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Coherence Angle in Heavy-Ion Reactions

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The angular cross-correlation function of cross-section fluctuations in heavy-ion reactions is investigated. It is demonstrated that in general the coherence angle is much larger than commonly assumed. Consequences for the identification of heavy-ion resonances are discussed.

The identification of isolated resonances in heavy-ion reactions at energies above the Coulomb barrier is made difficult by the fact that the width and amplitude of the observed structures are comparable to the structure predicted by statistical models of nuclear reactions (Ericson fluctuations). Criteria for the isolation of resonances are based on statistical-model predictions. In order to separate fluctuations from resonances, excitation functions for various reaction channels (channel-channel correlations) or, for a particular exit channel, excitation functions at various angles (angular cross-correlations) are usually studied. In purely statistical reactions, no correlations are expected between excitation functions for different exit channels. Also all correlations should vanish for excitation functions measured at scattering angles θ and θ' if $|\theta-\theta'|>\theta_c$ where θ_c is the coherence angle.

In heavy-ion reactions, the coherence angle is usually estimated using the "black-nucleus" approximation of Ref. 1. In this approximation, the magnitude of the coherence angle is $\theta_c=(kR)^{-1}$, where k is the wave number of relative motion and R is the nuclear radius. This estimate has been used, either implicitly or explicitly, to help separate resonance structure from statistical fluctuations (see, e.g., Refs. 2–7).

In the present note, we show that the black-nucleus approximation of Ref. 1 is not applicable to the description of heavy-ion reactions. Rather, due to the angular momentum dependence of the transmission coefficients and the level density, the cross section predicted by the statistical model is strongly localized in angular momentum resulting in a coherence angle $\theta_c \propto (\Delta L)^{-1}$, where ΔL is the width of the distribution of partial cross sections. This leads to values for the coherence angle which generally are very much larger than the prediction of the black-nucleus approximation. This will be quantitatively demonstrated in the following for two typical heavy-ion reactions. Furthermore, these studies show that the angular cross-correlation function provides a sensitive means to obtain information on the first and second moments of the distribution of partial cross sections in statistical reactions, which can be very useful partial cross sections in statistical reactions, which can be very useful, e.g., in mapping of the yrast line in highly excited compound nuclei.

The angular cross-correlation function $C(\theta,\theta')$ is defined as

$$C(\theta,\theta')=\frac{\langle\sigma(E,\theta)\sigma(E,\theta')\rangle}{\langle\sigma(E,\theta)\rangle\langle\sigma(E,\theta')\rangle}-1, \qquad (1)$$

where $\sigma(E, \theta)$ is the differential cross section and the angular brackets denote energy averages. It has been demonstrated in Ref. 1 that the cross-correlation function can be expressed in terms of the fluctuating part of the S matrix. The coefficients A are given by the expression

$$C(\theta, \theta') = \left[\sum_{\substack{S_i, S_f, M_i, M_f, \\ M_i', M_f'}} \left| \sum_{L_i L_f, J} A_{L_i L_f J}^{S_i S_f M_i M_f}(\theta) A_{L_i L_f J}^{S_i S_f M_i' M_f'}(\theta') \langle | S_{cc'}^{\mathrm{fl}} |^2 \rangle \right|^2 \right]$$

$$\times \left[\sum_{\substack{S_i, S_f, M_i, M_f, \\ L_i, L_f, J}} \left| A_{L_i L_f J}^{S_i S_f M_i M_f}(\theta) \langle | S_{cc'}^{\mathrm{fl}} |^2 \rangle \right|^2 \sum_{\substack{S_i, S_f, M_i, M_f, \\ L_i, L_f, J}} \left| A_{L_i L_f J}^{S_i S_f M_i M_f}(\theta') \langle | S_{cc'}^{\mathrm{fl}} |^2 \rangle \right|^2 \right]^{-1}, \tag{2}$$

where the initial channel is labeled by $c = S_i, M_i, L_i$, the final channel by $c' = S_f, M_f, L_f$; J is the total angular moment and $S_{cc'}^{\mathrm{fl}}$ is the fluctuating part of the S matrix. The coefficients A are given by the expression

$$A_{L_i L_f J}^{S_i S_f M_i M_f}(\theta) = |4\pi(2L_i + 1)|^{1/2} (S_f M_f L_f M_{L_f} | J M_i)(S_i M_i L_i 0 | J M_i) Y_{L_f M_f}(\theta, 0). \tag{3}$$

In Hauser-Feshbach theory,[8] $\langle | S_{cc'}^{\mathrm{fl}} |^2 \rangle$ is expressed in terms of optical-model transmission coefficients as

$$\langle | S_{cc'}^{\mathrm{fl}} | \rangle = T_c T_{c'} / \textstyle\sum_{c''} T_{c''}. \tag{4}$$

