Analysis of $\pi^- p \rightarrow \pi^+ \pi^- n$ below 1400 MeV and Chiral-Symmetry Breaking

R. Aaron
Department of Physics, Northeastern University, Boston, Massachusetts 02115

and

Richard A. Arndt
Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

and

J. B. Cammarata[(a)]
Physics Division, National Science Foundation, Washington, D. C. 20550

and

Duane A. Dicus
Center for Particle Theory, University of Texas, Austin, Texas 78712

and

Vigdor L. Teplitz[(a)]
Arms Control and Disarmament Agency, Strategic Affairs Division, Washington, D. C. 20451
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We construct a threshold K matrix for the dominant $PS11(\epsilon N)$ contribution to $\pi^- p \rightarrow \pi^+ \pi^- n$ that agrees with (1) the result of partial-wave isobar-model analyses above 1300 MeV, (2) the result of a single-arm–spectrometer experiment below 1300 MeV, and (3) the predictions of the effective Lagrangian at threshold. We find that the chiral-symmetry-breaking parameter ξ is -0.2 ± 0.3, and that by only 10 MeV above threshold, isobar production accounts for roughly one-third of the total cross section.

There are at least three different sources of information on $\pi N \rightarrow \pi\pi N$: (1) partial-wave, isobar-model analyses[1-3] of bubble-chamber data in the region $W \gtrsim 1300$ MeV; (2) a recent single-arm–spectrometer experiment[4] on the π^+ distribution (in $\pi^- p \rightarrow \pi^+ \pi^- n$) at several energies in the region $1240 \lesssim W \lesssim 1360$ MeV; (3) current-algebra predictions for the threshold behavior of the process.[5] This prediction depends on one parameter ξ, whose value ($\xi = 0$) is believed known from quark-model consideration.[6] The parameter ξ characterizes the chiral transformation properties of the chiral-symmetry-breaking term in the Lagrangian that must be present to give the pion its mass.[7] This Letter investigates the consistency of the information from these three sources. All analyses of the bubble-chamber data find a large imaginary part for the amplitude describing $PS11(\epsilon N)$ production[8] for $W \gtrsim 1300$ MeV. These results[1-3] are summarized in Fig. 1. In the isobar model, this amplitude must dominate at threshold since it is the only one of those believed to be important[1-3] which describes all particles in relative s waves.[9] Furthermore, at threshold, current algebra is believed to predict this amplitude. Because of the large imaginary part of the amplitude for $W \approx 1300$ MeV, it is clear that unitarity must produce large corrections to the current-algebra cross section predictions in the energy region from threshold to 1300 MeV.

In the region $1240 \lesssim W \lesssim 1300$ the cross sections deduced from the single-arm–spectrometer experiment are two to five times larger than the $\xi = 0$ chiral-symmetry predictions, although, as shown in Ref. 4, straightforward extrapolation to threshold suggests $\xi \cong 0$! In this Letter we find that large $PS11(\epsilon N)$ production through the tail of the Roper (1470) resonance persists toward $\pi\pi N$ threshold. We are, however, able to separate isobar production from the chiral-symmetry contribution, and we show that $\xi \cong 0$ implies a contribution from the latter that is consistent with the data, while other proposed values[6] such as $\xi = -2$ are ruled out.

K-matrix parametrization.—Initially we assume that the ϵN channel (with overall $I = J = \frac{1}{2}$) provides all the inelasticity in πN scattering at the energies under consideration. The major channels omitted in this approximation are $PP11(\pi\Delta)$ and $DS13(\pi\Delta)$. Thus, assuming coupled πN and ϵN channels, the isobar amplitude $f_{\epsilon N}$ may be written in terms of a 2×2 K-matrix K_{12} (with angular momentum and isotopic-spin in-

