

## Test of Integral- and Fractional-Charged-Quark Models

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Properties of  $\eta(549)$  and  $\eta'(958)$  are used to measure the quark charges. The result strongly favors fractional charges.

It is very difficult to construct experimental tests of the integral-charged-quark model<sup>1</sup> if the color threshold  $E_c$  is assumed to be far above presently accessible energies. The radiative widths of  $\eta(549)$  and  $\eta'(958)$  may be the only presently measured quantities which provide a test independent of  $E_c$ . The  $\eta'$  width has recently been measured in two mutually consistent experiments<sup>2</sup> and for the first time a complete analysis of the problem is possible.

Previous analyses<sup>3,4</sup> require problematical assumptions: They assume equality of octet and singlet partial conservation of axial-vector current (PCAC) constants,  $F_8 = F_1$ , and/or they rely heavily on the PCAC extrapolation for  $\eta$  and  $\eta'$ . Here I present an analysis which is independent of the values of  $F_1$  and  $F_8$  and the  $\eta$ - $\eta'$  mixing angle  $\theta$ . For the fractional-charged-quark model the result depends only on vector-meson dominance and flavor SU(3) symmetry (PCAC is *not* used) and is insensitive to a possible "glueball" component in the  $\eta'$ . For the integral-charged-quark model the result does depend on the PCAC extrapolation—but in a form which is less sensitive than earlier analyses—and would be affected by a large glueball component in  $\eta'$ .

I will define a quantity  $\xi^2$  such that  $\xi^2 = 1$  for fractional charges and  $\xi^2 = 4$  for integral charges. The experimental value is  $\xi^2 = 1.15 \pm 0.25$  which strongly favors the fractional-charged-quark model.

With a very large value of  $E_c$  the integral-charged-quark model might approximately resemble quantum chromodynamics in the presently accessible physics of the color-singlet sector. The electric current of the integral-charged-quark model is the sum of color-singlet and -octet components

$$J_I = J_1 + J_8, \quad (1)$$

where  $J_1$  is precisely the current of the fractional-charged-quark model

$$J_F = J_1. \quad (2)$$

Therefore, in the color-singlet sector first-order electromagnetic amplitudes are equal in the two

models.

For second-order amplitudes between color-singlet states we have

$$\langle J_I J_I \rangle_1 - \langle J_F J_F \rangle_1 = \langle J_8 J_8 \rangle_1. \quad (3)$$

The product of two octets contains a singlet projection so that the right-hand side may be non-vanishing and it may be possible to distinguish between the two models. But more than group theory is involved. If the right-hand side is of the same order as the terms on the left-hand side it means a "low"-energy amplitude probes the quanta of an arbitrarily high-energy scale. If we imagine writing a dispersion relation for  $\langle J_8 J_8 \rangle_1$  the integral over the absorptive part would be suppressed by powers of  $E_c$  and only a purely real contribution, like a fixed pole, could survive. In configuration space we require that the two currents interact instantaneously before the color and electric charge can fluctuate.<sup>5</sup> No convincing arguments have been presented<sup>6</sup> that these requirements are met in the parton model for  $\gamma N \rightarrow \gamma X$  or  $\gamma^* \gamma \rightarrow X$ .

The chiral anomaly<sup>7</sup> probes the ultraviolet limit of the theory<sup>8</sup> and is one context in which we can be sure that the right side of Eq. (3) is not suppressed. Since  $J_8$  is a flavor singlet we must consider the two-photon decay of a flavor-SU(3)-singlet pseudoscalar meson,  $X_1 \rightarrow \gamma\gamma$ . In quark-model folklore the  $\eta$  and  $\eta'$  are mixtures of singlet and octet  $\bar{q}q$  states:

$$\begin{aligned} \eta &= \cos\theta X_8 - \sin\theta X_1, \\ \eta' &= \sin\theta X_8 + \cos\theta X_1; \end{aligned} \quad (4)$$

hence our interest in their radiative decays.

