Horizon Problem and the Broken-Symmetric Theory of Gravity

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A way of avoiding the standard scenario (which many people find vexing on philosophical grounds) is suggested in which particles are in causal contact with only a limited number of other particles in the early universe.

There is something puzzling about the standard homogeneous, isotropic cosmology. Shortly after the big bang, the scale factor $R(t)$ in the homogeneous isotropic Robertson-Walker metric is growing like $t^{1/2}$. On the other hand, the region of space with which any given particle could have communicated with by signals not traveling faster than the speed of light has a size of the order of[1] t. Thus, at early times, the size of causally connected domains is much smaller than the effective size of the universe. This fact has profound physical, and perhaps even theological, implications. It is tempting to argue[2] that the apparent homogeneity and isotropy of the universe have to have been created in the very beginning.

Following Rindler[3] we could give a more precise formulation of the situation. In a Robertson-Walker[4] universe a photon emitted at $r=r_H$ and a time t_0 (and traveling in the radial direction) will be seen by an observer situated at $r=0$ at time t. These quantities are related by

$$\int_{t_0}^{t}\frac{dt'}{R(t')}=\int_0^{r_H(t,t_0)}\frac{dr}{(1-kr^2)^{1/2}}. \tag{1}$$

The proper distance from $r=0$ to $r=r_H$ is given by

$$d(t,t_0)=R(t)\int_0^{r_H(t,t_0)}\frac{dr}{(1-kr^2)^{1/2}}. \tag{2}$$

Thus, if the integral on the left-hand side converges as $t_0\to 0$ then our observer at time t can only see, or more precisely, it causally influenced by only, a domain of finite size of order $d(t,0)$. I refer to this as the horizon problem. In particular, in the standard radiation-dominated universe $R(t)\sim t^{1/2}$, so that[5] as $t_0\to 0$

$$r_H\sim t^{1/2} \text{ and } d\sim t.$$

I would like to remind the reader how the behavior of $R(t)$ is deduced. Consider Einstein's equations governing the expansion of the universe in the crude form

$$\dot{R}^2/R^2\sim G\rho. \tag{3}$$

The energy density ρ, in the very hot early universe when all particles could be considered as essentially massless, is given by

$$\rho\sim NT^4, \tag{4}$$

where N is a factor counting essentially the number of species. Entropy conservation yields

$$R^3T^3\sim \text{const.} \tag{5}$$

Putting Eqs. (3), (4), and (5) together one finds

$$\dot{R}^2\sim GN(1/R^2) \tag{6}$$

and thus $R\sim t^{1/2}$.

Physically, the so-called horizon problem occurs because shortly after the big bang the universe was expanding too fast. Clearly the universe could be made to slow down if gravitation grew weaker as we go back in time.[6] There have been a number of suggestions that the gravitation constant varies with time, beginning with Dirac's large-number hypothesis.[7] However, this hypothesis implies that $G\sim 1/t$ and gravity grows stronger as one goes back in time. It has been shown[8] that the latest formulation of this suggestion[9] is incompatible with the observed helium abundance.

Recently, it was proposed[10,11] that the concept of spontaneous symmetry breaking be introduced into gravitational theory. The gravitational constant G is effectively replaced by $(\epsilon\varphi^2)^{-1}$, where ϵ is a numerical coupling constant and where the "vacuum expectation value" of the scalar field φ is determined by minimizing a potential $V(\varphi)$, say of the form $-\mu^2\varphi^2+\lambda\varphi^4$. An interesting possibility is that the scalar field φ is also responsible for the breaking of a unified gauge interaction into strong, electromagnetic, and weak interactions. At high temperatures, the potential $V(\varphi)$ is to be modified by the addition of a term[12] $\sim+\lambda T^2\varphi^2$ where T is the temperature. (This is essentially a temperature-dependent mass correction.) Thus the gravitational "constant" G varies with temperature and hence with time. The variation is, however, completely negligible until the temperature reaches the order of the Planck mass $M_P\sim 10^{19}$ GeV, so that no possible incompatibility with such "late-time" cosmological constraints as helium abundance can arise. When the universe becomes hotter than the Planck temperature, G grows stronger, becoming infinite at a finite temperature.

It is the purpose of this Letter to point out that within the framework of this broken-symmetric theory of gravity it is actually possible, at the cost of complicating the theory slightly, to have the gravitational constant weaken with rising temperature.

