Frequency and Alignment Classification of Multiple Band Crossings

L. L. Riedinger[(a)], O. Andersen, S. Frauendorf[(b)], J. D. Garrett, J. J. Gaardhøje, G. B. Hagemann, B. Herskind, and Y. V. Makovetzky[(c)]

The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen, Denmark

and

J. C. Waddington

Institute of Physics, University of Bergen, Bergen, Norway, and McMaster University, Hamilton, Ontario L8S 4M1, Canada

and

M. Guttormsen and P. O. Tjøm

Institute of Physics, University of Oslo, Oslo, Norway

(Received 13 August 1979)

Experiments on ^{160}Yb indicate three negative-parity bands up to $I \geq 21\hbar$. All three experience a band crossing around a frequency $\hbar\omega = 0.36$ MeV, a value intermediate to those of the first and second yrast backbends. The $i_{13/2}$ band in ^{161}Yb is observed up to $\frac{49}{2}^+$; it also has a strong upbend at this intermediate frequency. The fingerprint of a band crossing based upon the crossing frequency and gain in aligned angular momentum is demonstrated and the results compared to cranking-model calculations to suggest quasiparticle assignments for the bands.

The general phenomenon of backbending in rotational bands of deformed nuclei is understood to result from a crossing of bands of different degrees of rotational alignment, as proposed by Stephens and Simon.[1] In the $Z = 64$–72 nuclei, the S (super or Stockholm) band, which crosses the ground-state band and causes the backbend, is found to have an extra angular momentum that can be attributed to the alignment of $i_{13/2}$ neutrons. Such crossings now have been observed in many nuclei. The present Letter, however, focuses on the interpretation of observed crossings in non-yrast rotational (side) bands in ^{160,161}Yb in terms of the selective rotational alignment of a few key particles. The fingerprint of a band crossing is the crossing frequency (ω_c) and the gain in aligned angular momentum (Δi), both of which are extracted from the experimental data and compared to values predicted in cranking-model calculations following the recent theoretical efforts of Bohr and Mottelson[2] and Bengtsson and Frauendorf.[3] Our in-beam–spectroscopy measurements demonstrate four sidebands in ^{160}Yb in addition to the yrast band; five, possibly six, crossings are found between these and bands of higher alignment. In ^{161}Yb, the yrast $i_{13/2}$ band is observed to bend up in a manner very similar to the three negative-parity bands in ^{160}Yb. We emphasize here the successful fingerprinting of these six or seven observed band crossings and demonstrate the basic importance of three special frequencies: $\hbar\omega_{c1} \approx 0.27$ MeV, the frequency of the first yrast backbend of ^{160}Yb, similar to the corresponding values for most nuclei in this region; $\hbar\omega_{c2} \approx 0.36$ MeV, the value for the band crossings in the three negative-parity sidebands in ^{160}Yb and for the yrast upbend in ^{161}Yb; and $\hbar\omega_{c3} \approx 0.42$ MeV, the frequency of the second yrast upbend in ^{160}Yb. The calculations can easily explain the significance of these three distinct frequencies, the first two associated with pair breaking and alignment of $i_{13/2}$ neutrons, the third with $h_{11/2}$ or $h_{9/2}$ protons. Furthermore, two or three of these basic alignment processes should affect all other two-quasiparticle bands in nuclei in this region.

The experiments were performed with the ^{147,148}Sm(^{16}O, $3n$)^{160,161}Yb reactions with beams of up to 82-MeV ^{16}O from the Norges Byggforsknings Institutt tandem accelerator and Pb-backed targets ≈ 1 mg/cm^2 thick. γ-ray coincidence data were accumulated with an array of four Ge(Li) and three NaI counters. Only events in which γ rays were detected in two Ge(Li) detectors plus at least one additional detector were recorded on magnetic tape. In all, 285 million Ge(Li)-Ge(Li) pairs were accumulated in the ^{160}Yb measurement. The structure of bands in ^{160}Yb is shown in the level scheme of Fig. 1. The yrast cascade extends to the 28^+ level, indicating band crossings at $I = 12\hbar$ and $I \gtrsim 28\hbar$. Beck *et al.*[4] indicate an assignment of an 865.8-keV line as the $30^+ \to 28^+$ transition,[5] but our coincidence data show that an 865.5-keV transition feeds the 16^+ level. The 744.3- and 780.1-keV transitions are the major

