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Resolution of Shubnikov–de Haas Paradoxes in Si Inversion Layers

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In two dimensions the frequency of magnetoconductivity oscillations measures the sum of Fermi-surface areas when the Landau levels are sharp. This explains why (100) Si inversion layers with two occupied subbands show only one frequency. It also explains why the frequency observed on (111) surfaces simulates a valley degeneracy of 2, even though six valleys are occupied.

In a recent experiment by Stallhofer, Kotthaus, and Abstreiter[1] it was shown that in Si(100) inversion layers under uniaxial stress two masses could be observed corresponding to simultaneous occupation of both subbands E_0 and $E_{0'}$. However, in a subsequent experiment by Gesch *et al.*,[2] in which cyclotron resonance and Shubnikov–de Haas (SdH) oscillations were observed on the same sample and under identical conditions, the oscillations showed only one frequency corresponding to all the electrons. One would have expected a superposition of two frequencies. In this Letter we show that this apparent discrepancy is to be expected in a two-dimensional system with sharp Landau levels. In the same way the long-standing problem of the unexpected (apparent) twofold valley degeneracy on (110) and (111) surfaces[3-7] can also be understood without resorting to an extremely high strain at the surface.[5]

On the (100) surface the electrons in the inversion layer are quantized into two different types of subbands[8]: The two valleys of the conduction band of Si that have their longitudinal mass perpendicular to the surface give rise to one set of subbands $(0, 1, 2, \ldots)$ which has a small effective mass $m = 0.19m_e$ for motion parallel to the surface. The other four valleys give rise to another set $(0', 1', 2', \ldots)$ which has a greater mass $m' = 0.42m_e$. Usually, only subband 0 is occupied, but application of uniaxial stress can move two of the four other valleys down in energy, so that the $0'$ subband gets populated. Furthermore, a magnetic field applied perpendicular to the surface causes the subbands to quantize into Landau levels which ideally are discrete:

$$E_{0N}=E_0+(N+\tfrac{1}{2})\hbar\omega_c\,,$$

$$E_{0'N}=E_{0'}+(N+\tfrac{1}{2})\hbar\omega_c'\,,\quad N=0,1,2,\ldots\,,$$

where E_0 ($E_{0'}$) is the bottom of subband 0 (0′) for zero magnetic field, and $\omega_c=eB/m$ ($\omega_c'=eB/m'$).

The number of states in each Landau level is

$$N_{LL}=2g_v(e/h)B$$

independent of effective mass. We neglect spin splitting for clarity, and $g_v=2$ is the valley degeneracy. The levels will in reality be broadened. In the following calculations, which are only meant to illustrate our argument, we assume a Gaussian shape with a constant width parameter Γ, so that the density of states is given by

$$D(E;B)=\frac{N_{LL}}{\Gamma(2\pi)^{1/2}}\left\{\sum_{N=0}^{\infty}\exp\left[-\frac{1}{2}\left(\frac{E-E_{0N}}{\Gamma}\right)^2\right]+\sum_{N=0}^{\infty}\exp\left[-\frac{1}{2}\left(\frac{E-E_{0'N}}{\Gamma}\right)^2\right]\right\}.$$

Furthermore, we take as a rough measure for the conductivity at temperature 0 the square of the density of states at the Fermi level E_F. Suppose that subband 0 contains n_0 electrons, and subband 0′, $n_{0'}$ electrons. If the Fermi level were constant, the conductivity would have maxima whenever $E_{0N}=E_F$ or $E_{0'N}=E_F$ and have two oscillation periods in $1/B$: $\lambda=2g_ve/hn_0$ and $\lambda'=2g_ve/hn_{0'}$ and one should be able to determine n_0 and $n_{0'}$ from the oscillations.

However, *the Fermi energy is not constant.* The total number of electrons n is constant, and this means that for sharp levels E_F gets locked to the Landau level that is being emptied with increasing B. When the level is completely empty, E_F jumps rapidly to the next lower Landau level from either subband 0 or 0′. At this field the conductivity is minimal. Since the number of states in each Landau level is independent of subband system, the minima will have a spacing $\lambda=2g_ve/hn$ reflecting the total number of electrons regardless of the relative population of the two subbands. In the lower frame of Fig. 1 this is illustrated for extremely small broadening. The square of the density of states at the Fermi level for constant total inversion-layer density is plotted as a function of $1/B$. The difference in subband energies $\Delta=E_{0'}-E_0$ is chosen so that 0.57 of the electrons are in subband 0 and 0.43 in subband 0′. Clearly the period of the oscillations reflects only the total number of electrons. The two spikes correspond to two Landau levels crossing each other at the Fermi level.

