Nuclear Density Oscillations

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Nuclear density oscillations coming from a semi-infinite nuclear matter calculation in the local-density approximation are compared with the density oscillations obtained by the Green's-function method of Thorpe and Thouless. A new, simpler method without use of Green's functions which easily leads to higher terms in density expansion is proposed. The appearance of the first "bump" in density on the rising density curve was justified. Connection with Regge-type oscillations is discussed.

A study of infinite nuclear matter—an infinite medium filled with equal numbers of protons and neutrons, interacting through the nuclear force and with the Coulomb force among protons assumed to be turned off—gives two important parameters of such a system: The nuclear-matter density (or a closely related quantity called the Fermi momentum), and volume energy. Also because of simplicity of this system, due mainly to its translational invariance, a study of it has been usually considered a first step towards understanding of finite nuclei.

A system with a medium of nuclear matter filling half of the space ($z>0$ for definiteness) and an empty medium at the other half of the space ($z<0$) is a logical candidate to be studied next. In this case the properties of the two media near the interface will change, but the important thing is that, far from the interface, all properties —which one can now regard as asymptotic properties of the new system, semi-infinite nuclear matter—will be the same. This will greatly help one in studying such properties of semi-infinite nuclear matter as the surface density and surface energy.

A good number of calculations for a semi-infinite nuclear matter can be found in the literature.[1] However, all of these calculations seem to suffer from one or more of the following shortcomings:

(1) Lack of numerical accuracy.

(2) Lack of a good two-body interaction and, in particular, lack of density dependence.

(3) Most of the calculations either do not take into account the dependence on the parallel component of the momentum along the surface ($k_\perp$) or average it somehow.

(4) Particularly as a result of (1), density oscillations are absent in results of all semi-infinite calculations (these are present in some slab calculations, though[2,3]).

In Ref. 4 an attempt was made to improve on the above. With use of a Sprung-Banerjee G-0 interaction (a Gaussian density-dependent interaction with range $\lambda=\frac{1}{2}$ fm),[5] a semi-infinite calculation in the LDA (local-density approximation) was performed which, among other things, clearly showed the density oscillations (Fig. 1). The present method is similar to that of Negele,[6] but in dealing with a semi-infinite system all sums over single-particle states turn into integrals over such states.

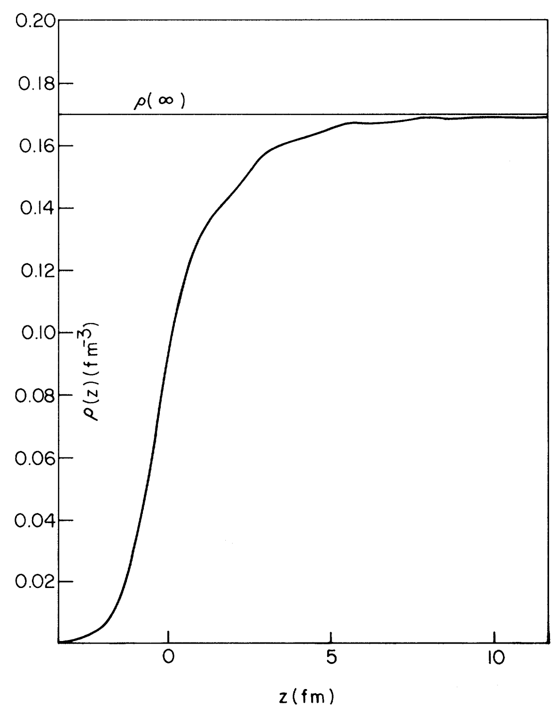

FIG. 1. Self-consistent density as a function of distance into the semi-infinite nuclear matter in the LDA after four iterations. Step size is 0.5 fm. A Fermi distribution potential was used to start the iterations as well as to help solve the integro-differential equation in subsequent iterations. Note that in this figure and also in Fig. 2 the origin is chosen arbitrarily at the half-density point after the calculation.

It is interesting to subtract Thorpe and Thouless' density oscillations[7] from the above density. One should get an almost smooth curve after such subtraction (almost smooth because of higher-order oscillatory terms and because of errors in calculation). To enhance the visibility of the behavior of the curves I further subtract a known smooth function which follows the general trend of these curves (Fig. 2).

In Thorpe and Thouless' paper there is no attempt at comparing the theory with a semi-infinite-nuclear-matter calculation and they only compare the theory with a simple model of ^{208}Pb. But as they have also noted, in such a case the surface oscillations interfere with central oscillations and oscillations due to the states of high angular momentum, and so they find that any comparison beyond checking the wavelength is not worthwhile.

I point out here that the density oscillations can be obtained without the use of Green's functions and by simply making an asymptotic expansion of the density. I start with the definition of the density (as used in Ref. 4),

$$\rho(z) = (4/\pi^2)\int_0^{k_F} k_\zeta\, dk_\zeta \int_0^{k_\zeta} f_{k_z,k_\zeta}{}^2(z)\, dk_z, \tag{1}$$

where $k_\zeta \equiv (k_F{}^2 - k_\perp{}^2)^{1/2}$ and f_{k_z,k_ζ}'s are the single-particle wave functions. Although the inclusion of k_ζ dependence in calculation of the density oscillations produces no difficulty in principle, one neglects it to keep the formalism simpler. Thus I may set $k_\zeta = k_F$. Applying the JWKB approximation, one gets