The black-nucleus approximation corresponds to setting $\langle | S_{cc'}^{\mathrm{fl}} |^2 \rangle$ to a constant for $L_i \leq kR$ and to zero for $L > kR$. The product $T_c T_{c'}$ indeed follows this behavior very closely. However, the denominator in Eq. (4) is approximately proportional to the compound-nucleus level density and, consequently, decreases rapidly with increasing total angular momentum. The combination of these two angular-momentum dependences produces a strong localization of the partial cross sections. This is shown in Fig. 1 for the reactions $^{12}C(^{16}O, ^{16}O)^{12}C$ and $^{12}C(^{16}O, \alpha)^{24}Mg$ at $E_{\mathrm{c.m.}} = 20$ MeV. Here, and in the following, calculations of the fluctuating S matrix have been performed using the code STATIS.[9] Also shown in Fig. 1 are the corresponding angular cross-correlations $C(90°, \theta')$ calculated using Eqs. (2) and (3). As expected, the cross-correlation pattern exhibits typical, diffractionlike features. The oscillations have a spacing approximately given by $\Delta\theta = \pi / L_0$, where L_0 is the angular momentum for which $\langle | S_{cc'}^{\mathrm{fl}} |^2 \rangle$ reaches its maximum. Contrary to the prediction of the black-nucleus approximation, the correlation function is large at angles corresponding to the second and higher maxima. Therefore, the relevant coherence angle θ_c is given by the half width at half maximum of the envelope of the correlation functions (dashed line in Fig. 1). The coherence angle θ_c is, then, proportional to $(\Delta L)^{-1}$, the width of the distribution of the partial cross sections. The angle θ_c is much larger than the half width θ_0 of the first oscillation, the value of which closely

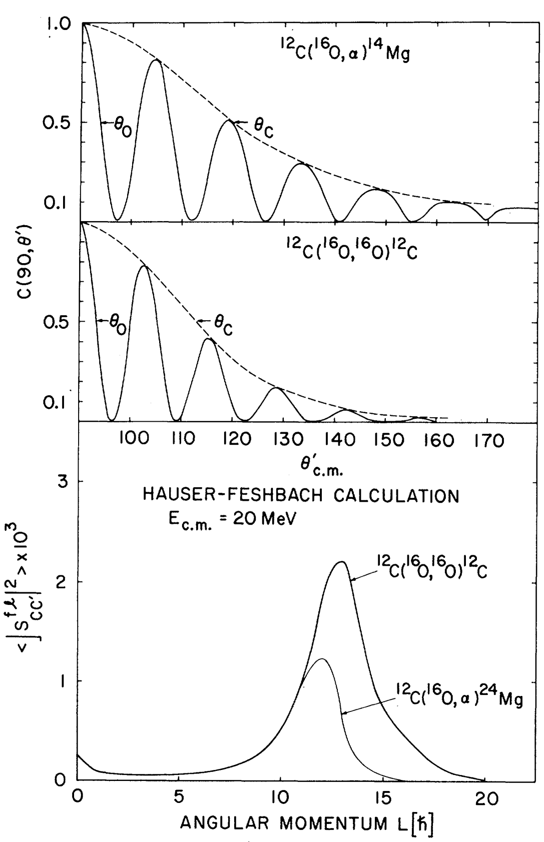

FIG. 1. The lowest panel shows the partial-wave distribution of the fluctuating cross section for the $^{12}C(^{16}O, ^{16}O)^{12}C$ and $^{12}C(^{16}O, \alpha)^{24}Mg$. The corresponding angular cross-correlation functions are shown in the two upper panels.

agrees with the predictions of the black-nucleus model, i.e., $\theta_0 \approx 1/L_0$. In this approximation, the higher maxima in the correlation function are very strongly suppressed.

Precisely these features are borne out by the experimental results of Ref. 10, where angular cross correlations for the reaction $^{12}C(^{16}O, \alpha)^{24}Mg$ near $E_{c.m.} = 13$ MeV were measured. In Fig. 2, the data for exit channels leading to the ground and first excited state in ^{24}Mg are compared to statistical-model calculations based on Eqs. (2) and (3). In order to be consistent with the correlation function experimentally determined in Ref. 10, the correlation function plotted in Fig. 2 is defined as

$$C_1(\theta) \sim \int_{\alpha_1}^{\alpha_2} d\theta' C(\theta, \theta'). \qquad (5)$$

The angular range $[\alpha_1, \alpha_2]$ has been taken from Table II of Ref. 10. The calculations are normalized to the data at $\theta = 0°$. Note that the present calculation does not include any contributions from direct interactions which will reduce the cross-correlation function without affecting its shape.

Good agreement is obtained between data and calculation, especially for the transition to the ground state in ^{24}Mg. It should be emphasized that the oscillations in the data for large angles may well be due to errors associated with the finite sample size and, consequently, the discrepancies observed at large angles should not be taken too seriously.