We now consider broadening. It is important to note that two mechanisms can broaden the measured signal. First, the intrinsic broadening of the Landau levels due to scattering will tend to reduce the oscillations and give additional structure because of the larger field ranges in which two Landau levels overlap at the Fermi energy. Second, one can expect broadening from inhomogeneous strain at the interface, which causes a fluctuating difference in subband energies and population ratio. This mechanism also reduces the oscillations, but tends to reduce even more the additional structure arising from overlapping levels. In the upper frame of Fig. 1 we illustrate these effects. The parameter for intrinsic broadening is chosen to be $\Gamma=0.25$ meV. The inhomogeneous broadening is simulated by taking an average over signals from nine subband

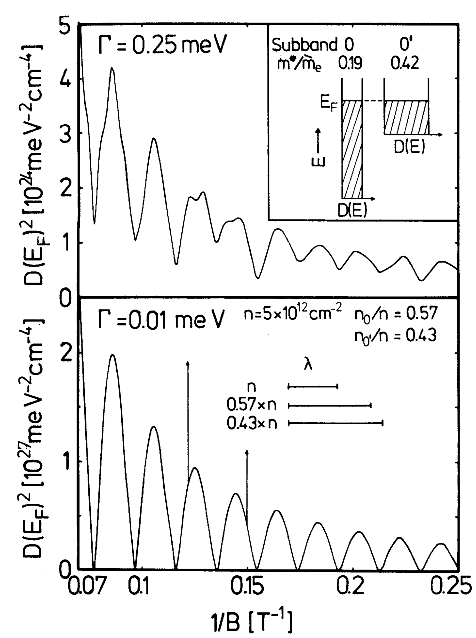

FIG. 1. Square of the density of states at the Fermi level as a function of reciprocal magnetic field when both subband 0 and 0′ are occupied. Lower frame: Very sharp Landau levels. Upper frame: Broader levels and inhomogeneous broadening as described in text. Inset: Density of states for $B=0$.

separations $\Delta^{(i)}=\Delta+(i-5)\times(1\text{ meV})$, $i=1,2,\ldots 9$, where Δ was chosen (as before) to give an average population ratio of 57/43. Comparison with the lower frame of the figure convincingly shows that the basic periodicity still counts all the electrons. The experimental fact[2,9] that the oscillation amplitude is considerably reduced when both subbands are occupied shows that strain inhomogeneities constitute the major broadening mechanism; they have no effect when only one subband is occupied. We can thus conclude that, as long as the intrinsic broadening is smaller than the smallest Landau-level separation, the SdH frequency will measure the total number of electrons, i.e., the sum of Fermi-surface areas.

Inhomogeneous strain can be expected to exist on all interfaces. On a (111) Si interface such a strain will split a subband of valley degeneracy 6 into three doubly degenerate subbands. If the Landau levels are sharper than this splitting, the Fermi level will again be locked to one Landau level until it is emptied and then rapidly move to the next lower one in energy as the magnetic field is increased. This will simulate a valley degeneracy of 2 even though all six valleys contain electrons. The only necessary condition is that the strain should split the valleys by more than the width of the levels; the rapid periodicity does not imply that the strain has moved four of the valleys above the Fermi level.[5]

In Fig. 2 we illustrate this effect: For a fixed magnetic field of $B=10$ T we show the square of the density of states at the Fermi level calculated as a function of density. We have assumed that the strain splits the six valleys into three pairs separated by 1 meV. In the lowest frame the intrinsic broadening is smaller than the valley splitting, and the oscillations clearly simulate a valley degeneracy of 2. In the upper frames we increase the broadening. When it is comparable to the splitting, the periodicity becomes that for a sixfold degeneracy. Experimental observations support this simple description: In samples which mimic double degeneracy, external stress enhances the signal,[5] because the already existing valley splitting is enhanced; the samples that show sixfold degeneracy[10,11] have undergone a treatment designed to reduce strain at the interface, but it also reduced the mobility by a factor of 2 or 3, which would increase the intrinsic broadening of the Landau levels.