Normally, when a symmetry is spontaneously broken,[13] we "intuitively" expect that as the temperature is raised the symmetry breaking tends to be restored. (In the example mentioned above φ^2 decreases as T increases.) However, it has been emphasized by Weinberg,[14] and more recently by Mohapatra and Senjanović,[14] that, quite consistent[15] with the rules of quantum field theory, the opposite behavior may occur. To see how this is possible, consider a theory with two scalar fields φ and η. Let the quartic terms in the potential $V(\varphi,\eta)$ read $\lambda_1\varphi^4+\lambda_2\eta^4+2\lambda_3\varphi^2\eta^2$. Positivity requires that $\lambda_1>0$, $\lambda_2>0$, and $\lambda_1\lambda_2>\lambda_3^2$. At high temperatures the potential $V(\varphi,\eta)$ is modified by the addition of various terms; the two relevant for our purposes read $\lambda_1T^2\varphi^2$ and $\lambda_3T^2\varphi^2$ coming from the Feynman graphs in Figs. 1(a) and 1(b), respectively. It is perfectly compatible with positivity to have $\lambda_3<0$ and larger in absolute magnitude than λ_1. In that case, we effectively add to V a term like $-T^2\varphi^2$ and thus $\varphi^2\sim T^2$ at high temperatures.

When the above mechanism is married to the broken-symmetric theory of gravity described in Ref. 10, we have the possibility[16] of the gravitational "constant" weakening like $1/T^2$. Since it appears to be characteristic of unified gauge theories to have more than one scalar field, elementary or dynamically produced, this is perhaps not a completely unlikely possibility.

It is, to put it mildly, an outrageous act of hubris for physicists to discuss the behavior of the universe at the Planck temperature. Surely, quantum gravitational effects, about which we know precious little, may be significant. The notion of a continuous space-time itself may be irrelevant, and it is not clear what thermal equilibrium may mean. Nevertheless, various people have attempted to discuss this regime. It is in this exploratory spirit of pushing the present theories as hard as we can and of cheerful though unrestrained speculation that I present the following discussion. As we shall see, the main result is essentially dimensional and thus may not be completely untrustworthy.

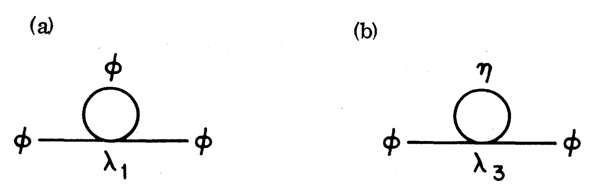

FIG. 1. Typical Feynman graphs giving corrections to the Higgs potential.

Allowing for gravitation to weaken as $1/T^2$ we see, after consulting Eqs. (3), (4), and (5), that the universe expands in this regime as

$$\dot{R}^2\sim N. \tag{7}$$

Thus,

$$R\sim N^{1/2}t. \tag{8}$$

This behavior is not surprising since, with $G\sim 1/T^2$, no dimensional parameter enters into the problem. We thus expect the $R\sim t$ behavior to hold also had we used the full coupled equations of motions[10] instead of the crude equation[17] in (3).

The behavior $R\sim N^{1/2}t$ is also just enough to make the Rindler integral on the left-hand side of Eq. (1) diverge. Thus, albeit only for a brief and fiery instant, every particle in the universe could be in causal contact with every other one. It may be unnecessary to set the universe off in a carefully prepared homogeneous and isotropic state.

It has sometimes been argued[18] that if the universe were somehow homogenized and isotropized from an initially chaotic state, the amount of entropy, or equivalently, the number of photons, produced would be excessive. However, this argument may be sidestepped if recent speculations[19] about the origin of matter are correct. According to these speculations the universe began with zero baryon number (or a net baryon number which quickly dissipated); baryons were produced later by grand-unification reactions. In this scenario, the ratio of the number of photons to the number of baryons in the universe is fixed by microphysics and has nothing to do with initial conditions[20] in the universe (unless the universe started with a baryon number too large to have dissipated quickly).

The main thrust of this Letter is in some sense philosophical. I propose a way to avoid a situation which many people find unappealing on philosophical or, if one prefers, theological grounds. There are no observable consequences to distinguish the proposed initial scenario from another. After a brief fiery instant, the universe settled down to a radiation-dominated $R \sim t^{1/2}$ expansion and then evolved according to the standard script, producing in turn baryons, nuclei, atoms, and molecules.

