes, we calculate the intrinsic phase angle φ introduced by either plate to the neutron beam to be 4580 deg or the phase equivalent of a 12.72-fringe displacement. The interferometer measurements were performed by alternately shuttling the plates from configuration (I) to configuration (II) with 10-min counting intervals. For maximum sensitivity to phase shifts, the phase flag was positioned so that the counting was done on the steep side of a fringe. This shuttling was performed many times, then data were similarly collected on the other side of the fringe. The plates were then interchanged and an equivalent set of data was acquired in the interchanged sense. The amplitude of the fringe display and the intensity change from configuration (I) to configuration (II) determine the phase-angle change θ.

Results for the LiF attenuators are summarized in the upper section of Table I where the indicated uncertainties represent the standard deviations of the measured quantities. The standard-deviation values are the larger of either the internal or external deviations, the two being in close agreement for all entries. Finite values for the measured phase shift are observed, with, however, a reversal of sign upon plate interchange. Using Eqs. (4), we obtain values for Δ and $\varphi_A - \varphi_B$ of $+0.08°$ ($\pm 0.30°$) and $+2.69°$ ($\pm 0.30°$), respectively. Thus the LiF plate data indicate no measurable value for the Shimony phase angle, and from Eq. (3), with $\tau = 1.123\times10^{-5}$ s as the transit time between plate positions separated by 28.4 mm, we obtain

$$b = (-1.0 \pm 3.7)\times10^{-13}\ \text{eV}.$$

The quite measurable value of $+2.69°$ ($\pm 0.30°$) for the intrinsic–phase-angle difference between the two LiF plates would correspond to a fractional thickness difference of 1 part in 1700, or a thickness difference of only 1.4 μm.

TABLE I. Measured phase shifts induced by attenuating plates in a neutron interferometer. The measured shifts yield values for Δ, the phase shift predicted by the nonlinear term in Eq. (1).

	Total neutrons counted BB	Total neutrons counted BF	θ_{meas}
LiF attenuating plates			
Normal arrangement (A downstream, B upstream)	118 272	584 219	$+5.53°$ ($\pm 0.79°$)
Interchanged arrangement (A upstream, B downstream)	85 941	446 784	$-5.22°$ ($\pm 0.92°$)
$\Delta = +0.08°$ ($\pm 0.30°$), $\varphi_A - \varphi_B = +2.69°$ ($\pm 0.30°$)			
Cd attenuating plates			
Normal arrangement	145 559	926 066	$-1.49°$ ($\pm 0.79°$)
Interchanged arrangement	150 916	987 776	$+0.60°$ ($\pm 0.76°$)
$\Delta = -0.22°$ ($\pm 0.27°$), $\varphi_A - \varphi_B = -0.52°$ ($\pm 0.27°$)			

Because of this instrumental asymmetry, it was decided to perform a second experiment with a pair of cadmium plates, where it was hoped that lesser asymmetry would be encountered. Because the absorption cross section for Cd is much larger than that for LiF, a much thinner plate is needed to attain the optimum e^{-1} transmission. This in turn means that the intrinsic phase angle introduced by passage of neutrons through the Cd plate is reduced to 132°, about 35 times smaller than that encountered with the LiF plates. It follows that less control of the thickness equivalence of two plates is called for in minimizing intrinsic phase differences. Accordingly, a thin sheet of cadmium was cold rolled to the desired thickness (0.086 mm) and various sections were studied by neutron transmission. Selected sections were then cut and mounted on picture-frame holders for use in the interferometer. Plates A and B gave transmission values of 0.378 55 and 0.378 94, respectively, with an uncertainty of 0.000 43.

A package of interferometer data was then obtained with the Cd plates in the same manner as in the LiF experiment, again with plate interchange, and the results are summarized in the lower section of Table I. The results for Cd indicate that the intrinsic plate asymmetry is now much reduced from that encountered in the first experiment and also that the Shimony phase angle Δ is again less than 1 standard deviation. Interpreting the results of the Cd-attenuator experiment in terms of Eq. (3), we obtain

$$b = (+2.3 \pm 2.9) \times 10^{-13} \text{ eV}.$$

Combining the results of the two experiments, we obtain

$$b(\text{total data}) = (+1.1 \pm 2.3) \times 10^{-13} \text{ eV}.$$

This establishes an upper limit on the magnitude of the nonlinear term in Eq. (1) (Ref. 4) which is about three orders of magnitude smaller than the upper limit value of 4×10^{-10} eV implied by the Lamb-shift experiments. It may be noted that the sensitivity attained in the present experiment implies that the interferometer technique can be exploited to measure very small energy modifications of a neutron beam on the order of 10^{-13} eV.

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[4]From the data obtained we are unable to make any interesting statements about limits on the magnitude of a general nonlinear term $F(|\psi|^2)\psi$, unless the form of the function F is specified and unless the absolute magnitude of $|\psi|^2$ is known.

Attenuation and Recombination of Quarks in Nuclear Matter

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Quark models of hadron production in the beam-fragmentation region are extended to production off nuclei by taking into account quark attenuation in nuclear matter. Simple expressions are derived for the A dependence of the production of beam fragments. They reproduce well the experimental data on hadron-nucleus and virtual-photon–nucleus collisions.

Recent experimental studies of interactions of high-energy leptons[1] and hadrons[2] with nuclear targets have confirmed previous observations[3] of a nuclear attenuation in nondiffractive production of hadrons with large x and small p_T.

In this Letter we include quark attenuation in nuclear matter in the standard quark models[4] of hadron production and derive simple expressions for the A dependence[5] of hadron production at large x and small p_T. The expressions are independent of the details of the hadronization mechanism, have no free parameter, and describe re-