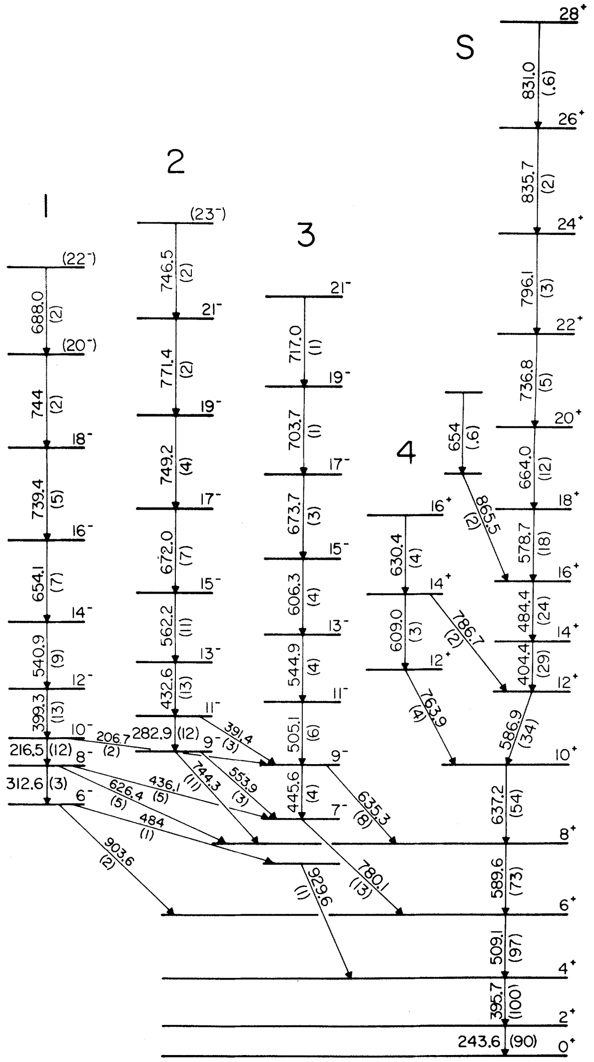

FIG. 1. Level scheme of ^{160}Yb. Energies are given in kiloelectronvolts and relative γ-ray intensities are shown in parentheses. These intensities generally result from the a_0 values deduced in our angular distribution measurements.

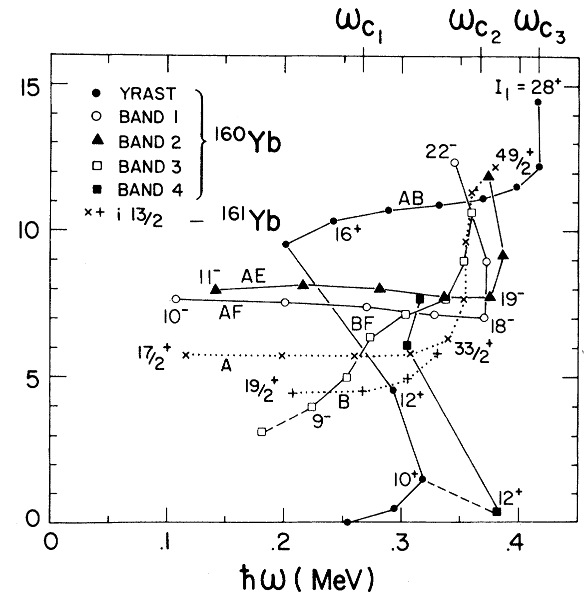

FIG. 2. Plot of the alignment vs rotational frequency for the bands. In calculating i for the ^{160}Yb bands, we use $\mathcal{G}_0$ = 16 MeV^{-1} and $\mathcal{G}$ = 90 MeV^{-3} (18 and 90 for ^{161}Yb) in parametrization of the ground-state band (see the text and Ref. 3).

