[b]On leave from Universidad Técnica Federico Santa Maria, Valparaiso, Chile.

[1]In this paper we analyze the electromagnetic and weak-interaction contributions to the (renormalized) quark running mass $m(p^2)$ as well as to the bare mass parameter $\lim_{p^2\to\infty} m(p^2)$ in the total Lagrangian. In contrast, the Cottingham formula yields the order-α perturbation to the quark or hadron mass renormalized only by the strong interactions. [See W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963); W. I. Weisberger, Phys. Rev. D 5, 2600 (1972); A. Zee, Phys. Rep. 3C, 129 (1972).] This quantity is logarithmically divergent and requires renormalization. See J. C. Collins, Nucl. Phys. B149, 90 (1979), and B153, 546(E) (1979). G. B. West, Los Alamos Scientific Laboratory Report No. LA-UR-79-1690 (to be published); J. Kiskis, private communication. We wish to thank M. Dine, G. P. Lepage, and K. Johnson for helpful discussions on this point.

[2]The regularity noted by Harari that the $\Delta I = 2$ mass differences such as $m_{\pi^\pm} - m_{\pi^0}$ or $m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0}$ can be computed in terms of a dispersion sum over low-lying resonances, but that $\Delta I = 1$ mass differences such as $m_p - m_n$, $m_{K^+} - m_{K^0}$, $m_{\Sigma^+} - m_{\Sigma^0}$, $m_{\Xi^+} - m_{\Xi^0}$ cannot, is due to the fact that the quark-mass contributions cancel for $\Delta I = 2$ mass differences. See H. Harari, Phys. Rev. Lett. 17, 1303 (1966).

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[5]This method is used in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw Hill, New York, 1965). For a more general approach see M. Baker and C. Lee, Phys. Rev. D 15, 2201 (1977).

[6]We neglect contributions to $m_s(p^2)$ beyond one-loop since these are higher order in $\alpha_s(p^2)$. Note that by using the Ward identity for the $\gamma q\bar{q}$ vertex, we can calculate $\partial m_s/\partial p^2$ directly as a function of renormalized quantities, with an overlapping-divergence-free skeleton expansion. Thus

$$\frac{\partial m_s(p^2)}{\partial \ln p^2} = \alpha_s(p^2) m_s(p^2) f\left(\alpha_s(p^2), \frac{m_s^2(p^2)}{p^2}\right),$$

and for large p^2, we only require $f(0,0)$.

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[9]Formally, the solution to Eq. (7) is given by the order-α contribution given in Eq. (8) plus a term $Am_s(q^2)$, which is the general solution to the homogeneous part of the integral equation

$$\delta m(p^2) = \frac{3}{4\pi}\int_{p^2}^{\infty} \frac{dk^2}{k^2} C_F \alpha_s(k^2)\,\delta m(k^2)$$

obtained by setting $\alpha \equiv 0$ in Eq. (7). Since this term has no dependence on α, such a contribution should be incorporated into the definition and normalization of $m_s(p^2)$ rather than the order-α electromagnetic perturbation (i.e., $A = 0$).

[10]Alternatively, one could have other irreducible representations of color SU(3) which yield $0 < \beta < 4$.

[11]If we assume that QED is imbedded in a grand unified theory which is asymptotically free, the effects of unification will prevent any singularity in $\alpha(p^2)$. See H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974), and Ref. 7.

Analytic Calculation of Higher-Order Quantum-Chromodynamic Corrections in e^+e^- Annihilation

William Celmaster[a] and Richard J. Gonsalves

Department of Physics, University of California, San Diego, La Jolla, California 92093

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Order-α_s^2 corrections to the annihilation of e^+e^- into hadrons are computed analytically. We briefly discuss the technique, which involves the extension to noninteger dimensions of Chebyshev expansions. In the energy range 15–30 GeV (five flavors) we find

$$R = 3\sum e_Q^2\{1 + \alpha_s(q)/\pi - 0.94[\alpha_s(q)/\pi]^2 + \dots\},$$

where α_s is the momentum-space subtracted strong-coupling constant. Our result agrees with that of Dine and Sapirstein, and Chetyrkin, Kataev, and Tkachov.

The calculation of high-order quantum-chromodynamic (QCD) corrections to $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ is of particular significance in at least two respects. First of all, both the theoretical and experimental determinations of R involve fewer parameters than an analysis of deep-inelastic scattering. The R analysis thus may be a more promising way to test high-order QCD predictions. Secondly, the perturbative expansion of R as a function of the strong-coupling constant, α_s, serves as a "test" of the theory itself. That is, in order for perturbative QCD to succeed as a theory it is necessary that higher-order terms tend to be smaller than the lowest-order terms.

