These values seem already sufficiently large. If not, it should be possible to raise T_e to 10^5 °K and η close to unity so that $\alpha\Omega\tau$ for all collisions becomes quite large.

Spatial nonuniformity of the wave field might conceivably have an unfavorable effect on the ion orbits, by introducing drifts. However, I expect that such drifts only cause the ions to precess about the z axis. Therefore, a separation device which is large compared to R in the transverse direction can accommodate most orbits. Nevertheless, it will be desirable to produce wave fields which are as uniform as possible in the transverse direction. A potential threat to the method might be instabilities associated with the large amplitude ion-cyclotron wave. However, the experiments reported in Ref. 1 do not seem to show such effects.

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Narrow Σ-Hypernuclear States

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It is shown that the spin-isospin dependence of low-energy $\Sigma N \to \Lambda N$ conversion leads to substantial quenching of nuclear-matter estimates of the widths of some Σ-hypernuclear states produced in (K^-,π) reactions, to a level below 10 MeV. The estimated widths compare favorably with those of the Σ-hypernuclear peaks recently observed at CERN for ^{7}Li, ^{9}Be, and ^{12}C. Tentative quantum number assignments are suggested for these states.

The unexpected observation[1] of narrow ($\Gamma \lesssim 10$ MeV) Σ-hypernuclear states at CERN in $(K^-,\pi^\mp)$ 0° production at 720 MeV/c on ^{7}Li, ^{9}Be, and ^{12}C has focused attention on the issue of widths expected for such states. As shown below, the nuclear-matter estimate for the width of the Σ yields values considerably in excess of the reported widths, since the conversion $\Sigma N \to \Lambda N$ proceeds strongly at low energies ($p_{\rm lab}^{\Sigma} \lesssim 200$ MeV/c), mostly through s-wave ΣN interaction.[2] However, this estimate does not account microscopically for the well-known *selectivity* of the conversion process at low energies to the spin and isospin of the decaying ΣN pair,[3] viz. $S=I$, $I=\frac{1}{2}$. In this Letter, we examine the consequences of such selectivity for Σ-hypernuclear states in *light* species ($A \lesssim 16$) and find substantial quenching, relative to the nuclear-matter estimate, for *some* of the p-shell states produced in (K^-,π) by the coherent substitution $(1p)_N \to (1p)_\Sigma$, in general agreement with the reported spectra. Heavy Σ hypernuclei are not expected to exhibit narrow-peak structure.

The nuclear-matter estimate for the Σ^- conversion width is given by

$$\Gamma \approx (V_{\Sigma^-p}\sigma_{\Sigma^-p\to\Lambda n})_{\rm av}\int_0^\infty \rho_p(r)|u_{n,l}^{\Sigma}(r)|^2 dr, \qquad (1)$$

where av denotes a Fermi average. The single-particle Σ orbital $|n,l_j\rangle$ is described by a radial wave function $u_{n,l}^{\Sigma}(r)$. Expression (1) can be derived by considering the *first-order* Σ^--nucleus optical potential $U_1 = t_{\Sigma^-p\to\Sigma^-p}\rho_p + t_{\Sigma^-n\to\Sigma^-n}\rho_n$ and identifying $-\Gamma_{n,l}/2$ with the expectation value of $\mathrm{Im}U_1$ in the Σ^- state $|n,l_j\rangle$. Using the optical theorem, and recalling that near threshold the total $\Sigma^- p$ cross section is dominated by $\Sigma^- p \to \Lambda n$, we obtain (1). The assumption of s-wave dominance for the low-energy ΣN interactions is implicit in the above derivation. Higher-order corrections to U_1 include, for instance, Pauli quenching. This is not expected to yield substantial modifications of (1) since for a $\Sigma^- p$ pair at rest, the ΛN relative momentum is about 290 MeV/c. Equation (1) is the generalization of the well-known expression,

$$\Gamma \approx (v\sigma)_{av}|\varphi(0)|^2, \quad \int |\varphi(r)|^2 d^3r = 1, \qquad (2)$$

for the *annihilation* of the two-body cluster described by an s-wave function $\varphi(r)$. The relationship between annihilation and meson-exchange mechanisms is discussed by Gal[4] and by Johnston and Law.[4]