transitions from the sidebands to the ground-state band. Multiplicity-gated angular-distribution measurements yield a_2 and a_4 values indicative of $I \rightarrow I-1$ pure or nearly pure dipole transitions. A conversion-electron measurement led to α_k values of $(0.30 \pm 0.08) \times 10^{-2}$ and $(0.20 \pm 0.09) \times 10^{-2}$ for the 744.3- and 780.1-keV transitions, compared to calculated values of (0.22, 0.54, or 1.20) $\times 10^{-2}$ for a 760-keV $E1$, $E2$, or $M1$ transition, respectively. The measured polarizations $(90^\circ - 0^\circ)/(90^\circ + 0^\circ)$ of the 744.3- and 780.1-keV lines are 0.48 ± 0.16 and 0.49 ± 0.22, respectively. From this we conclude that bands 2 and 3 have negative parity. Concerning band 1, the angular distribution of the 436.1-keV line indicated an $8 \rightarrow 7^-$ transition of $\approx 96\%$ quadrupole content, requiring negative parity for the parent level. The position of the level depopulated by the 929.6-keV line suggests that it could be the 5^- member of band 3, and we draw it accordingly in Figs. 1 and 2. Similar experiments on ^{161}Yb lead to the assignment of an aligned $i_{13/2}$ band up to $I = \frac{49}{2}$, in agreement with both the results of Ref. 6 for $I \leq \frac{37}{2}$ and the trend of the corresponding bands observed in ^{163,165}Yb (Ref. 7). Members of the unfavored part of the $i_{13/2}$ band in ^{161}Yb are observed up to $I = \frac{31}{2}$.

The aligned angular momentum, i, of each of the observed bands is plotted in Fig. 2, as a function of $\hbar\omega$, which is equal to half of the transition energy for $K = 0$ (see Ref. 3). As discussed in Refs. 2 and 3, i is defined as the difference in the angular momentum of the excited- and ground-state bands at a given rotational frequency, where the angular momentum of the ground-state band is taken to be[3] $(\mathcal{G}_0 + \omega^2 \mathcal{G}_1)\omega$. The first yrast backbend occurs at $\hbar\omega_{c1} \approx 0.27$ MeV, resulting in an alignment gain of approximately $10.6\hbar$. At the second upbend, $\hbar\omega_{c3} = 0.42$ MeV, the alignment increases by at least $3\hbar$. Since the $30^+ \rightarrow 28^+$ transition has not been assigned, the alignment of the crossing band remains unknown.

The alignments of the sidebands in ^{160}Yb are also rather high, even at the lowest observed frequencies for bands 1 and 2. In contrast, band 3 begins with low $i \simeq (3-4)\hbar$ but rapidly gains alignment. This gain in i probably results from the crossing of a $K=0$ octupole band with a two-quasiparticle band of larger i, similar to that observed at $I=11^-$ in the $N=88$ nuclei.[8]

A remarkable fact is that these three sidebands experience a band crossing around $\hbar\omega_{c2}=0.36$ MeV, between the frequencies of the first (0.27 MeV) and second (0.42 MeV) yrast backbends. Furthermore, the yrast $i_{13/2}$ band in ^{161}Yb shows a crossing at a very similar frequency; in this case, an estimate of the alignment gain in the crossing ($\Delta i \approx 6.5\hbar$) can be extracted. In order to understand these bands and the significance of $\hbar\omega_{c2}$, we have calculated the quasiparticle (qp) energies of a rotating nucleus.[2,3] The deformations, ϵ_2 and ϵ_4 (calculated by the Strutinsky method[9]), the pairing gap Δ and the chemical potential λ are assumed to be independent of ω. The qp energies in the rotating frame, $e'(\omega)$, are plotted versus $\hbar\omega$ in Fig. 3. Here we use the occupation-number representation employed in Ref. 3; i.e., if a level (e.g., A) is occupied, the conjugate level ($-A$), obtained by reflection about $e'=0$ and by changing the signature, α, to $-\alpha$, must be free. Therefore, half of the levels are always occupied, and a qp excitation corresponds to occupying a level at e_i and freeing its conjugate partner, with a resulting change of energy of e_i. At the lowest frequencies, the ground-state band corresponds to the configuration where all the $e'>0$ levels are empty; the S band then results from placing one qp in A and one in B. The S band (only even spins) lowers in energy relative to the ground-state band as a result of rotation, until they cross at $\hbar\omega=0.23$ MeV. After the crossing, the S band is the vacuum state. The ground-state band has A and B empty, and $-A$ and $-B$ filled; for $\hbar\omega>0.23$ MeV, this $-A-B$ configuration lies at a higher energy than AB. The alignments are obtained from the slopes in Fig. 3, $i=-de'/d\omega$.