We have estimated the energy dependence of the coherence angle for the two reactions under consideration in the energy range $15 < E_{c.m.} < 35$

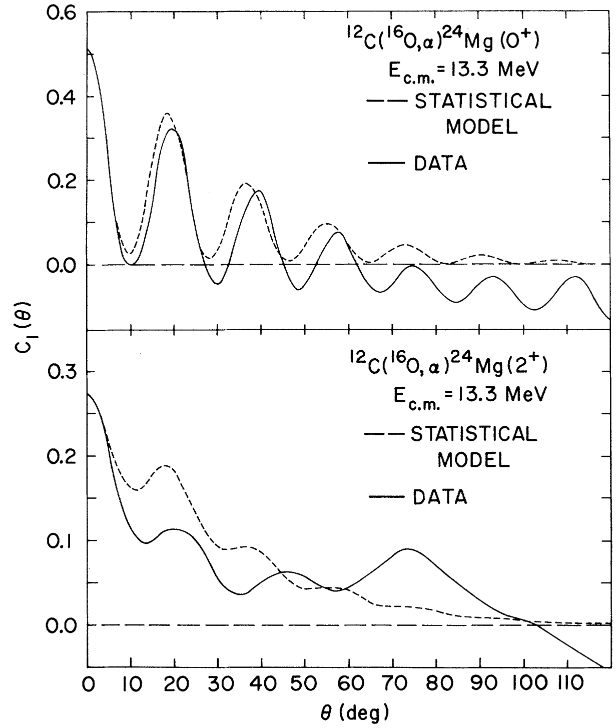

FIG. 2. Comparison of calculated angular cross-correlation functions with the data of Ref. 10. For details, see text.

MeV. The parameters for the transmission coefficients used in the calculations were adjusted to reproduce the measured fusion cross section.[11] The dependence of ΔL, L_0, θ_c, and θ_0 on energy are presented in Table I, along with the fusion cross section. It is obvious from this table that for energies not too close to the Coulomb barrier,

TABLE I. Energy dependence of θ_0, θ_c, ΔL, and L_0 for the reactions $^{12}C(^{16}O, ^{16}O)^{12}C$ and $^{12}C(^{16}O, \alpha)^{24}Mg$. The transmission coefficients have been adjusted to yield the total fusion cross section as given in the last column. The variations of ΔL with energy for the elastic scattering case are mainly due to the variation in the fusion cross section.

	$E_{c.m.}$ (MeV)	θ_0 (deg)	θ_c (deg)	ΔL	L_0	σ_{fusion} (mb)
$^{12}C(^{16}O, ^{16}O)^{12}C$	15	3.6	19	5.2	10.8	850
	20	3.2	22	3.3	12.8	920
	25	2.9	18	5.3	13.2	870
	30	2.6	19	5.0	16.0	900
	35	2.4	19	3.9	17.3	900
$^{12}C(^{16}O, \alpha)^{24}Mg$(g.s.)	15	4.3	27	2.5	9.8	850
	20	3.7	30	2.6	11.8	920
	25	3.3	26	3.0	12.6	870
	30	3.0	23	3.4	14.2	900
	35	2.7	25	3.2	15.0	900

the width ΔL is much smaller than L_0, resulting in a comparatively large coherence angle of $18° \lesssim \theta_c \lesssim 30°$, to be compared with the value in the black-nucleus approximation, $2° \lesssim \theta_0 \lesssim 5°$. From the previous calculations, one can obtain a rough estimate of the coherence angle by the empirical relation

$$\theta_c \approx 1.4/\Delta L. \tag{6}$$

The above considerations have an important bearing on the identification of heavy-ion resonances. Because of the generally small peak-to-valley ratio of these resonances and the usually strong statistical fluctuations of the nonresonant background, statistical tests for positive identification of resonant structures are essential. This is usually done by comparing measured excitation functions for various exit channels or for many angles. The large coherence angle obtained above, however, means that the number of statistically independent angles at which excitation functions can be compared, is very limited. This restriction is especially severe for reactions leading to heavy-ion exit channels, since the cross sections are usually dominated at forward angles by direct processes thereby restricting the interesting angular range.

In summary, we have shown that the angular cross-correlation function for fluctuating heavy-ion reactions has a characteristic diffractionlike pattern with a coherence angle of $\theta_c \approx 1.4(\Delta L)^{-1}$. This coherence angle is up to an order of magnitude larger than previously assumed. Therefore, identifications of resonant structures based on comparison of excitation functions at various angles need to be reconsidered.

Furthermore, it should be pointed out that nearly no experimentally determined angular cross-correlations exist in the literature. As is shown above, such data could be used to deduce information on the distribution of partial cross sections in fluctuating reactions. This may help to provide quantitative understanding of the physics of the evaporation of complex fragments.

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