Determination of the Quark Decay Function without Phenomenological Input

Van Chang and Rudolph C. Hwa
Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, Oregon 97403
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On the basis of quantum chromodynamics (QCD) and the recombination model the quark decay function is calculated. The inclusive distribution for a quark and an antiquark in a quark jet can be determined by the perturbative method in QCD. The hadronization of quarks and antiquarks is described by a recombination function that is completely fixed in a recent study of the structure function. No free parameters are involved and the agreement with the data is excellent.

Theoretical investigations of the quark decay function $D(x,Q^2)$ in the framework of quantum chromodynamics (QCD) have studied mainly the Q^2 dependence of the x distribution.[1] In such studies the x distribution itself at some low Q^2 must first be taken from experiments as the boundary value. The evolution equation[2] then specifies the x distribution at higher Q^2. In this paper we go beyond the usual procedure and calculate $D(x,Q^2)$ at some high Q^2 without any phenomenological input at low Q^2. There are no adjustable parameters, and the result agrees well with experiment.

A skeptic would with good reason immediately question the feasibility of such a calculation before the resolution of the confinement problem. Indeed, the hadronization of quarks requires a knowledge of the wave function of the quarks in a hadron at low Q^2. Despite the absence of a calculational scheme for confinement, recent investigation of the nucleon structure function has revealed a way of extracting the wave function of the "constituent" quarks.[3] This possibility gives impetus to the recombination model for the hadronization problem,[4] since the recombination function is now known explicitly without undetermined parameters.[5]

Our procedure therefore has two parts which, in skeleton form, are as follows. First, we calculate the inclusive distribution $F(x_1,x_2,Q^2)$ for a quark at x_1 and antiquark at x_2 in a quark jet initiated at Q^2. This can be done reliably in perturbative QCD. Then we calculate the pion inclusive distribution $D(x,Q^2)$ in the framework of the recombination model,[4]

$$xD(x,Q^2) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} F(x_1,x_2,Q^2) R(x_1,x_2,x), \qquad (1)$$

where $R(x_1,x_2,x)$ is the recombination function for forming a pion at x. Implicit in (1) is a ${Q_0}^2$ at which recombination takes place, and to which the Q^2 degradation of the initial quark must lead by gluon bremsstrahlung and $q\bar{q}$ pair creation. Its value is already determined in a separate analysis,[3] although its precision is not crucial to our result here.

We discuss $R(x_1,x_2,x)$ first since it plays the key role in the hadronization of quarks, and yet it is not generally known. But even before that, we must review the concept of valons.[3] The bridge between "constituent" quarks in the bound-state problem of hadrons and "current" quarks as probed in deep inelastic scattering is the recognition of valence-quark clusters, called valons, which are the valence quarks dressed by QCD virtual processes. Thus a valon contains a valence quark and its own sea quarks and gluon cloud. There are three valons in a nucleon and two in a meson, just like the constituent quarks. But when probed at high Q^2 they have internal structure completely determined by calculable methods in QCD. That is, the valon structure functions themselves have no uncalculable complications associated with confinement, the latter attributed entirely to the momentum distribution of the valons in a hadron, $G_{v/h}(y)$, where y is the momentum fraction of a valon. In the case of a nucleon it is shown in Ref. 3 how $G_{v/N}(y)$ can be determined from the structure function $xF_3(x)$ as probed by neutrino scattering at high Q^2. It is straightforward to extend the method to the meson case; based on the structure function of the pion as revealed in lepton-pair production,[6] one can infer that the two-valon exclusive distribution for a pion is[5]

$$F_{v_1v_2/\pi}(y_1,y_2) = y_1y_2G_{v_1v_2/\pi}(y_1,y_2) = y_1y_2\delta(y_1+y_2-1). \qquad (2)$$

This is the distribution at Q_0, the value of Q at which the internal structure of valons cannot be resolved. We adopt the value $Q_0 = 0.82$ GeV determined in Ref. 3, in which the strong interaction scale is taken to be $\Lambda = 0.74$ GeV. The valon distribution as well as the value of Q_0 is determined directly from high-Q^2 data with use of leading-order perturbative QCD analysis without relying on any theoretical extrapolation to low Q^2. In that sense Q_0 is an effective value.

Equation (2) describes the wave function of a pion in valon coordinates, more precisely $|\langle v_1 v_2 | \times | \pi\rangle|^2$. The complex conjugate of the amplitude then describes the recombination process; the probability for recombining two valons at x_1 and x_2 to form a pion at x is, therefore,

$$R(x_1,x_2,x)=\frac{x_1x_2}{x^2}\left(\frac{x_1}{x}+\frac{x_2}{x}-1\right). \tag{3}$$

This is exactly the form suggested earlier[4] based on counting rules,[7] except that the normalization is now known precisely.