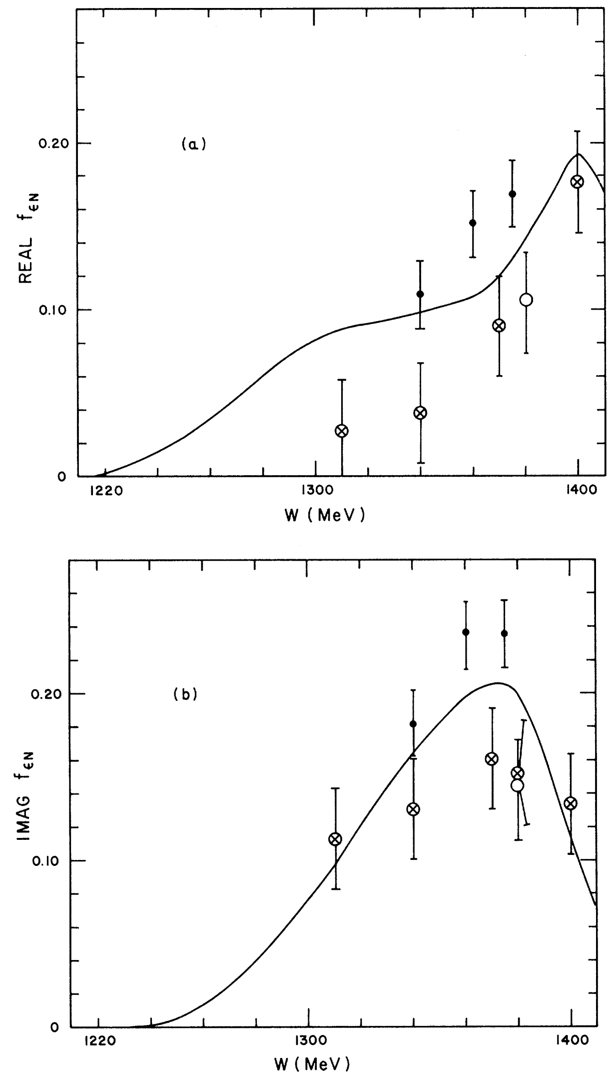

FIG. 1. The (a) real and (b) imaginary parts of the $PS11(\epsilon N)$ amplitude vs c. m. energy. The data points are from various isobar-model analyses: circles with crosses, D. H. Herndon *et al.*, Ref. 1; open circles, R. S. Longacre and J. Dolbeau, Ref. 2; closed circles, Ref. 3. The curves are our fits for the five-parameter case with $\xi = 0$.

dices suppressed). Algebraic manipulation yields

$$f_{\epsilon N} = K_{12} \exp(i\delta_{\pi N}) \cos(\delta_{\pi N})/(1 - iK_{22}). \tag{1}$$

The quantity $\delta_{\pi N}$ appearing in the above equation is the *complex* pion-nucleon ($P11$) phase shift.[10] The matrix K is real and symmetric; its elements are labeled by channels, with "1" referring to πN and "2" referring to ϵN. We parametrize K_{12} and K_{22} in the forms:

$$K_{12} = K_{12}{}^{CA} + \rho \sum_{n=N_1}^{N_2} \alpha_n \left(\frac{W - W_T}{100}\right)^n, \tag{2}$$

$$K_{22} = \sum_{n=1}^{N22} \beta_n \left(\frac{W - W_T}{100}\right)^{n+2}, \tag{3}$$

where W (expressed in MeV) is the overall c.m. energy, $W_T = m_N + 2m_\pi$ is the single-pion production threshold, and the current-algebra contribution to K_{12} can be shown to be

$$K_{12}{}^{CA} = \lambda(\xi) p^{3/2} (W - W_T) \times 10^{-8}, \tag{4}$$

$$\lambda(\xi) = \sqrt{10}(4.16 - 1.84\xi), \tag{5}$$

where p is the three momentum in MeV/c of the initial particles in the c.m. system. In Eq. (4) $K_{12}{}^{CA}$ is the chiral-symmetry prediction of Ref. 5. The total cross section for $\pi^- p \to \pi^+ \pi^- n$ may now be written

$$\sigma = (4\pi\hbar/p)^2 \times \tfrac{4}{9} |f_{\epsilon N}|^2 + \ldots, \tag{6}$$

where $\frac{4}{9}$ is the appropriate sum of squares of Clebsch-Gordan coefficients.

K-matrix fits.—The above K-matrix formalism connects the bubble-chamber analyses[1-3] and the single-arm-spectrometer results.[4] We use Eqs. (2) and (3) in Eq. (1). We begin by fitting the free K-matrix parameters only to the $PS11(\epsilon N)$ amplitude obtained from partial-wave analyses of the $N\pi\pi$ system (Fig. 1). We consider two cases $\xi = 0$ and $\xi = -2$. (The value $\xi = +1$ is already disfavored by K_{e4} decay data.[11]) In Table I we give the calculated cross sections at 1262 MeV predicted by the parameters obtained by fitting to the bubble-chamber analysis $PS11$ amplitude using standard χ^2 techniques. As can be seen, the values of σ are sensitive to whether the sum in Eq. (2) starts off as $(W - W_T)$ or $(W - W_T)^2$. Nevertheless, the $\xi = 0$ results bracket the experimental result of 62 ± 4 μb, while $\xi = -2$ results do not.

We now fit the amplitudes (see Fig. 1) constrained by the total cross sections of Ref. 4.