The problem is, therefore, to calculate R_2 in

$$R(s) = 3\sum e_Q^2\left[1+\frac{\alpha_s(s)}{\pi}+R_2\left(\frac{\alpha_s(s)}{\pi}\right)^2+\dots\right],$$

where e_Q are quark charges and $\sqrt{s}$ is the center-of-mass energy of the e^+e^- system. The coefficient R_2 has very recently been evaluated numerically by Dine and Sapirstein[1] and analytically by Chetyrkin, Kataev, and Tkachov.[1] They find a rather large value ($R_2 \simeq 7.4-0.4N_f$ for N_f quark flavors) if renormalization is done by minimal subtraction.[2] However, as those authors observe, R_2 is renormalization-scheme dependent and they present their results in a scheme ($\overline{MS}$) where R_2 is much smaller. We return to the scheme dependence later.

The complexity of the above computation warrants an independent confirmation. In this Letter we present an analytical calculation which yields

$$R_2 = \{\tfrac{2}{3}\zeta(3)-\tfrac{11}{12}-\tfrac{1}{6}[\ln(4\pi)-\gamma_E]\}N_F+\{-11\zeta(3)+\tfrac{365}{24}+\tfrac{11}{4}[\ln(4\pi)-\gamma_E]\}, \tag{1a}$$

$$\simeq 7.3587-0.4409N_F \tag{1b}$$

in agreement with Refs. 1. We also present exact partial results which serve to check the accuracy of the numerical integration routines used by Dine and Sapirstein.[1] In contrast to the position-space technique of Chetyrkin, Kataev, and Tkachov,[1] we work directly in momentum space; we also use the momentum of the virtual photon rather than an auxiliary mass as an infrared cutoff. The calculational method presented here should prove useful in future high-order calculations and in checking results such as those of Ref. 3.

We perform our calculation by making use of an N-dimensional generalization of the Chebyshev expansion technique employed by Rosner and others.[4] Since this method is (to the best of our knowledge) novel we describe it briefly here.[5] The appropriate generalization of the Chebyshev polynomials (i.e., polynomials which are orthogonal with respect to angular measure in four dimensions) is the family of Gegenbauer polynomials $C_n^\alpha(x)$, where $\alpha=(N-2)/2$ and N is the number of dimensions of space-time (occurring in the dimensional regularization program). The fundamental orthogonality relation[6] is

$$\int_{-1}^{1} dx(1-x^2)^{\alpha-1/2}C_m^\alpha(x)C_n^\alpha(x)=\delta_{nm}\frac{\pi}{n!}\frac{2^{1-2\alpha}\Gamma(n+2\alpha)}{(\alpha+n)[\Gamma(\alpha)]^2}$$

from which one derives

$$\int d\Omega_N C_n^\alpha(\cos\theta_1)C_m^\alpha(\cos\theta_1)=\delta_{mn}\frac{2\pi^{2-\epsilon/2}\Gamma(n+2-\epsilon)}{n!(n+1-\epsilon/2)\Gamma(1-\epsilon/2)\Gamma(2-\epsilon)}, \tag{2}$$

where $N=4-\epsilon$ and

$$\int d\Omega_N=\int_0^{2\pi}d\varphi\int_0^\pi \sin\theta_{N-2}\,d\theta_{N-2}\cdots\int_0^\pi \sin^{N-2}\theta_1\,d\theta_1.$$

These results are extended from integer to noninteger dimensions by simply allowing N to be noninteger in the above formulas. Also of use in our calculations are various addition theorems, as well as the convolution formula[6]

$$\int d\Omega_N(\hat{k})\,C_n^\alpha(\hat{k}\cdot\hat{p})C_n^\alpha(\hat{k}\cdot\hat{q})=\delta_{nm}C_n^\alpha(\hat{p}\cdot\hat{q})\frac{2\pi^{(2-\epsilon/2)}}{(n+1-\epsilon/2)\Gamma(1-\epsilon/2)}. \tag{3}$$

In the work of Rosner and others,[4] one of the key elements is the expansion of propagators in terms of Chebyshev polynomials. Then the angular parts of Feynman integrals are trivially done by use of the orthogonality relations [Eqs. (2) and (3) in four dimensions] leaving only simple radial integrals to perform. We use the N-dimensional analog of this technique, namely[7]

$$\frac{1}{(\vec{k}_1-\vec{k}_2)^2}=\frac{1}{k_>^2}\sum_{n=0}^{\infty}Z^n G_n(Z^2)C_n^\alpha(\hat{k}_1\cdot\hat{k}_2), \tag{4}$$

where $Z = k_< / k_>$. $G_n(x)$ are related to hypergeometric functions and can be expanded in powers of ϵ.[5] In fact, writing $G_n(x) = 1 + \epsilon H_n(x) + \epsilon^2 K_n(x) + \ldots$ we compute, for instance,

$$H_n(x) = \frac{1}{2}\left\{ \sum_{j=1}^{n} \frac{1}{j} + \sum_{j=1}^{\infty} \frac{(n+1)x^j}{j(j+n+1)} \right\}.$$

