pp collisions the ρ production rate is known experimentally; based on that rate it is possible to infer that the probability of formation of ρ is 0.45 times that of π.[11] If we use this ratio to compute the contribution from ρ production and subsequent decay to the quark fragmentation function, the net pion distribution is that shown by the solid curve in Fig. 2. The agreement with the data is striking.

The success of our calculation lends support to both the theory of quark proliferation as prescribed by QCD and the model for hadronization in the framework of valons and recombination. The model that we have adopted for the hadronization of emitted gluons as expressed in (6) and (8) can be justified only *a posteriori*. It implies total gluon conversion to pions as suggested in the naive inside-outside quark-gluon diagram for e^+e^- annihilation. The evolution parameter s depends crucially on Q_0/Λ which is extracted from structure functions and is not free to vary. The largeness of s accounts for precocious scaling and the sharp damping in x in Fig. 2. Our result offers a strong indication for the correct mechanism of hadronization of quarks.

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Inner-Shell Electron Capture by a Swift Bare Ion: Second Born Effects

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The second Born cross section for electron capture from a hydrogenlike atom of atomic number Z_T by a bare ion of atomic number Z_P and speed v is evaluated for $(Z_P e^2/\hbar v) << 1$ and Z_T arbitrary. The ratio of the second to first Born cross sections increases rapidly as Z_T increases; for $(Z_T e^2/\hbar v) \gtrsim 1$ this ratio is very large. These results indicate that second- and higher-order Born terms must be considered in calculating the cross section for inner-shell electron capture (or the time-reversed process).

It is well known that, within the nonrelativistic framework, the second Born term dominates over the first for electron capture by an ion whose impact speed v (velocity $\vec{v}$) is large.[1-3] The asymptotic expansion in powers of $e^2/\hbar v$ of the first and second Born terms, for electron capture from a hydrogenlike atom by a bare ion, is presumably valid for $(Z_T e^2/\hbar v)$ and $(Z_P e^2/\hbar v) \ll 1$ where $Z_T e$ and $Z_P e$ are the charges of the target nucleus T and the projectile P, respectively. However, if Z_T or $Z_P \gg 1$ these inequalities are not satisfied until v is so large that relativistic effects are significant (and the cross section is so small that its measurement is of little interest at present). I have evaluated the second Born term for ground-state–to–ground-state electron capture from a hydrogenlike atom by a bare ion under the single assumption $(Z_P e^2/\hbar v) \ll 1$, with Z_T arbitrary except that $(Z_T e^2/\hbar c) \ll 1$ and $(v/c)^2 \ll 1$ so that relativistic effects are negligible. I find that the ratio of second to first Born cross sections increases rapidly with increasing Z_T

and for $(Z_T e^2/\hbar v) \gtrsim 1$ this ratio is very large. This indicates that second- and higher-order Born terms must be considered in calculating the cross section for inner-shell electron capture by light projectiles.

Let φ_i and φ_f, respectively, represent the initial and final states i and f of the electron, and let ϵ_i and ϵ_f denote the energy eigenvalues. Let $\hbar\vec{K}_i$ and $\hbar\vec{K}_f$, respectively, denote the initial momentum of P and the final momentum of $(P+e)$ in the center-of-mass frame of $(P+T+e)$. Define $\alpha \equiv M_T/(M_T+m)$ and $\beta \equiv M_P/(M_P+m)$, where m, M_T, and M_P are the masses of the electron, T, and P. Define

$$\vec{K} = \beta\vec{K}_f - \vec{K}_i,\ \vec{J} = \alpha\vec{K}_i - \vec{K}_f; \tag{1}$$