We now give numerical estimates for the width Γ. The $\Sigma^- p \to \Lambda n$ total cross sections, as fitted by Nagels *et al.*,[3] are well described for $v \lesssim v_F$ by the form

$$v\sigma = (v\sigma)_0/(1+\alpha v) \qquad (3)$$

with $(v\sigma)_0 = 65$ mb and $\alpha = 20$. The Fermi averaging of (3) for Σ^- at rest yields $(v\sigma)_{av} = 14$ mb for $k_F = 250$ MeV/c. The Σ^- atomic data,[5] fitted by an effective scattering length $\bar{a}_{\Sigma^- N} = -(0.35 \pm 0.04) - i(0.19 \pm 0.03)$ fm, yield another estimate:

$$(v\sigma)_{av} = 8\pi |\mathrm{Im}\bar{a}_{\Sigma^- N}|/\mu_{\Sigma N} = 18 \text{ mb}, \qquad (4)$$

which we adopt here. Rather than evaluate the radial overlap integral in Eq. (1) we solved the Σ-nucleus Schrödinger equation for a complex potential U, with $\mathrm{Im}U = -(v\sigma)_{av}\rho_p(r)/2$, and with a three-parameter Fermi distribution[6] to describe both $\rho_p(r)$ and $\mathrm{Re}U$. The resulting widths are sensitive mainly to $\mathrm{Im}U(0)$, and are insensitive to $\mathrm{Re}U(0)$; the value $\mathrm{Re}U(0) \sim -26$ MeV is suggested by the Σ^- atomic-data analysis.[5] In ^{12}C we thus obtain

$$\Gamma_{1s} \approx 23 \text{ MeV}, \quad \Gamma_{1p} \approx 13 \text{ MeV}, \qquad (5)$$

where the $1s$ state is bound by 9.6 MeV (13.2 MeV if Coulomb attraction is added for Σ^-). The $1p$ state is unbound, and the value of Γ_{1p} in (5) was reached by increasing $|\mathrm{Re}U(0)|$ by about 10%, in addition to including Coulomb attraction, which combine to produce a barely bound $1p$ state. In ^{40}Ca we obtain

$$\Gamma(\text{MeV}) \approx 28(1s), \quad 23(1p), \quad 18(1d), \quad 16(2s). \qquad (6)$$

A similar procedure has been reported by Batty[5] to produce widths in the range of 28–18 MeV for Σ states in ^{208}Pb.

The nuclear-matter estimate given above for Γ is valid only for medium-weight and heavy nuclei, where spins and isospins are close to saturation. This is not the case for light nuclei since the $\Sigma N \to \Lambda N$ conversion process, operative only for $I=\frac{1}{2}$, is dominated[3] by $^3S_1 \to {}^3S_1$ and $^3S_1 \to {}^3D_1$ components for $p_\Sigma \lesssim 250$ MeV/c. We must therefore incorporate this selectivity of spin and isospin explicitly into the calculation of the Σ conversion width. The expectation value of

$$\sum_{i \in \mathcal{P}} \delta(\vec{r}_i - \vec{r}_{\Sigma^-})$$

is to be replaced by the expectation value of

$$\sum_i \delta(\vec{r}_i - \vec{r}_\Sigma)(\tfrac{3}{4} + \tfrac{1}{4}\vec{\sigma}_i \cdot \vec{\sigma}_\Sigma)(\tfrac{1}{3} - \tfrac{1}{3}\vec{\tau}_i \cdot \vec{t}_\Sigma), \qquad (7)$$

where the index i runs over all nucleons, as appropriate to a *charge-independent* formulation. The Σ isospin operator is here normalized to $\vec{t}_\Sigma^{\,2} = 2$. The matrix elements of (7) were evaluated for several Σ-hypernuclear states in the p shell which are expected to be significantly excited in low-momentum-transfer (small-q) strangeness exchange. We demonstrate the relevant arguments for the relatively simple case of ^{12}C.