If (3) is to be regarded as a simple but reliable summary of the hadronization process of quarks (which incidentally has been successful in describing low-p_T reactions), it can be put to a stringent test in the present problem of determining $D(x,Q^2)$. For large Q^2, the perturbative QCD method is reliable for calculating the proliferation of quarks and antiquarks in a jet as the Q^2 value of the initial quark degrades toward Q_0^2 at the valon level. The distribution $F(x_1,x_2,Q^2)$ refers to a quark and an antiquark at Q_0^2 on the one hand, but the same $q\bar{q}$ pair is to be regarded as two valons, on the other hand, in the context of hadronization as described by $R(x_1,x_2,x)$ in (1). The duality of these roles is an essential aspect of the valon model which, as mentioned above, serves as a bridge between the hard and soft processes. Equation (1) is analogous to an (inverse) expression for the structure function of a hadron: a convolution of valon distribution (at Q_0) and valon structure function (evolution from Q_0 to Q).[3]

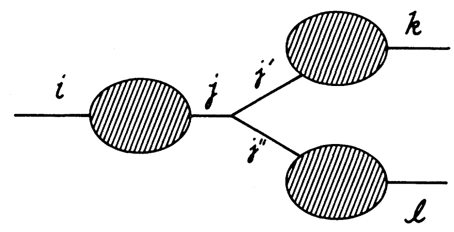

FIG. 1. Schematic diagram for the inclusive distribution of a quark and an antiquark in a quark jet. Gluons and quarks leaking from the blobs are not indicated.

The stage is now set for the discussion of the main task of the calculation, i.e., the determination of $F(x_1,x_2,Q^2)$. We shall consider the leading order of perturbative QCD only. The justification for its meaningfulness for the Q^2 degradation all the way down to a low Q_0^2 is rooted in the definition of Q_0^2, which is the effective value for the start of Q^2 evolution of the valon structure function also in the leading-order approximation, as mentioned above. That is, in both the decay function here and the structure function earlier[3] we are consistent in using the same effective evolution parameter

$$s=\ln[(\ln Q^2/\Lambda^2)/(\ln Q_0^2/\Lambda^2)], \tag{4}$$

where Q_0 was adjusted to render the *leading* order results to agree with high-Q^2 νN data.

The inclusive distribution for a q and $\bar{q}$ must involve the explicit consideration of one bifurcation point, as depicted in Fig. 1, where a blob represents a G function for Q^2 degradation, i.e., the evolution function due to emission of gluons and quarks which are not shown explicitly in the diagram. Using perturbative QCD and consistent with jet calculus,[8] we have

$$x_1x_2G_{i\to kl}(x_1,x_2,Q^2,Q_0^2)=\sum_{jj'j''}\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\frac{\alpha(k^2)}{2\pi}\int_{x_1+x_2}^{1}dy\,G_{i\to j}(y,Q^2,k^2)\times\int_{x_1}^{y-x_2}\frac{dz}{y}P_{j\to j'}\left(\frac{z}{y}\right)\frac{x_1}{z}G_{j'\to k}\left(\frac{x_1}{z},k^2,Q_0^2\right)\frac{x_2}{y-z}G_{j''\to l}\left(\frac{x_2}{y-z},k^2,Q_0^2\right), \tag{5}$$

where the subscripts refer to partons of various types (quark, antiquark, gluon). $P_{j\to j'}$ is the decay function associated with the bifurcation vertex.[2] $G_{i\to j}(y,Q^2,k^2)$ is the distribution of parton j in parton i carrying momentum fraction y as it evolves from Q^2 to k^2.[1,2,9] $G_{i\to kl}$ is the two parton distribution defined similarly.[8] Since the moments of the G functions possess explicit expressions in terms of the anomalous dimensions, it is more convenient to work with the double-moment equation of (5), which due to the convolution theorem involves only a sum of products of moments. All factors in the moment equation are familiar quantities in renormalization-group analysis and are straightforward to evaluate. Because of the many possible parton types that can contribute before and after the bifurcation vertex, and to the various singlet and nonsinglet terms for each G function, there are all together 42 terms, which are too numerous to be exhibited here. The result is a function only of s, defined in (4).