It turns out that for our calculations[5] it suffices to know $H_n(x)$, $K_j(0)$, and $dK_j(0)/dx$, where $j = 0$, 1, 2, and 3. After using Eqs. (2) and (3) to perform the angular integrations we are left with radial integrals which involve only powers of the internal momenta. Those are trivially done following the prescription for dimensional regularization.[8] The resulting answers are simple sums which reduce to $\zeta(2)$ or $\zeta(3)$. [Actually, after a volume factor is factored out, only $\zeta(3)$'s appear.]

The actual calculation which must be done is described by Dine and Sapirstein[1] and reviewed here. For ease of comparison we use their notation. R can be calculated by dispersion methods[9] from

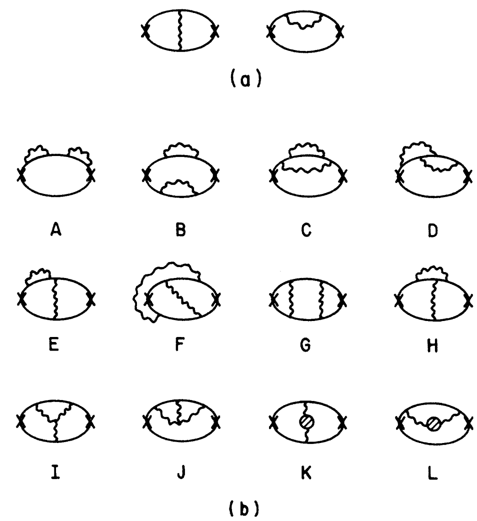

FIG. 1. Graphs contributing to R through order g^4 (Ref. 1).

$$\Pi_{\mu\nu}(q) = i(g_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2) = \int d^4x\, e^{iqx} \langle 0 | T\{J_\mu(x) J_\nu(0)\} | 0 \rangle .$$

By a renormalization-group argument it can be shown that

$$\Pi(q^2, g, \mu) = \Pi(\mu^2, \bar{g}(t), \mu) + \int_0^t \frac{\partial}{\partial g^2}[g^2 C_1(t')]\, dt' , \tag{5}$$

where μ is the mass parameter used in the minimal subtraction procedure,[2] $t = \frac{1}{2}\ln(q^2/\mu^2)$, $\bar{g}(t)$ is the running coupling constant satisfying $\bar{g}(0) = g$, and C_1 is the coefficient of the simple pole of the unrenormalized vacuum polarization. After taking the imaginary part of Π one finds that the result of expanding the right-hand side of (5) gives

$$R = 3\sum e_Q^2 \left\{ 1 + \frac{2C_{11}}{C_{10}} \bar{g}^2 + \frac{1}{C_{10}} \left(3C_{12} - 2\beta_0 \frac{\partial \Pi}{\partial \bar{g}^2}\bigg|_{\bar{g}=0} \right) \bar{g}^4 + \ldots \right\} , \tag{6}$$

where C_{1j} is defined by $C_1 = \sum C_{1j} g^{2j}$. C_{11} and C_{10} are well-known quantum-electrodynamic results.[10] $\partial\Pi/\partial \bar{g}^2|_{\bar{g}=0}$ is the sum of the diagrams in Fig. 1(a) and C_{12} is the sum of simple pole parts of the diagrams in Fig. 1(b).

In order to simplify the analytic calculation we computed Π from $\Pi_\mu{}^\mu$ noting that $\Pi_\mu{}^\mu(q) = iq^2(3-\epsilon)\Pi(s)$. Feynman gauge was used for the QCD propagators. An alternative method would be to extract the coefficient of $q^\mu q^\nu$ in individual diagrams. This was the method used by Dine and Sapirstein.[1] The two methods of calculation can be compared only for gauge-invariant sets. We do this in Table I. The agreement is remarkable.