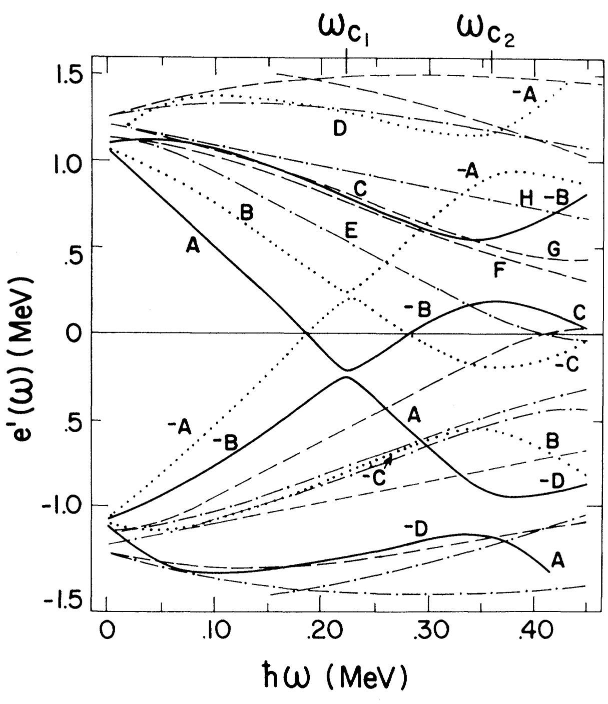

FIG. 3. Cranking-model calculations of e', the neutron qp energy in the rotating frame, vs $\hbar\omega$. The parameters used in this calculation are $\epsilon_2=0.2$, $\epsilon_4=-0.02$, $\Delta=1.06$ MeV, $\lambda=6.38\hbar\omega_0$, and $N=90$. At $\omega=0$ levels A and B, C and D, E and F, and G and H correspond to the [651,$\frac{3}{2}$], [660,$\frac{1}{2}$], [532,$\frac{3}{2}$], and [521,$\frac{3}{2}$] Nilsson states, respectively. These letters are assigned to levels of similar wave functions through a band crossing, contrary to the format of Ref. 3.

The qp assignments for the measured bands are listed in Table I. For ^{161}Yb, the observed $i_{13/2}$ states correspond to levels A and B, the two signatures of the [651,$\frac{3}{2}$] Nilsson state; the i values

TABLE I. Predicted quasiparticle levels and band crossings (from Fig. 3).

Conf.[a]	$\alpha\pi$[b]	i ($\hbar$)	Crossing band Conf.	$\hbar\omega_c$ (MeV)	Δi ($\hbar$)
		^{161}Yb			
A	+1/2+	5.8	ABC	0.36	6.6
B	−1/2+	4.1	ABD	0.37	7.2
E^c	+1/2−	3.0	ABE	0.23	9.9
		^{160}Yb			
GSB	0+		AB	0.23	9.9
S:AB	0+		$AB2p$	0.43	6.0
2:AE	1−	9.9	$ABCE$	0.36	6.6
1:AF	0−	8.8	$ABCF$	0.36	6.6
3:BF	1−	7.8	$ABDF$	0.37	7.2
GSB	0+	6.1	4:BC	0.36	6.6

[a]Configurations listed in order of increasing energy.

[b]The signature, α, is defined such that $I=\alpha+$ an even number (Ref. 3). For a multiquasiparticle state, α is sum of single-quasiparticle values; $\alpha=0$ and 1 correspond to even and odd spins.

[c]Although not reported in this Letter, band E is observed also and is very similar to that seen in ^{163,165}Yb (Ref. 7).

(see Fig. 2) agree very well with the predictions. In ^{160}Yb, these two levels combine to make the S band, AB. The alignments, spins, and relative energy spacings of the three negative-parity bands are well explained by couplings of A or B with E and F, the $[532, \frac{3}{2}]$ levels. The AF and BE couplings are predicted to be close in energy and i; one probably lies higher in reality because of residual interactions not included here.