If we set k in (5) to be a quark, l an antiquark, what is then obtained represents only a part of the $q\bar{q}$ distribution we need. That is because emitted gluons are missed; they carry momenta that are not recovered by the pions in the above calculation. To account for them, we convert each gluon into a $q\bar{q}$ pair, and add to the $q\bar{q}$ distribution obtained above. In spirit, the procedure is equivalent to changing the Fock space from a basis involving vectors of the type $|q_1,q_2,\ldots;\bar{q}_1,\bar{q}_2,\ldots;g_1,g_2,\ldots\rangle$ to another basis with vectors $|q_1,q_2,\ldots;\bar{q}_1,\bar{q}_2,\ldots\rangle$ where the gluons (g_i) do not appear explicitly. Fock space in the latter basis can be spanned by pion states where the absolute squares of the expansion coefficients are the recombination functions.

It is therefore necessary to repeat the calculation of (5) but with (a) $k=q$, $l=g$, (b) $k=\bar{q}$, $l=g$, and (c) $k=g_1$, $l=g_2$. Now, for each gluon in the distribution we further convolute with a pair-creation function so that in the specific case (a) above, for example, we have

$$G_{i\to q\bar{q}'}{}^{(a)}(x_1,x_2,Q^2,Q_0^2) = \int_{x_2}^{1-x_1}\frac{dy}{y}G_{i\to qg}(x_1,y,Q^2,Q_0^2)\bar{P}_{g\to q'\bar{q}'}(x_2/y), \quad (6)$$

where

$$\bar{P}_{g\to q'\bar{q}'}(z)=(3/2f)[z^2+(1-z)^2],$$

and f is the number of flavors. For case (c) above, convolution over both gluon momenta must obviously be taken. $\bar{P}$ describes the probability for a quark (or antiquark) of a definite flavor to have a momentum fraction z of the parent gluon (averaged over initial color and spin, and summed over final color and spin, as with all G functions).[2] The normalization is chosen to satisfy the sum rule

$$\sum_{j=1}^{f}\int_0^1 dz\, z\bar{P}_{g\to q_j\bar{q}_j}(z)=\tfrac{1}{2} \quad (7)$$

so that the $g\to q\bar{q}$ conversion exhausts the gluon momentum, i.e., on the average half of it is deposited in a quark and the other half in an antiquark. In our numerical computation we use $f=3$, since it is more reasonable for the value of Q^2 at which experimental data will be compared with our result.

Adding the various contributions yields

$$F_{i\to q\bar{q}'}(x_1,x_2,s)=x_1x_2[G+G^{(a)}+G^{(b)}+G^{(c)}] \quad (8)$$

for every flavor of initial and final quarks. The substitution of (3) and (8) into (1) then gives the meson inclusive distribution in a quark jet. To relate to the experimental distribution measured in e^+e^- annihilation,

$$N^{\pi^\pm}(x)=(2\sigma_T)^{-1}d\sigma^{\pi^\pm}/dx,$$

we calculate

$$N^{\pi^\pm}(x)=\tfrac{2}{3}D_u{}^{\pi^\pm}(x)+\tfrac{1}{6}[D_d{}^{\pi^\pm}(x)+D_s{}^{\pi^\pm}(x)]. \quad (9)$$

There are no free parameters in the calculation. The value of s is specified by (4). For the values of Λ and Q_0 determined in Ref. 3 ($\Lambda=0.74$ GeV, $Q_0=0.82$ GeV), we have $s=2.94$ for $Q=5.2$ GeV. The result of our calculation for this value of s is shown by the dashed curve in Fig. 2. Although it is slightly below the data,[10] it is remarkably close in both shape and normalization, considering that it is not a fit.

The agreement with data can be improved if we take resonance contribution into account. The recombination model is at present inadequate for the determination of the relative rates of production of resonances versus stable particles. In

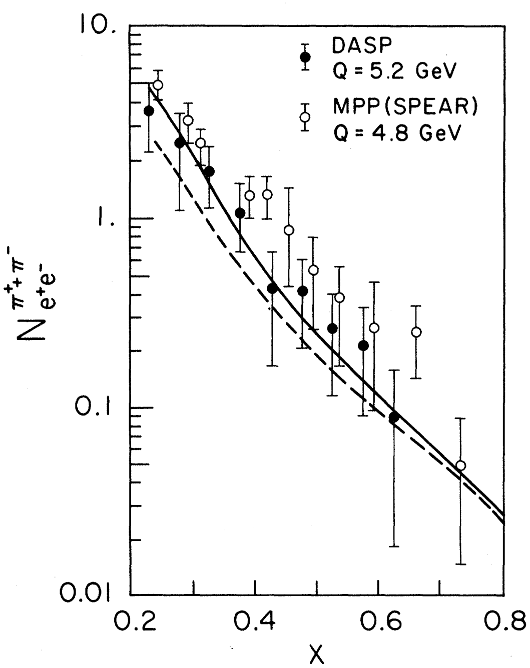

FIG. 2. Quark decay function vs momentum fraction of pions. Curves are theoretical results with (solid) or without (dashed) resonance contribution. Data are from Ref. 10.

pp collisions the ρ production rate is known experimentally; based on that rate it is possible to infer that the probability of formation of ρ is 0.45 times that of π.[11] If we use this ratio to compute the contribution from ρ production and subsequent decay to the quark fragmentation function, the net pion distribution is that shown by the solid curve in Fig. 2. The agreement with the data is striking.