We have shown the results for only one strain value. The inhomogeneity in magnitude and direction of the strain will reduce the signal and will average out extra structure coming from overlapping Landau levels in the same way as for the (100) surface. The component with $\frac{1}{3}$ the fundamental frequency seen in the lower frames of Fig. 2 also disappears, if the strains are large enough. Introduction of spin splitting of the levels does not alter the picture qualitatively. In particular, experiments in which the spin splitting is changed by tilting the magnetic field will not show structure from overlapping spin levels of different valleys when the strain is inhomogeneous.

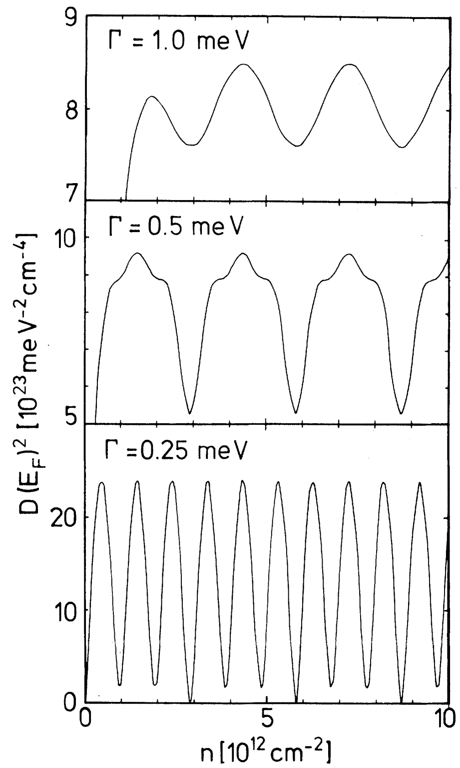

FIG. 2. Square of the density of states at the Fermi level as a function of inversion layer density n of a (111) Si inversion layer for three values of the Landau-level width parameter Γ. $B=10$ T. The three pairs of valleys are assumed to be split by 1 meV. At $n=10^{13}$ cm^{-2}, $E_F=11$ meV for six valleys filled. If only two valleys were filled, E_F would be 33 meV.

Obviously our description applies in the same way to fourfold-degenerate subbands, whether they be investigated on (110) surfaces[3,4,7] or whether they be brought down through stress on (100)[2,12] or (111) surfaces. Only when the mobility is relatively low has the fourfold degeneracy been observed.[11] In other cases the intrinsic broadening appears to have been smaller than the strain-induced valley splitting since "twofold degeneracy" was reported.

We have pointed out that the usual interpretation of SdH measurements is not valid when the Landau levels are sharp. As the width of the level grows, the motion of the Fermi level becomes less and less pronounced. If just one electron system has low mobility, the Fermi level will be fairly constant, so that the usual superposition of oscillations can be observed. We believe this is the case in measurements reported on p-channel Si inversion layers[13] and on other semiconductor materials.[14]

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Role of Clusters in the Approach to Localization of Josephson-Coupled Granular Lead Films

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The superconducting properties of thin granular lead films provide evidence for the existence of clusters which mitigate electrostatic charging effects and allow Josephson coupling to remain favorable in samples with normal-state sheet resistance near the maximum metallic resistivity of 30 000 Ω/sq. The disappearance of zero-resistance transitions together with the precipitous onset of temperature broadening in this range is therefore most likely associated with the localization of electronic states.

The resistive transitions of thin two-dimensional (2D) granular superconducting film typically exhibit a broadening in temperature[1] as the normal-state resistance per square $R_\square^N$ approaches the maximum metallic resistivity[2] of 30 000 Ω/sq. Recent measurements on thin films of Pb, Sn, and Al also confirm that a transtiion to zero resistance will not occur if $R_\square^N$ is greater than 30 000 Ω/sq.[3] In this report we describe experimental observations on thin (~300 Å) granular lead films with $R_\square^N \lesssim 30\,000$ Ω/sq which provide new and convincing evidence for the presence of clusters[4,5] which we shall demonstrate play an important role in maintaining Josephson coupling across regions of the film containing a relatively large number of grains. The appearance on the I-V characteristics of uniformly spaced voltage steps, separated by twice the energy gap (2Δ) of lead, is a direct manifestation of the fact that Josephson phase-slip processes are occurring along the oxide boundaries separating these macroscopic-size clusters of grains. Our data show that the clusters decrease in size with increasing $R_\square^N$ and yet remain large enough to mitigate the effects of electrostatic charging[6] for films with $R_\square^N \simeq 30\,000$ Ω/sq. We argue that the absence of transi-