It is generally presumed that  $X_1$  is not a Goldstone boson in the massless quark limit. But  $\partial A_1$ , the singlet axial divergence, is an acceptable interpolating field for  $X_1$  and we can certainly use the standard current-algebra techniques to compute  $X_1 \rightarrow \gamma\gamma$  at the off-shell point  $m_{X_1} = 0$ . The U(1) problem arises in (1) whether we can trust the extrapolation to the physical amplitudes and (2) whether  $\eta$  and  $\eta'$  contain a significant pure gluon component in addition to the  $\bar{q}q$  states  $X_1$

and  $X_8$ . Since  $m_{\eta'} = 958$  MeV, it is clear that the extrapolation is potentially dangerous, regardless of the nature of the chiral limit. And the apparent smallness of the mixing angle ( $\theta = -11^\circ$  from the naive mass formula) suggests that  $\eta'$  may contain a substantial purely gluonic component.

In the standard current-algebra formulation<sup>3,4</sup> these problems are ignored. The low-energy theorems for  $X_i \rightarrow \gamma\gamma$  are determined to all orders in perturbation theory<sup>9</sup> by just the lowest-order triangle diagram:

$$\mathcal{F}_8(0) = -[\alpha/(3)^{1/2}\pi](1/F_8), \quad (5)$$

$$\mathcal{F}_1(0) = -2(2)^{1/2}\xi(F_8/F_1)\mathcal{F}_8(0). \quad (6)$$

The factor  $\xi$  in Eq. (6) is given by  $\xi = 1$  for fractional charges and  $\xi = 2$  for integral charges. [The factor 2 is due to the right-hand side of Eq. (3): In the integral-charged-quark model,  $\mathcal{F}_1(0) \propto \langle X_1 | J_1 J_1 | 0 \rangle + \langle X_1 | J_8 J_8 | 0 \rangle = 2 \langle X_1 | J_1 J_1 | 0 \rangle$ .] The  $\eta$  and  $\eta'$  amplitudes are then obtained from (5) and (6) using the mixing (4). If one assumes  $\theta = -11^\circ$  and  $F_1 = F_8 = F_\pi$  (so-called "nonet symmetry"), then for the fractional- (integral-) charged-quark model the result is  $\Gamma(\eta \rightarrow \gamma\gamma) = 390$  (720) eV and  $\Gamma(\eta' \rightarrow \gamma\gamma) = 6.1$  (26) keV. The experimental values<sup>10</sup> are  $\Gamma(\eta \rightarrow \gamma\gamma) = 324 \pm 46$  eV and  $\Gamma(\eta' \rightarrow \gamma\gamma) = 5.9 \pm 1.6$  keV.

These results favor the fractional-charged-quark model. But they are not compelling, not only for the reasons associated with the U(1) problem but also because they rely too strongly on the assumption that  $F_1 = F_8$ . If instead we let  $F_1 \cong 2F_8$ , the factor  $\xi = 2$  in Eq. (6) is canceled and the results favor the integral-charged-quark model.  $F_1 = F_8$  is not a consequence of any symmetry but depends on dynamics. In particular, in the naive quark model it would follow from the equality of the  $X_1$  and  $X_8$  wave functions, which would be plausible if  $\eta$  and  $\eta'$  were ideally mixed like  $\omega$  and  $\phi$ . But  $\eta$  and  $\eta'$  are far from being ideally mixed: While  $(m_\omega^2 + m_\phi^2)/2m_K^{*2} = 1.04$ , we have  $(m_\eta^2 + m_{\eta'}^2)/2m_K^{*2} = 2.48$ . The deviation from one measures the deviation from ideal mixing. For  $\eta$  and  $\eta'$  the deviation is of order 1. In the naive quark model it means that the difference in the  $X_1$  and  $X_8$  binding energies is large, of the same order as the masses. So the wave functions may be very different as may  $F_8$  and  $F_1$ . To decide between the two quark-charge assignments we want a method which does not require the dubious "nonet-symmetry" assumption.

One possibility<sup>4</sup> is to use the low-energy theo-

rems for  $\eta \rightarrow \pi\pi\gamma$  and  $\eta' \rightarrow \pi\pi\gamma$ , also determined by the chiral anomaly, to determine  $F_8/F_1$ . But the dipion in both decays is in the  $\rho$  channel and strongly dominated by the  $\rho$ . There is no unambiguous prescription for how to include the effect of the  $\rho$  on the extrapolation. Different choices give quite different results and none gives a satisfactory description of  $\Gamma(\eta' \rightarrow \pi\pi\gamma)/\Gamma(\eta' \rightarrow \gamma\gamma)$ . A large component of  $\eta' \rightarrow \pi\pi\gamma$  might be "elementary"  $\rho\gamma$  production and be unrelated to the low-energy theorem for  $\eta' \rightarrow \pi\pi\gamma$ .