The frequencies of various band crossings can also be deduced from the theoretical energy diagram of Fig. 3 and are listed in Table I. For example, the ground-state and S (AB) bands are predicted to cross at $\hbar\omega = 0.23$ MeV; the observed value is 0.27 MeV. The calculations predict another important crossing, associated with the pair breaking and rotational alignment of the next most aligned pair of $i_{13/2}$ neutrons, BC, at $\hbar\omega_{c2} = 0.36$ MeV and $\Delta i = 6.6\hbar$. In the ^{161}Yb yrast band, level A is filled and thus the $0 \rightarrow AB$ crossing is blocked. Instead the first band crossing comes at 0.35 MeV with the correct Δi ($\sim 6.5\hbar$) to be interpreted as $A \rightarrow ABC$. This crossing is only an upbend compared to the sharp backbend of the ground band in ^{160}Yb. The calculations reproduce this feature: The interaction between the crossing levels (e.g., A and $-B$) at $\hbar\omega_{c1}$ is much smaller than at $\hbar\omega_{c2}$ (C and $-B$ or A and $-D$). This interaction is an oscillating function of the neutron number,[3,10] which explains why only the $N = 91$ and 93 Yb and Er isotopes show evidence of this $A \rightarrow ABC$ crossing.

The fundamental importance of the BC alignment process is demonstrated by considering the side bands in ^{160}Yb. Bands AE, AF, and BF all upbend around $\hbar\omega = 0.36$ MeV. The measured $\Delta i \gtrsim 4\hbar$ for each is compatible with the $6.6\hbar$ expected for the fingerprint of a BC or AD crossing. In each case, the presence of a particle in level A or B prevents a crossing with the most highly aligned 2-qp configuration, AB, at $\hbar\omega_{c1}$, and so the first crossing is delayed until $\hbar\omega_{c2}$ and involves the alignment of BC (or AD – an equivalent pair). This alignment process should be quite common in 2-qp sidebands in other nuclei where one of the qp occupies a low-lying $i_{13/2}$ orbital A or B. As in the case of the odd-N nuclei,[3,10] the sharpness of this crossing should be an oscillating function of N.

The theoretical diagram of Fig. 3 shows that the continuation of the ground-state band after $\hbar\omega_{c1}$ $(-A-B)$ should also cross BC at 0.36 MeV. Indeed, this could be the interpretation of band 4 in Fig. 1. If the 763.9-keV line is viewed as the $12 \rightarrow 10$ ground-state-band (GSB) transition, then the next two transitions are indicative of a band crossing of $\hbar\omega \approx 0.34$ MeV and $\Delta i \geq 7\hbar$, the correct fingerprint for the BC alignment process. The sharper crossing compared to those in the sidebands could indicate a need for slightly different ϵ_2, ϵ_4, or λ values in the ground-state band and sidebands, which would affect the interaction between the crossing levels. It is true that the frequency sequence and the pattern of interaction strengths of the observed crossings are not precisely reproduced in the calculations. However, it is gratifying that this rather simple model, which assumes a rotation-independent potential, explains so well the observed bands and crossings.

The second yrast crossing, $\hbar\omega_{c3} \approx 0.42$ MeV and $\Delta i \geq 3\hbar$, is compatible with the alignment of a pair of $h_{11/2}$ protons (0.43 MeV, $6.0\hbar$), as discussed in Refs. 3 and 11 for ^{158}Er. Each of the sidebands in ^{160}Yb and the $i_{13/2}$ band in ^{161}Yb should experience a similar alignment process. The observation of a second band crossing with the correct frequency and alignment fingerprint in each case would represent further verification of these calculations which explain so impressively the maze of bands and crossings observed here in the light Yb isotopes.

The authors wish to acknowledge the many enlightening discussions with Professor A. Bohr and Professor B. R. Mottelson. This work has been supported by the Danish National Science Research Council. Two of us (P.O.T. and M.G.) appreciate the support from the Nordic Committee for Accelerator-Based Research.

[a]Permanent address: Department of Physics, University of Tennessee, Knoxville, Tennessee 37916.

[b]Permanent address: Central Institute for Nuclear Research, Rossendorf, Democratic Republic of Germany.

[c]Permanent address: Institute for Nuclear Research, Kiev, U.S.S.R.

[1]F. S. Stephens and R. S. Simon, Nucl. Phys. A183, 257 (1972).

[2]A. Bohr and B. R. Mottelson, in Proceedings of the International Conference on Nuclear Structure, Contributed Papers, Tokyo, 1977 (unpublished); J. Phys. Soc. Jpn. 44 (1978), Suppl., p. 157.