The success of our calculation lends support to both the theory of quark proliferation as prescribed by QCD and the model for hadronization in the framework of valons and recombination. The model that we have adopted for the hadronization of emitted gluons as expressed in (6) and (8) can be justified only *a posteriori*. It implies total gluon conversion to pions as suggested in the naive inside-outside quark-gluon diagram for e^+e^- annihilation. The evolution parameter s depends crucially on Q_0/Λ which is extracted from structure functions and is not free to vary. The largeness of s accounts for precocious scaling and the sharp damping in x in Fig. 2. Our result offers a strong indication for the correct mechanism of hadronization of quarks.

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[1]J. F. Owens, Phys. Lett. 76B, 85 (1978), and Phys. Rev. D 18, 1501 (1978); T. Uematsu, Phys. Lett. 79B, 97 (1978).

[2]G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977).

[3]R. C. Hwa, Phys. Rev. D (to be published).

[4]K. P. Das and R. C. Hwa, Phys. Lett. 68B, 459 (1977).

[5]R. C. Hwa, University of Oregon Report No. OITS-122 (unpublished); see also R. C. Hwa, in Proceedings of the Kazimierz Symposium on Hadron Structure and Multiparticle Production, Poland, May 1979 (unpublished).

[6]C. B. Newman *et al.*, Phys. Rev. Lett. 42, 951 (1979).

[7]R. Blankenbecler and S. J. Brodsky, Phys. Rev. D 10, 2973 (1974).

[8]K. Konishi, A. Ukawa, and G. Veneziano, Phys. Lett. 78B, 243 (1978), and 80B, 259 (1978), and Nucl. Phys. B157, 45 (1979).

[9]For a summary see, for example, T. A. DeGrand, Nucl. Phys. B151, 485 (1979).

[10]R. Brandelik *et al.*, Nucl. Phys. B148, 189 (1979), and data presented by H. F. W. Sadrozinski, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energy, Hamburg, 1977,* edited by F. Gutbrod (DESY, Hamburg, Germany, 1977).

[11]R. G. Roberts, R. C. Hwa, and S. Matsuda, J. Phys. G 5, 1043 (1979).

Inner-Shell Electron Capture by a Swift Bare Ion: Second Born Effects

Robin Shakeshaft
Physics Department, Texas A & M University, College Station, Texas 77843
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The second Born cross section for electron capture from a hydrogenlike atom of atomic number Z_T by a bare ion of atomic number Z_P and speed v is evaluated for $(Z_P e^2/\hbar v) << 1$ and Z_T arbitrary. The ratio of the second to first Born cross sections increases rapidly as Z_T increases; for $(Z_T e^2/\hbar v) \gtrsim 1$ this ratio is very large. These results indicate that second- and higher-order Born terms must be considered in calculating the cross section for inner-shell electron capture (or the time-reversed process).

It is well known that, within the nonrelativistic framework, the second Born term dominates over the first for electron capture by an ion whose impact speed v (velocity $\vec{v}$) is large.[1-3] The asymptotic expansion in powers of $e^2/\hbar v$ of the first and second Born terms, for electron capture from a hydrogenlike atom by a bare ion, is presumably valid for $(Z_T e^2/\hbar v)$ and $(Z_P e^2/\hbar v) \ll 1$ where $Z_T e$ and $Z_P e$ are the charges of the target nucleus T and the projectile P, respectively. However, if Z_T or $Z_P \gg 1$ these inequalities are not satisfied until v is so large that relativistic effects are significant (and the cross section is so small that its measurement is of little interest at present). I have evaluated the second Born term for ground-state–to–ground-state electron capture from a hydrogenlike atom by a bare ion under the single assumption $(Z_P e^2/\hbar v) \ll 1$, with Z_T arbitrary except that $(Z_T e^2/\hbar c) \ll 1$ and $(v/c)^2 \ll 1$ so that relativistic effects are negligible. I find that the ratio of second to first Born cross sections increases rapidly with increasing Z_T