The use of PCAC for  $\eta$  and  $\eta'$  is at best on the empirical footing of vector-meson dominance.  $\rho$ ,  $\omega$ , and  $\phi$  dominance succeed at the  $\lesssim 25\%$  level.<sup>11</sup> It should be more reliable to use the well-tested vector-meson-dominance approximation to relate  $\rho \rightarrow \eta\gamma$  and  $\eta' \rightarrow \rho\gamma$  with  $\eta \rightarrow \gamma\gamma$  and  $\eta' \rightarrow \gamma\gamma$  than to extrapolate the four low-energy theorems for  $\eta/\eta' \rightarrow \gamma\gamma/\pi\pi\gamma$ . The analogous relationship<sup>12</sup> between  $\omega \rightarrow \pi_0\gamma$  and  $\pi_0 \rightarrow \gamma\gamma$  is accurate to 7% in the rate.

Saturating with  $\rho$ ,  $\omega$ , and  $\phi$ , using SU(3) symmetry, and taking  $\phi$  and  $\omega$  to be ideally mixed [just as in Ref. (11)], I find

$$\begin{aligned} \mathcal{F}_1 &= \xi \sum_{V=\rho,\omega,\phi} (e/f_V) \Re(X_1 \rightarrow V\gamma) \\ &= \xi \cdot \frac{4}{3} (e/f_\rho) \Re(X_1 \rightarrow \rho\gamma). \end{aligned} \quad (7)$$

The important feature is the factor  $\xi$ . For fractional charges  $\xi = 1$  and Eq. (7) makes no use of current algebra or PCAC. But for integral charges  $\xi = 2$  and Eq. (7) incorporates part of the content of the low-energy theorem (6). As explained below Eq. (6) the factor  $\xi = 2$  occurs because  $\mathcal{F}_1(0)$  receives equal contributions from  $\langle X_1 | J_1 J_1 | 0 \rangle$  and  $\langle X_1 | J_8 J_8 | 0 \rangle$  while only the former is dominated by  $\rho$ ,  $\omega$ , and  $\phi$ . The only use of PCAC is to assume that the ratio  $\langle X_1 | J_1 J_1 | 0 \rangle / \langle X_1 | J_8 J_8 | 0 \rangle = 1$  extrapolates smoothly from the low-energy point. This is clearly a more mild assumption than is required by the current-algebra analysis: There is no extrapolation of the magnitudes of the amplitudes (5) and (6) and no assumption about  $F_1$ ,  $F_8$ , or  $\theta$ . It is easy to turn Eq. (7) into an experimental determination of  $\xi$ . We use  $\mathcal{F}_1^2 = \mathcal{F}_\eta^2 + \mathcal{F}_{\eta'}^2 - \mathcal{F}_8^2$  and the analogous relationship for  $\Re(X_1 \rightarrow \rho\gamma)$ . By SU(3) symmetry  $\mathcal{F}_8^2 = \frac{1}{3} \mathcal{F}_{\pi_0}^2$  and  $\Re(X_8 \rightarrow \gamma\rho)^2 = \frac{1}{3} \Re(\omega \rightarrow \pi_0\gamma)^2$ . Finally then<sup>13,14</sup>

$$\begin{aligned} \xi^2 &= \left(\frac{3f_\rho}{4e}\right)^2 \frac{\mathcal{F}_\eta^2 + \mathcal{F}_{\eta'}^2 - \frac{1}{3}\mathcal{F}_{\pi_0}^2}{\Re_{\eta'\rho\gamma}^2 + \Re_{\rho\eta\gamma}^2 - \frac{1}{3}\Re_{\omega\pi_0\gamma}^2} \\ &= 1.15 \pm 0.25, \end{aligned} \quad (8)$$

to be compared with

$$\xi^2 = \begin{cases} 1 & \text{for fractional-charged-quark model} \\ 4 & \text{for integral-charged-quark model.} \end{cases} \quad (9)$$