[3]R. Bengtsson and S. Frauendorf, Nucl. Phys. A314, 27 (1979), and Nucl. Phys. A327, 139 (1979).

[4]F. A. Beck, E. Bozek, T. Byrski, C. Gehringer, J. C. Merdinger, Y. Schutz, J. Styczen, and J. P. Vivien, Phys. Rev. Lett. 42, 493 (1979).

[5]This transition is erroneously listed as 965.8 keV in Table I of Ref. 4.
[6]R. L. Hershberger, K. E. G. Löbner, R. Lutter, D. Proetel, and F. Riess, Universität München Annual Report, 1975, p. 78.
[7]L. Richter, H. Backe, E. Kankeleit, F. Weik, and R. Willwater, Phys. Lett. 71B, 74 (1977).
[8]F. W. N. De Boer, L. K. Peker, P. Koldewijn, and J. Konijn, Z. Phys. A 284, 267 (1978).
[9]S. G. Nilsson, C. F. Tsang, A. Soviczewski, Z. Szymanski, S. Wycech, C. Gustafson, I. L. Lamm, P. Möller, and B. Nilsson, Nucl. Phys. A131, 1 (1969).
[10]R. Bengtsson, I. Hamamoto, and B. Mottelson, Phys. Lett. 73B, 259 (1978).
[11]A. Faessler and M. Ploszajczak, Phys. Lett. 76B, 1 (1978).

Spontaneous Singularity in Three-Dimensional, Inviscid, Incompressible Flow

Rudolf H. Morf[(a)]
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

Steven A. Orszag
Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Uriel Frisch
Observatoire de Nice, Centre National de la Recherche Scientifique, F-06300 Nice, France
(Received 10 December 1979)

The results obtained by series-analysis techniques applied to the time evolution of the inviscid Taylor-Green vortex support the conjecture that vortex lines may be stretched an infinite amount in a finite time.

The classical theorems[1] of Kelvin and Helmholtz imply that, in an inviscid incompressible fluid of constant density, vortex lines move with the fluid and vorticity is amplified proportional to the stretching of a vortex line element. These theorems are central to the understanding of the dynamics of high-Reynolds-number flows.

For boundary-free flow, the Kelvin and Helmholtz theorems imply that an initially smooth, inviscid flow remains smooth so long as vortex lines are stretched only a finite amount. Indeed, the restriction of the flow to two space dimensions precludes vortex-line stretching so global regularity follows.[2] However, in three dimensions, vortex lines can twist, tangle, turn, and stretch. It is conceivable that flow velocities remain bounded and, still, a singularity of the flow appears spontaneously after a finite time in the interior of the flow.[3] Segments of vortex lines could develop infinite length by becoming intricately wound up and twisted without the end points of the segment being separated by an infinite distance. These properties of inviscid flow have been the subject of some speculation in the past.[4-6] In this Letter we offer evidence of the correctness of the conjecture that spontaneous singularities occur in three-dimensional inviscid incompressible flows. While the results to be presented below are clearly not a rigorous proof of singularity, they provide the first quantitative data that support the existence of the putative singularity.

Our results are obtained by solving the three-dimensional Euler equations as power series in time t with the simple initial conditions introduced by Taylor and Green.[7] The resulting power-series expansions are analyzed using techniques developed for the study of singularities in critical phenomena.[8] The flow is the solution of the incompressible Euler equations which are, in Fourier representation,[4]

$$\frac{\partial u_\alpha(\vec{k};t)}{\partial t} = -i\sum_{\beta,\gamma=1}^{3} k_\beta\left(\delta_{\alpha\gamma}-\frac{k_\alpha k_\gamma}{k^2}\right)\times\sum_{\vec{p}} u_\beta(\vec{p};t)\,u_\gamma(\vec{k}-\vec{p};t)\,. \quad (1)$$

The initial conditions in real space are[5,7,9]

$$v_1(x_1,x_2,x_3;t=0)=\cos x_1 \sin x_2 \cos x_3\,, \quad (2)$$

$$v_2(x_1,x_2,x_3;t=0)=v_1(x_2,-x_1,x_3;t=0)\,, \quad (3)$$

$$v_3(x_1,x_2,x_3;t=0)=0\,. \quad (4)$$