I conclude by entertaining another speculation. It is tempting to suggest that the theory of the world, including gravitation, contains no dimensional parameter. We simply allow only quartic terms in the scalar-field potential. More precisely, let the Lagrangian of the world be

$$\mathcal{L} = \sqrt{g}\left[\tfrac{1}{2}\epsilon\varphi^2 R + \tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \lambda\varphi^4 + \mathcal{L}_m(\psi, A, \varphi, g)\right].$$

(For the sake of simplicity, I have introduced only one scalar field.) $\mathcal{L}_m$ denotes the usual gauge-theory Lagrangian involving fermion fields ψ, gauge fields A_μ, scalar field φ, and of course the metric $g_{\mu\nu}$. It has been shown[21] that, in flat space, there is spontaneous symmetry breaking in this theory and $\varphi \neq 0$. It seems to me quite appealing to begin with a dimensionless theory, generate the gravitational coupling, and break the gauge Lagrangian into strong, electromagnetic, and weak components with the same mechanism. This "unified" picture is, however, not without some difficulties. The mass scale of gravitation appears to be substantially higher than the mass scale at which the unified gauge theory breaks up (at least according to the most recent estimates[22]), which in turn is much higher than the mass scale at which the electroweak interaction breaks up. This is the so-called hierarchy problem.[23]

I thank S. Bludman for repeatedly emphasizing to me the importance of the horizon problem and for clarifying, on a number of occasions, some of the fine points of cosmology; S. Barr for a stimulating remark; and D. Boulware for the useful conversation. I should also record that the conversation with D. Boulware took place while the author was visiting the Aspen Center for Physics.

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Notes added.—J. Barrow has kindly pointed out to the author that if the universe was not created in a homogeneous isotropic state then effects such as shear anisotropy, which were not taken into account in this Letter, dominate. The situation could become much more complicated. The dissipation of anisotropy has recently been examined by J. B. Hartle and B. L. Hu. For a summary, see B. L. Hu, Harvard University Center for Astrophysics Report No. 1263 (to be published).

A weakening gravitational constant has also been considered by A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. 30, 479 (1979). I thank A. D. Linde for a correspondence on this and other points.

[1]I use particle-physics units $\hbar = c = 1$; also I measure temperature in units of energy (or mass), i.e., $k = 1$.

[2]J. D. Barrow, Nature 272, 211 (1978), and references therein; C. Misner, K. Thorne, and J. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), p. 815.

[3]W. Rindler, Mon. Not. Roy. Astron. Soc. 116, 663 (1956). For an exposition see, e.g., S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), p. 489ff; Misner, Thorne, and Wheeler, Ref. 2, p. 740.

[4]My notation:

$$d\tau^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta\, d\varphi^2\right),$$

$k = +1$, 0, or -1.

[5]For simplicity we take a flat universe: $k = 0$.

[6]This is most easily seen by invoking time reversal and imagining the universe collapsing.

[7]P. A. M. Dirac, Nature 139, 323 (1937); Weinberg, Ref. 3, p. 619ff.

[8]D. Falik, Astrophys. J. 231, L1 (1979); D. N. Schramm and G. Steigman, to be published.

[9]V. Canuto and S.-H. Hsieh, Astrophys. J. 224, 302 (1978).

[10]A. Zee, Phys. Rev. Lett. 42, 417 (1979). See also L. Smolin, Nucl. Phys. B160, 253 (1979).

[11]The introduction of scalar fields into gravity has a long history. However, none of the earlier work appears to incorporate the notion of spontaneous symmetry breaking. The literature is vast and includes C. Brans and R. Dicke, Phys. Rev. 124, 925 (1961); C. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (N.Y.) 59, 42 (1972); F. Gürsey, Ann. Phys. (N.Y.) 24, 211 (1963); R. Penrose, Proc. Roy. Soc. London, Ser. A 284, 159 (1965); M. Omote, Lett. Nuovo Cimento 2, 58 (1971), and Phys. Rev. D 11, 2746 (1975); P. A. M. Dirac, Proc. Roy. Soc. London, Ser. A 333, 403 (1973); P. G. O. Freund, Ann. Phys. (N.Y.) 84, 440 (1974); S. Deser, Ann. Phys. (N.Y.) 59, 248 (1970); T. Matsuki, Prog. Theor. Phys. 59, 235 (1978) (we thank Professor H. Terazawa for calling our attention to this reference); R. Wagoner, Phys. Rev. D 1, 3209 (1970) (we thank Professor R. Wagoner for calling our attention to this reference).