The  $\pm 0.25$  uncertainty is due primarily to the experimental uncertainty<sup>15</sup> in  $\Gamma(\rho \rightarrow \eta\gamma)$ . The substantial uncertainties in the  $\eta'$  rates tend to cancel in Eq. (8). The agreement with the fractional-charged-quark model is similar to the success of the relationship between  $\Gamma(\pi_0 \rightarrow \gamma\gamma)$  and  $\Gamma(\omega \rightarrow \pi_0\gamma)$  and is at a plausible level for an exercise in SU(3) symmetry and vector-meson dominance.

Equation (8) does not take account of a possible "glueball" admixture in  $\eta$  and  $\eta'$ . Let  $G_0$  be the hypothetical pure glueball  $J^P = 0^-$  state. Then the physical states  $\eta$ ,  $\eta'$ , and  $G$  would be mixtures of  $\eta_8$ ,  $\eta_1$ , and  $G_0$ . The state  $G$  presumably remains to be identified at a mass above the  $\eta'$ . Since  $m_\eta \cong m_{\eta_8} = (\frac{4}{3}m_K^2 + \frac{1}{3}m_\pi^2)^{1/2} = 580$  MeV,  $\eta$  is probably strongly dominated by  $\eta_8$ .  $\eta'$  would then contain most of the small remaining  $\eta_8$  component. The heavier state  $G$  would be made almost completely of  $G_0$  and  $\eta_1$ .

If one neglects the very small  $\eta_8$  component in  $G$ , the mixing would be described by two angles:

$$\begin{aligned} \eta &= c_1\eta_8 - s_1c_2\eta_1 + s_1s_2G_0, \\ \eta' &= s_1\eta_8 + c_1c_2\eta_1 - c_1s_2G_0, \\ G &= s_2\eta_1 + c_2G_0, \end{aligned} \quad (10)$$

where  $c_i \equiv \cos\theta_i$  and  $s_i \equiv \sin\theta_i$ . In the fractional-charged-quark model the *unmixed* gluon state  $G_0$  would have no coupling to photons,<sup>16</sup>  $\mathcal{F}_{G_0} = \mathfrak{M}_{G_0} = 0$ , in which case we find

$$c_2^2\mathcal{F}_1^2 = \mathcal{F}_\eta^2 + \mathcal{F}_{\eta'}^2 - \mathcal{F}_G^2$$

and

$$c_2^2\mathfrak{M}_1^2 = \mathfrak{M}_\eta^2 + \mathfrak{M}_{\eta'}^2 - \mathfrak{M}_G^2.$$

Taking the ratio of these equations the factor  $c_2^2$  cancels and the result is again Eq. (8). So in the fractional-charged-quark model Eq. (8) is a plausible result even if  $\eta'$  does have a large glueball component.

In the integral-charged-quark model gluons carry electric charge so that  $\mathcal{F}_{G_0}$  and  $\mathfrak{M}_{G_0}$  need not vanish. A large value of  $\theta_2$  in Eq. (10) might induce large corrections in Eq. (8). For this reason and because of the need to extrapolate the ratio  $\langle X_1 | J_1 J_1 | 0 \rangle / \langle X_1 | J_8 J_8 | 0 \rangle = 1$  to the  $\eta$  and  $\eta'$  mass shells, the derivation of Eq. (8) is on a less secure footing for the integral-charged-quark model than for the fractional-charged-quark model. But there is no reason to expect

these effects to cause the integral-charged-quark model to agree with the experimental result recorded in Eq. (8). That result is rather compelling evidence in favor of the fractional-charge assignment.

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<sup>9</sup>In renormalizable formulations of the Han-Nambu model [J. Pati and A. Salam, Phys. Rev. D **8**, 1240 (1973); T. P. Cheng and F. Wilczek, Phys. Lett. **53B**, 269 (1974); and M. Furman and G. Komen, Nucl. Phys. **B84**, 323 (1975)] the validity of the low-energy theorem is not affected by the existence of charged gluons. This was shown at the two-loop level by an explicit calculation [M. Chanowitz, Phys. Rev. D **9**, 503 (1974)] and subsequently to all orders in perturbation theory [W. Bardeen, Nucl. Phys. **B75**, 246 (1974)]. In another context, these results are essential to assure the anomaly cancellation and therefore the renormalizability of the unified theories of weak and electromagnetic interactions.