Setting $C_{2A} = 3$, $C_{2F} = \frac{4}{3}$, and $T(R) = \frac{1}{2}$, and using Eq. (6), we finally arrive at the value for R_2 given in Eq. (1). As mentioned earlier, this value appears to be larger than one would like for a reasonable perturbative expansion. However, it has been noted[11,12] that the minimal subtraction scheme does not appear to be optimal for obtaining small high-order coefficients. In fact, in Ref. 11 it was shown that a more appropriate renormalization scheme is one in which a momentum-space subtraction is performed on the gluon propagator and trigluon vertex. When the subtraction is done in Landau gauge, we get $\alpha_{\rm mom} = \alpha_{\rm min}[1 + (\alpha_{\rm min}/\pi)K(N_F) + \ldots]$, where $K(N_F) = 11.9661 - 1.1798 N_F$. R can be rewritten in terms of $\alpha_{\rm mom}$ as

$$R(s) = 3\sum e_Q^2 \left[1 + \frac{\bar{\alpha}_{\rm mom}(s)}{\pi} + \left(\frac{\bar{\alpha}_{\rm mom}(s)}{\pi} \right)^2 (0.7389 N_F - 4.6374) \right].$$

TABLE I. (a) Analytical results for gauge-invariant subsets of diagrams of Fig. 1 compared with the numerical results of Dine and Sapirstein, Ref. 1. Given is the coefficient of $1/\epsilon$ when a factor

$$\left[3\sum e_Q^2\left(\frac{e^2}{4\pi^2}\right)\left(\frac{g^2}{4\pi^2}\right)^2\{\Gamma(1-\epsilon/2)\}^{-3}(4\pi)^{3\epsilon/2}\left(\frac{-q^2}{\mu^2}\right)^{-3\epsilon/2}\right]$$

has been extracted from each subset. (b) The finite part of the sum of Fig. 1(a) when a factor

$$\left[3\sum e_Q^2\left(\frac{e^2}{4\pi^2}\right)\left(\frac{g^2}{4\pi^2}\right)\{\Gamma(1-\epsilon/2)\}^{-2}(4\pi)^{\epsilon}\left(\frac{-q^2}{\mu^2}\right)^{-\epsilon}\right]$$

has been extracted.

Diagrams	Group Weight	Analytic Result	Ref. 1
(a)			
A + B + E/2	C_{2F}^2	$119/96 - \zeta(3) \simeq 0.03753$	0.0376
C + E/2 + G	C_{2F}^2	$-7/32 \simeq -0.21875$	-0.2177
D + F + H	$C_{2F}^2 - \frac{C_{2F}C_{2A}}{2}$	$-1 + \zeta(3) \simeq 0.20206$	0.2015
I + J	$C_{2F}C_{2A}$	$-41/24 + 3\zeta(3)/2 \simeq 0.09475$	0.0948
K + L	C_{2F}	$-113/108\, C_{2A} + 22/27\, N_F T(R)$ $\simeq -0.04458\, C_{2A} + 0.00672\, N_F$	$-0.0447\, C_{2A} + 0.00679\, N_f$
(b)			
I(a)		$55/48 - \zeta(3) \simeq 0.056224$	-0.0564

Finally, it is noteworthy that the order-α_s^2 to R is essentially the same calculation as the order-α_s^2 correction to the width $\Gamma(Z^0 \to \text{hadrons})$.[13] Because of the expected large cross section for Z^0 production, it may actually be possible to measure this width to an accuracy of order α_s^2.[14]

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[(a)] Present address: Argonne National Laboratory, Argonne, Ill. 60439.

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Search for Prompt Neutrinos and New Penetrating Particles from 28-GeV Proton-Nucleus Collisions

A. Soukas, P. Wanderer, and W.-T. Weng
Brookhaven National Laboratory, Upton, New York 11973

and

M. Bregman, M. Claudson, J. LoSecco,[(a)] L. Rivkin, J. Roeder, S. Russek, L. Sulak,[(a)] P. Timbie, and M. Yudis
Harvard University, Cambridge, Massachusetts 02138

and

T. A. Gabriel
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

and

R. S. Galik, J. Horstkotte, J. Knauer,[(b)] M. Levine, and H. H. Williams
University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 13 November 1979)

This paper describes a search for weakly or semistrongly interacting particles produced in the collision of 4.9×10^{18} 28-GeV protons with a thick copper and brass beam stop. 104 events have been observed; their characteristics are similar to those produced by neutrinos arising from proton interactions in a 15-cm-thick brass target. However, compared to the number expected, an excess of 48 events is found with uncertainties of ± 10 (statistical) and ± 12 (systematic). At most ten events (68% confedence level) are attributable to beam losses.

We have searched for directly produced neutrinos, neutrinos from the decay of short-lived ($\leq 10^{-11}$ sec) parents, or new penetrating neutral particles produced by 28.5-GeV protons from the Brookhaven National Laboratory alternating-gradient synchrotron (AGS) in a thick copper and brass target. Neutral particles were detected in our neutrino detector.[1] The flux of neutrinos from the decay of long-lived mesons is suppressed through the use of such a high-density, thick target, greatly increasing the likelihood of observing promptly produced particles in the detector.

For the new-particle search, protons were transported in vacuum to a copper and brass beam dump 1.0 m long and 0.3 m^2 in cross section followed by a 30-m-long iron absorber. Two special runs were made for normalization and background measurements. In the normalization