[12]D. A. Kirzhnitz, Pis'ma Zh. Eksp. Theor. Fiz. 15, 745 (1972) [JETP Lett. 15, 529 (1972)]; D. A. Kirzhnitz and A. Linde, Phys. Lett. 42B, 471 (1972); S. Weinberg,

Phys. Rev. D 9, 3357 (1974); L. Dolan and R. Jackiw, Phys. Rev. D 9, 2904 (1974); C. Bernard, Phys. Rev. D 9, 3312 (1974); D. A. Kirzhnitz and A. D. Linde, Ann. Phys. (N.Y.) 101, 195 (1976). For a review, see A. D. Linde, Rep. Prog. Phys. 42, 389 (1979).

[13]It is a standard problem that spontaneous symmetry breaking yields an enormous cosmological constant [M. Veltman, unpublished; J. Dreitlein, Phys. Rev. Lett. 33, 1243 (1974)]. However, this effect is negligible compared to the energy density of order NT^4 when N is large (see S. Bludman and M. Ruderman, Phys. Rev. Lett. 38, 255 (1977).

[14]Weinberg, Ref. 12; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 42, 1651 (1979).

[15]Indeed, real systems with this behavior actually exist. An example often quoted is that of Rochelle salt. F. Jonq and G. Shirane, *Ferroelectric Crystals* (Pergamon, Oxford, 1962), p. 280.

[16]The scalar curvature couples to both φ^2 and η^2. However, at high temperatures the behavior of G is dominated by φ^2 and η^2 is going to zero.

[17]In particular, I did not include a vacuum energy of the order $-(\lambda_3 + \lambda_1)^2\lambda_1^{-1}T^4$ which is negligible provided that N is large. Similarly, the curvature correction ($k \neq \pm 1$, footnote 4) is negligible.

[18]Barrow, Ref. 2, and references therein; S. Bludman, private communication.

[19]M. Yoshimura, Phys. Rev. Lett. 41, 381 (1978), and to be published; S. Dimopoulos and L. Susskind, Phys. Rev.D 18, 4500 (1978); D. Toussaint, S. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 19, 1036 (1979); S. Weinberg, Phys. Rev. Lett. 42, 850 (1979); J. Ellis, M. K. Gaillard, and D. Nanopoulos, Phys. Lett. 80B, 360 (1978); A. Y. Ignatiev, N. Y. Krosnikov, V. A. Kuzmin, and A. N. Tavkhelidze, Phys. Lett. 76B, 436 (1978); S. Barr, G. Segrè, and A. Weldon, University of Pennsylvania Report No. UPR-0127T, 1979 (to be published); D. Nanopoulos and S. Weinberg, to be published; A. Yildiz and P. Cox, to be published.

[20]This point has also been made by S. Turner, Nature 281, 549 (1979).

[21]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[22]M. Marciano, Phys. Rev. D 20, 274 (1979); T. Goldman and G. Ross, to be published.

[23]S. Weinberg, Phys. Lett. 82B, 387 (1979); K. T. Mahanthappa, M. A. Sher, and D. G. Unger, Phys. Lett. 84B, 113 (1979), and references therein.

Radiative Width of the ρ^- Meson

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The excitation of high-energy pions in the nuclear Coulomb field has been investigated. The data, analyzed assuming the presence of both electromagnetic and strong contributions to coherent production of $\pi^-\pi^0$ systems, yield a decay width for $\rho^- \to \pi^-\gamma$ of 67 ± 7 keV.

A long-standing difficulty in our understanding of radiative transitions between vector-meson and pseudoscalar-meson states has been the small value of $\Gamma_\gamma = 35 \pm 10$ keV measured for the decay width of $\rho^- \to \pi^-\gamma$.[1] Although several attempts have been made to reconcile the expected theoretical prediction[2] of ~90 keV with the results of this measurement, these have been far from satisfying.[3] In order to substantiate the results of the previous experiment, we undertook to repeat the measurement, but at a higher energy, where background from nonelectromagnetic processes is less significant.

We have determined Γ_γ using the inverse process $\pi^-\gamma \to \rho^-$, where the Coulomb field of a heavy nucleus (of charge Z) is employed as a source of photons. The specific reaction we studied was

$$\pi^- + Z \to \pi^- + \pi^0 + Z. \qquad (1)$$

The experimental setup, shown in Fig. 1(a),