TABLE I. Cross section at 1262 MeV in microbarns. N_1, N_2, and N_{22} are defined by Eqs. (2) and (3).

N_1	N_2	N_{22}	$\sigma(\xi=0)$	$\sigma(\xi=-2)$	σ(expt)
1	2	2	93	131	62 ± 4
2	3	2	43	91	

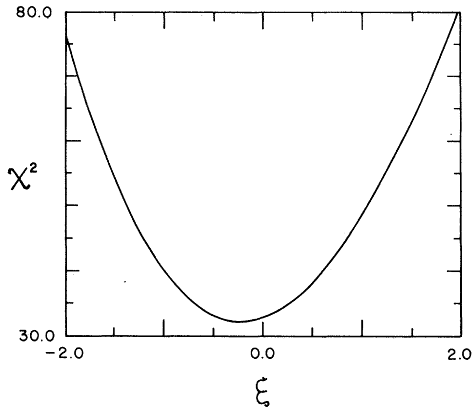

FIG. 2. X^2 vs ξ for nineteen degrees of freedom with $N_1=1$, $N_2=3$, and $N_{22}=2$.

The χ^2 is plotted in Fig. 2 for a five-parameter fit ($N_1=1, N_2=3, N_{22}=2$), and yields[12] $\xi=-0.2\pm0.3$ This is our central result. Physically, the reason that $\xi=-2$ is ruled out is that large values are obtained from the bubble-chamber analyses for imaginary $f_{\epsilon N}$ for $W>1300$ MeV. This result, with any reasonable extrapolation, implies both a nonnegligible imaginary part for $W<1300$ MeV and a large, rapidly varying real part. These two effects by themselves explain the excess of the cross section measured by the spectrometer experiment below 1300 MeV, over the pure $\xi=0$ chiral-symmetry contribution; a nonnegligible $\xi<0$ would then predict too large a cross section.

Additional contributions and corrections.—(i) Our isobar-model amplitude includes the full s-wave $\pi\pi$ scattering amplitude, which, above threshold, differs considerably from its scattering-length approximation (the latter appears in the current-algebra expression). This difference in dependence on $\pi\pi$ subenergy decreases the overlap between current-algebra and isobar contributions, and tends to lower the cross section predicted from adding these two contributions in the T matrix below the obtained from Eq. (2). (ii) In Ref. 3 we gave explicit curves showing the chiral-symmetry prediction for production of p-wave pions. Near threshold, the full current-algebra amplitude for $\pi(\vec{Q})+N(-\vec{Q})\to\pi(\vec{q}_1)+\pi(\vec{q}_2)+N$ is of the form[13]

$$T^{CA}=\alpha\vec{\sigma}\cdot\vec{Q}+\beta\vec{\sigma}\cdot(\vec{q}_2-\vec{q}_1). \qquad (7)$$

The contribution of p-wave pions to the total cross section, represented by a term proportional to β^2 [Eq. (7)] tends to cancel the decrease in cross section resulting from (i) above. We have taken these cross-section corrections into account in the fits given above; they did not significantly affect the results for ξ. (iii) We have considered the contribution for the $PP11(\pi\Delta)$ and $DS13(\pi\Delta)$ isobar amplitudes[14] to the total cross section and find it to be quite small (~2%). In addition, there is further cancellation from the negative overlap of these waves with the chiral background.

π^+ angular distribution.—The single-arm-spectrometer experiment shows the π^+ peaking in the direction of the initial π^- beam. As pointed out in Ref. 3, this property is shared by the bubble-chamber data; for example, at 1360 ± 10 MeV, 1400 $\pi^-p\to\pi^+\pi^-n$ events give[15]

$$R=\frac{\sigma(z>0)-\sigma(z<0)}{\sigma(z>0)+\sigma(z<0)}=0.14, \qquad (8)$$

where $\sigma(z>0)$ is the total cross section for π^+ production in the direction of the incoming π^-. The chiral-symmetry amplitude (4) predicts, near threshold, $R\cong0.63(\sigma^S\sigma^A)^{1/2}/(\sigma^S+\sigma^A)$. σ^S (and σ^A) are the cross sections for producing pion pairs with even (and odd) two-body c.m. J values. These are plotted in Fig. 7(a′) of Ref. 3. Including $PS11(\epsilon N)$ isobar production in R by replacing σ^S by $\sigma-\sigma^A$, implies $R\approx0.20$ to within a few MeV of threshold (with a linear decrease from there toward threshold). This value of R is of the order obtained from the single-arm-spectrometer data.[16] R is very sensitive to isobar production; it is enhanced by $PS11(\epsilon N)$ production, and above 1260 MeV, it is decreased by admixtures of $PP11(\pi\Delta)$ and $DS13(\pi\Delta)$ of the few percent discussed earlier. Detailed analysis of R could provide important information on the small isobar amplitudes, and conceivably, on $\pi\pi$ scattering.