$\hbar\vec{K}$ is the momentum transferred to P, averaged over the internal motion of $(e+P)$ in state f, and $\hbar\vec{J}$ is the momentum transferred to T, averaged over the internal motion of $(e+T)$ in state i. Let V_{Pe} and V_{Te} denote the interactions of the electron with P and T, respectively. Corrections of order m/M_T and m/M_P will be neglected throughout; these corrections include the effect of the internuclear potential in the integrated cross section σ. Within the second Born approximation the cross section is[4]

$$\sigma \equiv (2\pi\hbar^2 v^2)^{-1}\int_\xi^\infty |T|^2 K\,dK, \tag{2}$$

where the lower limit

$$\xi = |(mv/2\hbar) + (\epsilon_f - \epsilon_i)/v\hbar|, \tag{3}$$

and $T = T_1 + T_2$ where, with a tilde denoting a Fourier transform,

$$T_1 = -(2\pi)^3[(\hbar^2/2m)K^2 - \epsilon_f]\tilde{\varphi}_f{}^*(\vec{K})\tilde{\varphi}_i(-\vec{J}), \tag{4}$$

$$T_2 = \int d^3p \int d^3q\, \tilde{\varphi}_f{}^*(-\vec{p})\tilde{V}_{Te}(\vec{q})(1/D)\tilde{V}_{Pe}(\vec{p}+\vec{K})\tilde{\varphi}_i(\vec{q}-\vec{J}), \tag{5}$$

$$D = \epsilon_f - \hbar\vec{q}\cdot\vec{v} - (\hbar^2/2m)(\vec{q}-\vec{p})^2 + i\eta, \tag{6}$$

where η is infinitesimally small but positive. T_1 is just the Brinkman-Kramers amplitude. T_2 is the second-order Born amplitude, obtained by replacing the Green's function for three interacting particles by the Green's function for three *noninteracting* particles.

Assume now that $(Z_P e^2/\hbar v) \ll 1$. Since the main contribution to the integral over $\vec{p}$ in Eq. (5) comes from the region where $\tilde{\varphi}_f(\vec{p})$ is nonnegligible, i.e., the region $|\vec{p}| \lesssim Z_P/a_0$, and since $K > \xi \gg Z_P/a_0$, the approximation

$$\tilde{V}_{Pe}(\vec{p}+\vec{K}) \sim \tilde{V}_{Pe}(\vec{K}) \tag{7}$$

is justified. The integral over $\vec{p}$ can then be evaluated exactly. With i and f both $1s$ states I find, using atomic units $(e = m = \hbar = 1)$,

$$T_2 \sim \frac{-2^5 Z_P{}^{5/2} Z_T{}^{7/2}}{\pi K^2}\int \frac{d^3q}{q^2}\,\frac{1}{[Z_T{}^2 + |\vec{q}-\vec{J}|^2]^2}\,\frac{1}{q^2 + (\gamma + Z_P)^2}, \tag{8}$$

$$\gamma^2 = 2\vec{q}\cdot\vec{v} + Z_P{}^2 - i\eta. \tag{9}$$

With the polar axis chosen to be along $\vec{v}$, the integration over the azimuthal angle of $\vec{q}$ can be done exactly, leaving a two-dimensional integral to be evaluated using the computer. (Actually, integration over one more variable can be done in the present case.)

The amplitude T_2 of Eq. (5), together with the approximation of Eq. (7), corresponds to the double-scattering mechanism proposed by Thomas.[5] Thus, working in the laboratory frame, the electron initially has momentum $\hbar(\vec{q}-\vec{J})$, but receives momentum $-\hbar\vec{K}$, with an amplitude $\tilde{V}_{Pe}(\vec{K})$, via a collision with P. The electron emerges from this collision with momentum $\hbar\vec{q} + m\vec{v}$ (note that $\vec{J}+\vec{K} = -m\vec{v}/\hbar$) and then scatters from T, receiving momentum $-\hbar\vec{q}$ with an amplitude $\tilde{V}_{Te}(\vec{q})$. Hence the electron finally has momentum $m\vec{v}$ and is moving with the same velocity as P (whose motion is barely altered after the first collision) so that capture can subsequently occur with ease. Since $m/M_T \ll 1$ the second collision is elastic. Therefore the electron propagates on the energy shell between the two collisions if

$$|\hbar\vec{q} + m\vec{v}|^2/2m = mv^2/2,$$

that is, if

$$\hbar\vec{q}\cdot\vec{v} + (\hbar^2/2m)q^2 = 0. \tag{10}$$

This condition is equivalent to the requirement that the energy denominator D of Eq. (6) nearly vanish, since in the region where D nearly vanishes ϵ_f and the terms in $\vec{p}$ are small compared to $\hbar\vec{q}\cdot\vec{v}$. The contribution to T_2 from the region of integration where D nearly vanishes is large and purely imaginary.