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<sup>12</sup>M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Lett. **8**, 261 (1962).

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<sup>14</sup>According to Ref. 10 the effect of finite-width corrections is to renormalize the  $\rho$ - $\gamma$  coupling at  $k^2 = 0$  to  $f_{\rho\pi\pi}^2/4\pi = 2.95 \pm 0.06$ . I am grateful to D. Yennie for

correspondence on this point.

<sup>15</sup>Andrews *et al.* (Ref. 13). They cite a phase ambiguity which I have resolved by choosing the solution consistent with SU(3) symmetry. For this solution  $\Gamma(\rho \rightarrow \eta\gamma)$

$= 50 \pm 13$  keV. (The other solution would give  $\xi^2 = 0.8 \pm 0.2$ .)

<sup>16</sup>I am grateful to Tom Weiler for emphasizing this to me.

## Measurement of $\pi^-p \rightarrow \pi^- \pi^+ n$ near Threshold and Chiral-Symmetry Breaking

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$d^2\sigma/d\Omega dT$  for  $\pi^+$  mesons produced in  $\pi^-p \rightarrow \pi^- \pi^+ n$  was measured at seven incident energies between 203 and 357 MeV and the integrated reaction cross section was calculated. The matrix element, when extrapolated to threshold and compared with soft-pion calculations, determined the chiral-symmetry-breaking parameter  $\xi = 0.05 \pm 0.26$ , which is consistent with the Weinberg Lagrangian. The large hard-pion contributions at 203 MeV demonstrated the absolute necessity for comparing at threshold.

Soft-pion calculations for several processes are unambiguously determined with but one free parameter, the pion decay constant  $f_\pi$ , and have been found reasonably consistent with measurements. Pion decay, the Goldberger-Treiman relation,  $\pi N$  S-wave scattering lengths, and the  $\pi\pi$  P-wave isovector scattering length all yield values for  $f_\pi$  between 82 and 94 MeV.<sup>1</sup> This degree of success is remarkable since the hypotheses of soft-pion theory are strictly valid only for pions with vanishing four-momenta, not for physical pions with mass. Variations in  $f_\pi$  may reflect both the differing importance of the  $m_\pi/M_N$  corrections to the various soft-pion predictions and the limited accuracy in the measurements.

Soft-pion calculations for other processes to which  $\pi\pi$  scattering can contribute depend on an additional parameter  $\xi$  which is determined by the nature of the chiral-symmetry breaking. The validity of soft-pion calculations for these processes has not been critically tested. Single pion production in pion-nucleon scattering  $\pi N \rightarrow \pi\pi N$ , which depends on the  $\pi\pi$  S-wave isoscalar and isoscalar scattering lengths, and hence on  $\xi$ , is

such a process and has been calculated by several authors.<sup>2-5</sup> Although the charge state  $\pi^-p \rightarrow \pi^- \pi^+ n$  is amenable to measurement, significant comparison with experiment to determine  $\xi$  has been elusive. The limited accuracy with which the small cross sections near threshold could formerly be measured have made extrapolation of the matrix element to threshold, where comparison must be made, less than convincing. Nevertheless, calculations with  $\xi$  between  $\pm 1$  have agreed roughly with experiments. In an alternative analysis,<sup>6</sup> comparison of the nonresonant  $SP11(\epsilon N)$  wave of an isobar model, fitted to bubble-chamber events between 324 and 396 MeV with the  $SP11(\epsilon N)$  portion of the soft-pion prediction gave  $\xi = -0.3 \pm 1.6$ .

This Letter reports a study of the reaction  $\pi^-p \rightarrow \pi^- \pi^+ n$  with the improved accuracy made possible by the intense  $\pi$  beams at the Clinton P. Anderson Meson Physics Facility (LAMPF) in an experiment specifically designed for extrapolation to threshold and comparison to soft-pion calculations. The doubly differential cross section  $d^2\sigma/d\Omega dT$  for  $\pi^+$  mesons produced was measured