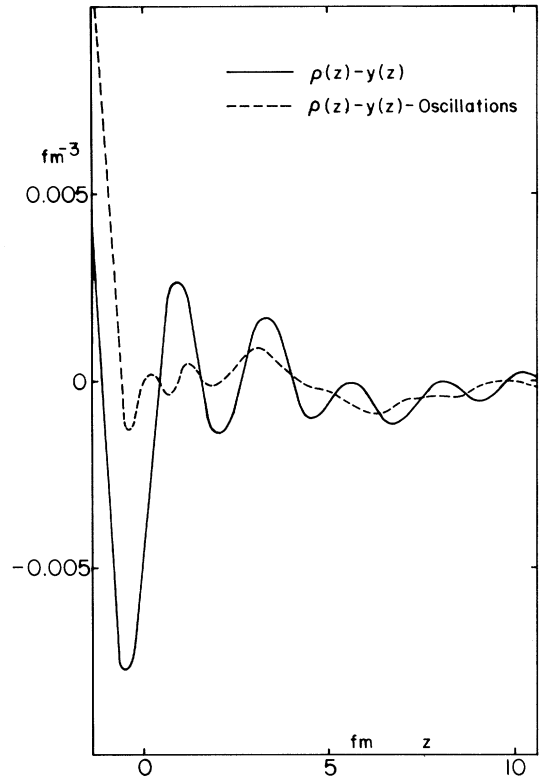

FIG. 2. The full curve is density oscillations obtained by subtracting a "smooth curve," $y(z) = 0.169(1 - e^{-[(z+1.59)/1.24]^2}) + 0.005(z - 0.27)e^{-[(z-0.27)/3]^2}$, from the density of Fig. 1. The dotted curve is the full curve after having subtracted the Thorpe-Thouless oscillations [first term in formula (4)] from it.

$$\rho(z) \simeq -\frac{1}{\pi^2}\int_{k_{z_0}(z)}^{k_F} (k_F{}^2 - k_z{}^2)\frac{k_z}{K_{k_z,k_F}(z)}\cos[2(\textstyle\int_{z_0(k_z)}^{z} K_{k_z,k_F}(z')\,dz' + \frac{1}{4}\pi)]\,dk_z + \text{nonoscillatory terms}, \tag{2}$$

where

$$K_{k_z,k_\zeta}(z) \equiv [k_z{}^2 + U_{k_z,k_\zeta}(\infty) - U_{k_z,k_\zeta}{}^{\text{eq}}(z)],^{1/2}$$

with U^{eq} the equivalent one-body potential, $z_0(k_z)$ the turning point for wave function with momentum k_z, and $k_{z_0}(z)$ the momentum for the wave function with turning point z. With use of the asymptotic expansion

$$\int A(k)e^{iB(k)}\,dk = \frac{e^{iB(k)}}{iB'(k)}\sum_{n=0}^{\infty}(-1)^n\left[\frac{d}{dk}\left(\frac{1}{iB'(k)}\right)\right]^n A(k), \tag{3}$$

which can also be easily arranged in increasing in inverse powers of the derivative B', the formula (2) can be written as

$$\rho(z) \simeq \frac{k_F{}^2}{2\pi^2 K_{k_F,k_F}(z)}\,\frac{\cos[2(\int_{z_0(k_F)}^{z} K_{k_F,k_F}(z')\,dz' + \frac{1}{4}\pi)]}{\{\int_{z_0(k_F)}^{z}[\partial K_{k_F,k_F}(z')/\partial k_z]_{k_z=k_F}\,dz'\}^2} + \text{higher-order oscillatory terms}$$

$$+ \text{nonoscillatory terms}. \tag{4}$$

The first term and also the lower limits in the formula (3) do not contribute to the oscillations. According to Regge theory,[8] pairing would produce another kind of oscillation around the surface of the semi-infinite system. However, the wavelength of such oscillations is of the order of 6.5 fm, which is much larger than wavelength of the Thrope-Thouless-type oscillations. The two kinds of oscillations are thus quite distinguishable and the two types of calculations can be considered complementary.[8]

Of particular interest is the appearance of the first bump on density distribution long before the regime of nuclear matter is reached. This might be surprising but it is not caused by a mistake in calculation. It corresponds to the first zero of the highest-lying momentum state in the Fermi sea.[4] It agrees with considerations based on a Thomas-Fermi calculation in a linear potential.[9] It is also in agreement with considerations due to diffuseness of the one-body potential.[4] Furthermore, it can also occur in a slab calculation,[3] if the range of the two-body interaction is chosen long enough. However, in this last case the Skyrme-type interaction in Ref. 3 was found not good enough to produce accurate density oscillations, as has already been pointed out.[10]

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Collective Sideward Flow of Nuclear Matter in Violent High-Energy Heavy-Ion Collisions

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Angular and energy distributions of fragments emitted from fast nucleus-nucleus collisions (Ne→U at 250, 400, and 800 MeV/N) are calculated with use of nuclear fluid dynamics. A characteristic dependence of the energy spectra and angular distributions on the impact parameter is predicted. The preferential sideward emission of reaction fragments observed in the calculation for nearly central collisions seems to be supported by recent experimental data.

Sideward emission of nuclear matter was one of the first predictions of fluid-dynamical model calculations of fast nuclear collisions, implying considerable transformation of the incident kinetic energy into compression and heat energy.[1-6] The outflow of matter from the highly compressed "shock" region reflects a large transverse-momentum transfer. Quite early experiments[2,7] supported this hypothesis: Preferential sideward emission of (mainly) α particles was observed in the irradiation of 4π-sr particle-track detectors with light nuclei at $E_{\rm lab} = 0.2-4.2$ GeV/N when selecting nearly central collisions (only azimuthally symmetric many-pronged stars). This was inter-