If $(Z_T e^2/\hbar v) \ll 1$ then $J \gg Z_T/a_0$ and the integrand of Eq. (5) peaks where $\tilde{\varphi}_i(\vec{q}-\vec{J})$ is non-negligible, i.e., in the region $|\vec{q}-\vec{J}| \lesssim Z_T/a_0$. In this case $\tilde{V}_{Te}(\vec{q})$ can be approximated by $\tilde{V}_{Te}(\vec{J})$ and, using Eq. (7), the integrations in Eq. (5) can be performed to give in atomic units

$$T_2 \sim 2^5\pi(Z_T Z_P)^{5/2}K^{-4}[v^2 - K^2 + i2(Z_T + Z_P)v]^{-1}. \tag{11}$$

This result has been obtained many times previously, with an unimportant difference in the imaginary component, namely, $2(Z_T + Z_P)v \rightarrow 2(Z_T + Z_P)K$. Note that the right-hand side of Eq. (11) is largest when $K = v$; the denominator D nearly vanishes when $\vec{q} = \vec{J}$ and $K = v$.

If $(Z_T e^2/\hbar v) \gtrsim 1$ the initial orbital momentum of the electron is important, the significant values of J are not large compared to Z_T/a_0, and the integrand of Eq. (5) does not peak in the region $\vec{q} \approx \vec{J}$. Rather, the integrand peaks for *all* $\vec{q}$ satisfying Eq. (10). The significant region of phase space is therefore much larger than when $(Z_T e^2/\hbar v) \ll 1$; I will return to this point below.

For $(Z_P e^2/\hbar v) \ll 1$ and i and f both $1s$ states,

$$T_1 \sim -2^5\pi(Z_T Z_P)^{5/2}K^{-6}, \tag{12}$$

in atomic units. In Fig. 1 the ratio R of the cross section computed from the sum $T_1 + T_2$ to the cross section computed from just T_1 is plotted as a function of Z_T, with $Z_P = 1$. T_1 and T_2 were evaluated according to Eqs. (12) and (8). R is shown for energies of 2.5 MeV/u $(e^2/\hbar v = 0.1)$ and 10 MeV/u $(e^2/\hbar v = 0.05$. Evidently R rises rapidly with increasing Z_T and for $(Z_T e^2/\hbar v) \gtrsim 1$ second Born effects are overwhelming. For Z_T fixed, R tends to about 0.3 as v increases beyond $Z_T e^2/\hbar$.

The reason that R increases as Z_T increases, for fixed v, is the following: Note from Eq. (10) that the significant values of the momentum transfer, $-\hbar\vec{q}$, in the second collision lie between 0 and $2m\vec{v}$. Therefore the second collision is a "soft" one in the sense that *the Coulomb differential cross section for the second scattering is not small* compared to the area $\pi(a_0/Z_T)^2$ of the target atom. This also implies that the electron does not have to make a head-on collision with T so that the electron *does not have to scatter into a narrow solid angle in the first collision.* (In-

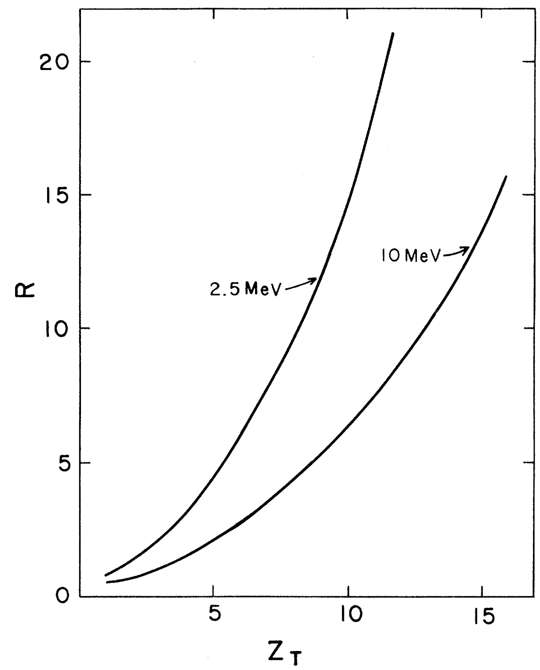

FIG. 1. The ratio R of the second to first Born cross sections vs Z_T, for $Z_P = 1$ and for energies of 2.5 and 10 MeV/u.