With $q \approx 130$ MeV/c at $p_K = 720$ MeV/c, the strongest Σ-hypernuclear excitations expected at 0° are $0^+(p_{3/2}^{-1}, p_{3/2})$ and $1^-(p_{3/2}^{-1}, s_{1/2})$ for both $I=\frac{1}{2}$ and $\frac{3}{2}$ in ^{12}C$(K^-,\pi^-)^{12}_{\Sigma}$C and for $I=\frac{3}{2}$ in ^{12}C$(K^-,\pi^+)^{12}_{\Sigma}$Be. We stress that the same $I=\frac{3}{2}$ states are excited in both (K^-,π^-) and (K^-,π^+). The $0^+(1s^{-1}, 1s)$ states are not expected to exhibit an experimentally meaningful signature because of their large $1s_N^{-1}$ width, and the $2^+(p_{3/2}^{-1}, p_j)$ states require $q \approx 200$ MeV/c for their observation, in analogy[7,8] with $^{12}_{\Lambda}$C. In a jj description of ^{12}C, the forward cross sections for excitation of these $J^\pi = 0^+, 1^-$ states are given by

$$(K^-,\pi^+):\ 4|M_J|^2|f_{p\to\Sigma^-}|^2, \quad I=\tfrac{3}{2}, \qquad (8a)$$

$$(K^-,\pi^-):\ \tfrac{8}{3}|M_J|^2|f_{n\to\Sigma^0} - f_{p\to\Sigma^+}/\sqrt{2}|^2, \quad I=\tfrac{3}{2}, \qquad (8b)$$

$$(K^-,\pi^-):\ \tfrac{4}{3}|M_J|^2|f_{n\to\Sigma^0} + \sqrt{2}f_{p\to\Sigma^+}|^2, \quad I=\tfrac{1}{2}, \qquad (8c)$$

where M_J are distorted-wave integrals and the f's are forward amplitudes. At 720 MeV/c, $|f_{n\to\Sigma^0}| \gg |f_{p\to\Sigma^+}|$ and the $I=\frac{3}{2}$ state is expected to be produced twice as strongly as the $I=\frac{1}{2}$ state in (K^-,π^-). Although the production process occurs via $n \to \Sigma^0$, the strong charge-exchange process $\Sigma^0 p \to \Sigma^+ n$ renders this particle basis meaningless and implies isospin classification, with the result that two states, not just one, are produced for each value of J^{π}. Of course, if the $I=\frac{3}{2}$ and $I=\frac{1}{2}$ states are almost degenerate with each other, large mixings may occur.

Our findings are as follows: (i) The width of the 1^- states is not significantly changed relative to the nuclear-matter estimate [the latter is here obtained by retaining only the spin-isospin independent term in (7)]. For oscillator wave functions, the width of the $I=\frac{1}{2}$ state is increased by a factor 1.17 whereas that of the $I=\frac{3}{2}$ state decreases by a factor 0.86. Hence, these states are most likely too broad to be identified. (ii) The width of the coherent 0^+ states significantly changes from the nuclear-matter breakdown of $\Gamma=\Gamma_s+\Gamma_p$, the subscripts denoting conversion within the appropriate nucleon shells, into

$$\Gamma(I=\tfrac{1}{2})=\Gamma_s+\tfrac{12}{7}\Gamma_p, \quad \Gamma(I=\tfrac{3}{2})=\Gamma_s. \tag{9}$$