The isobar-model analysis of the spectrometer and bubble-chamber data favors the value $\xi=0$. This value implies that the chiral invariance of the phenomenological Lagrangian is broken by a term which transforms according to the $(\frac{1}{2},\frac{1}{2})$ representation of chiral $SU(2)\otimes SU(2)$. This result is consistent with the quark model,[6] hard-pion current algebra,[17] and the $I=0$ $\pi\pi$ scattering length extracted from K_{e4} decay (as discussed in Ref. 3). We note that the result of this analysis ($\xi=0$) agrees with that of Ref. 3 in which the size of the $SP11(\epsilon N)$ amplitude was used to determine ξ. The present analysis leads to the following picture of the $\pi^-p\to\pi^+\pi^-n$ amplitude: Within a few MeV of threshold, the amplitude is well represented by the real contributions of tree diagrams evaluated with the phenomenological Lagrangian with $\xi = 0$. In this region the amplitude has only $PS11(\epsilon N)$ wave. By 10 MeV above threshold, interference with dispersive corrections to the $PS11(\epsilon N)$ current-algebra amplitude accounts for 30% of the cross section, and production of p-wave pions accounts for 1% of the cross section. By 50 MeV above threshold, the imaginary part of the $PS11(\epsilon N)$ amplitude and the dispersive correction to the (real) chiral-symmetry contribution, are both as large as the chiral-symmetry contribution itself. Furthermore, by this energy, p-wave πN production is of the same order and the $PP11(\pi\Delta)$ and $DS13(\pi\Delta)$ contributions are large enough to affect the π^+ angular distributions, although not the cross section.

In summary, the domain in which the corrections to the chiral-symmetry cross-section prediction are less than 10% extends to only a few MeV above threshold. Nevertheless, the low- and high-energy data are sufficiently correlated by unitarity that one can obtain the chiral-symmetry-breaking parameter ξ by fitting the known π-production data up to 1400 MeV in parameterized K-matrix formalism. Our result is $\xi = -0.2 \pm 0.3$.

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[a]Permanent address: Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Va. 24061.

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[6]Within quantum chromodynamics ξ is expected to be zero; this result follows from calculating current commutators in a quark model as discussed by M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), and J. F. Gunion, P. C. McNamee, and M. D. Scadron, Nucl. Phys. B123, 445 (1977). J. Schwinger has suggested $\xi = +1$ [Phys. Lett. 24B, 473 (1967)] and $\xi = -2$ [in *Proceedings of the Seventh Hawaii Topical Conference on Particle Physics, 1977*, edited by J. Okada (Univ. of Hawaii, Honolulu, 1977)].

[7]S. Weinberg, Phys. Rev. 166, 1568 (1968).

[8]The standard notation used here is [L (initial), L (final), twice total I, twice total J].

[9]We believe that πS_{11} and πS_{31} production can be neglected because (i) δ_{S11} is greater than $|\delta_{S31}|$, and both are considerably smaller than $\delta_{\pi\pi}$ over their respective phase spaces; (ii) Fig. 2 of Ref. 3 shows that, in the bubble-chamber data, $\pi\pi$ isobar production is much larger than πN isobar production and that the latter is consistent with an $I = \frac{3}{2}$ assignment.

[10]V. S. Zidell, R. A. Arndt, and D. A. Roper, unpublished. Note that δ of $W = m_N + 2m_\pi$ is $\approx 1.4°$ so that $|f_{\epsilon N}|^2$ at threshold is just the current-algebra contribution to K_{12} to one part in 10^4.

[11]See, for example, E. P. Tryon, Phys. Rev. D 10, 1595 (1974); L. Rosselet *et al.*, Phys. Rev. D 15, 574 (1977).

[12]The result for the four-parameter fit is $\xi = +0.3 \pm 0.2$.

[13]β is less model dependent than α; in particular, it does not depend on ξ.

[14]We fit these amplitudes using the same methods described in the text for the $PS11(\epsilon N)$. The $PS11(\pi\Delta)$ was treated as an independent production channel in that we did not consider its coupling to $PS11(\epsilon N)$.

[15]Here z is the cosine of the angle of the π^+ momentum vector with respect to the momentum vector of the incident π^-; this is the negative of the z of Ref. 3.

[16]We are grateful to G. Rebka for making available to us the angular distribution data from the spectrometer experiment of Ref. 4.

[17]R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. 175, 1820 (1968).