deed, from the equation preceding Eq. (10) one sees that the vector tip of the momentum $\hbar\vec{q} + m\vec{v}$ of the electron between collisions can lie *anywhere* on a sphere of radius mv.) Neither of these two emphasized statements is true when $(Z_T e^2/\hbar v) \ll 1$, as emphasized in Ref. 3; for then the second collision is a "hard" one, in the sense that the Coulomb differential cross section for the second scattering is small compared to the target area $\pi(a_0/Z_T)^2$, because the significant values of $\vec{q}$ are $\vec{q} \approx \vec{J}$, but $J \gg Z_T/a_0$.

For $(Z_T e^2/\hbar v) \gtrsim 1$ third- and higher-order Born terms are probably just as important as the second-order Born term, and the result obtained in the second Born approximation might be very inaccurate (too large). All orders in V_{Te} can be built in by use of the *Coulomb* Green's function rather than the *free* Green's function (used here) in the definition of the second-order Born term T_2. Alternatively, one could build in V_{Te} to all orders by use of the impulse approximation, as suggested by Briggs.[6] However, as Briggs has emphasized, in arriving at the impulse-approximation amplitude an approximation is made which is not fully consistent. Furthermore, this amplitude is rather difficult to evaluate without making further approximations; it might be no more difficult to evaluate the second-order Born amplitude with the Coulomb Green's function, this latter procedure being fully consistent.

Cross sections for electron capture from hydrogenlike atoms of high atomic number have been calculated by Olson[7] using a classical trajectory Monte-Carlo method. Newton's equations were solved essentially exactly, and not perturbatively, so that the only error lies in the classical approximation.

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Turbulence near Onset of Convection

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New long-term (days) measurements of the evolution of turbulence for a Rayleigh-Bénard system with aspect ratio $\Gamma = 4.72$ reveal a turbulent state, with a threshold near the critical Rayleigh number R_c, which consists of a random background time dependence and rare, randomly spaced, major events. These events are discussed in terms of the analog of a particle under the influence of a stochastic driving force and in a potential with two minima.

It was reported previously[1,2] that the sequence of events leading to nonperiodic, or turbulent, flow in a horizontal layer of fluid heated from below (Rayleigh-Bénard system) is qualitatively altered by changing the aspect ratio $\Gamma \equiv L/d$ of the sample (L is the radius and d is the height of the cylindrical container). Particularly surprising was the observation that nonperiodic behavior occurred at Rayleigh numbers R immediately above the critical Rayleigh number R_c for the onset of flow when Γ was large ($\Gamma = 57$). This experimental result is difficult to reconcile with the stability analysis of Schlüter, Lortz, and Busse[3] who predicted on the basis of the deterministic equations of motion of the laterally infinite system that there should be a range of R above R_c over which a fluid velocity field consisting of a time-independent system of rolls will be stable. At an intermediate value of Γ ($\Gamma = 4.72$), experiments over time periods of many hours had led us to believe that the fluid flow was time independent for $R \lesssim 2R_c$, and had demonstrated that the system was *obviously* turbulent for greater R.[1,2,4] This result also differs from the analysis of the deterministic equations of motion for the laterally infinite system which predicts[5] that the first instability of the steady convection should be to a periodic state and should occur only when $R \gtrsim 5R_c$. Indeed, the experiment revealed that structure in the broadband spectrum evolves for $R \geqslant 4.7R_c$ at frequencies which are consistent with the predicted periodic state[1,2]; but the bifurcation which yields this state occurs only *after* the system is already turbulent. In the present Letter we report new results which were obtained for the medium-aspect-ratio system ($\Gamma = 4.72$) on a time scale of many days rather than many hours. They show that rare, randomly spaced, major events occur for R well below $2R_c$ where we previously thought the fluid flow to be stationary. The new results indicate that the onset of chaotic behavior occurs very close to R_c even for medium-aspect-ratio systems, and the data suggest that the time scale of the turbulence diverges exponentially as $R - R_c$ vanishes. We will discuss the observations in terms of a model with an external stochastic driving force.