Thus, the $I=\frac{3}{2}, 0^+$ state is narrower than given by the nuclear-matter estimate by a factor $\Gamma_s/(\Gamma_s+\Gamma_p)$, which for oscillator wave functions assumes the value 0.41. This state, excited significantly in both (K^-,π^-) and (K^-,π^+), is predicted to have a conversion width of about 5 MeV; adding a few MeV for the Σ-particle decay width,[9] it is likely to have a width smaller than 10 MeV. In the absence of identifying angular distributions, we tentatively assign the observed ${}^{12}_{\Sigma}$C and ${}^{12}_{\Sigma}$Be structures[1] to be the $0^+, I=\frac{3}{2}$ state. The $0^+, I=\frac{1}{2}$ state, excited only in (K^-,π^-), should be much broader.

The general lesson of the ^{12}C example is that for $|n,l_j\rangle$ Σ states where the corresponding $|n,l_j\rangle$ nuclear shell is not closed, the Γ_l component of Γ may be substantially reduced as a result of spin-isospin correlations. For heavy nuclei, the effect of such reduction is smaller, except possibly for *coherent* strangeness analog states. Substantial reductions may be achieved for $1p$ Σ states in the p shell. Table I summarizes our calculations for coherently produced Σ $1p$ states on ^{7}Li and ^{9}Be. These states are described by the dominant LS component of the target nucleus, $^2P_{3/2}$, with $I=0,1,2$. A similar schematic description has been shown[10] to reproduce qualitatively the observed features of ^{9}Be$(K^-,\pi^-)^9_{\Lambda}$Be. The forward cross sections for excitation of these $\frac{3}{2}^-$ states at $p_K=720$ MeV/c are given approximately by $n_c^{\mp}|M_0|^2|f_{N\to\Sigma^0}|^2$ for $(K^-,\pi^{\mp})$, respectively. The sum of n_c^- (n_c^+) equals the number of $1p$ neutrons (protons) in the target. Lastly, we note that since the central proton density[6]

TABLE I. Number $n_c^{\pm}$ of $1p$ nucleons for the coherent excitation, $(1p)_N \to (1p)_{\Sigma}$, of Σ-hypernuclear $\frac{3}{2}^-$ states in $(K^-,\pi^{\pm})$ reactions, respectively, at 0° on ^{7}Li and ^{9}Be for $p_K=720$ MeV/c. The conversion width Γ relative to the nuclear-matter estimate $\Gamma_{\rm nm}$ is also shown.

Target nucleus	${}^A_{\Sigma}Z$ structure	(I_N, I)	n_c^-	$n_c^{\pm}$	$\Gamma/\Gamma_{\rm nm}$
^{7}Li	$\left(\{(5/9)^{1/2}\,{}^2S[2]+(4/9)^{1/2}\,{}^2D[2]\}\otimes{}^2p_{\Sigma}\right)_{2p_{3/2}}$	(0, 1)	3/2	0	0.9
		(1, 0)	1/6	0	2.7
		(1, 1)	0	0	2.0
		(1, 2)	1/3	1	0.7
^{9}Be	Lower peak[a]	(0, 1)	5/4	0	0.8
	Upper peak[b]	(0, 1)	3/4	0	1.2
		(1, 0)	1/3[c]	0	1.9[d]
		(1, 1)	0	0	2.0[e]
		(1, 2)	2/3[c]	2	1.6[d]
					1.3[e]
					1.0[d]
					0.7[e]

[a] $\left(\{(8/15)^{1/2}\,{}^1S[4]-(7/15)^{1/2}\,{}^1D[4]\}\otimes{}^2p_{\Sigma}\right)_{2p_{3/2}}$.

[b] $\left(\{(2/3)^{1/2}\,{}^{(2S_N+1)}P[3,1]+(1/3)^{1/2}\,{}^{(2S_N+1)}D[3,1]\}\otimes{}^2p_{\Sigma}\right)_{2p_{3/2}}$, with $S_N=1$ for $I_N=0$ and $S_N=0,1$ for $I_N=1$.

[c] Distributed according to $(2S_N+1)$ for $S_N=0,1$.

[d] $S_N=0$.

[e] $S_N=1$.

in ^{7}Li and ^{9}Be is lower than for ^{12}C, the nuclear-matter estimate (1) for Γ_{1p} yields values smaller than given in (5). With oscillator densities we estimate $\Gamma_{1p} = 7, 9$ MeV in ^{7}Li, ^{9}Be, respectively.

In $_\Sigma^7$Li there is only one state, with $I = 2$, whose conversion width is expected to be significantly quenched, by a factor 0.7. Its formation, however, is only about one-sixth of the overall coherent $(1p)_N \rightarrow (1p)_\Sigma$ production in (K^-, π^-). This same state, with different isospin projection, may more readily be excited in (K^-, π^+).

In $_\Sigma^9$Be two peaks are reported,[1] about 10 MeV apart from each other, similar to the case of $_\Lambda^9$Be. Since the conversion width of only one of the components of the upper $_\Sigma^9$Be peak is significantly quenched, we tentatively assign the observed $_\Sigma^9$Be peaks to $(S_N, I_N; I) = (1, 1; 2)$ for the upper one and $(0, 0; 1)$ for the lower one, with formation cross section ratio of $\frac{2}{5}$, in accordance with the observation[1] that the lower peak is more abundantly produced in (K^-, π^-). The $(1, 1; 2)$ state is then that observed in (K^-, π^+). If all the $_\Sigma^9$Be states listed in Table I were relatively narrow, the formation cross-section ratio would be $\frac{7}{5}$ in favor of the upper peak, a situation prevailing[10] in $_\Lambda^9$Be.

Table II gives our predictions for an ^{16}O target, The nature of the $_\Sigma^{12}$O $0^+(1p^{-1}, 1p)$ states depends critically on the strength of the effective one-body Σ spin-orbit potential relative to the 6-MeV $p_{3/2}^{-1}$-$p_{1/2}^{-1}$ nuclear spin-orbit splitting. We differentiate in Table II between two representative possibilities: (i) The Σ spin-orbit potential is small, in which case the 0^+ states formed have the approximate structure $(p_{3/2}^{-1}, p_{3/2})$ and $(p_{1/2}^{-1}, p_{1/2})$, roughly 6 MeV apart from each other for each of the values $I = \frac{1}{2}, \frac{3}{2}$. The width of the $I = \frac{3}{2}$ states is seen from Table II to be partly quenched, in particular that of the main $(p_{3/2}^{-1}, p_{3/2})$ excitation. However, the superposition of these two $I = \frac{3}{2}$ states may overlap the 6 MeV spacing between them, so that their appearance as separate peaks, say in (K^-, π^+), is uncertain. (ii) The Σ spin-orbit potential is comparable to that of a nucleon, in which case only *one* 0^+ state (with spectroscopic assignment $^1S_0$) is expected[10] to be formed for each of the values $I = \frac{1}{2}, \frac{3}{2}$ in (K^-, π^-). The width of the $I = \frac{3}{2}$ state is substantially quenched (0.3) and it may be seen as a narrow structure in (K^-, π^+).

TABLE II. Number $n_c^\pm$ of $1p$ nucleons involved in the coherent excitation of Σ-hypernuclear 0^+ states in $(K^-, \pi^\pm)$ reactions, respectively, at 0° on ^{16}O for $p_K = 720$ MeV/c, and conversion width ratio $\Gamma/\Gamma_{\rm nm}$. We show two representative cases, corresponding to weak or strong Σ spin-orbit coupling.

$_\Sigma^A Z$ structure	I	n_c^-	n_c^+	$\Gamma/\Gamma_{\rm nm}$
$({}_Np_{3/2}^{-1} \otimes {}_\Sigma p_{1/2})_{0^+}$	1/2	4/3	0	1.3
	3/2	8/3	4	0.6
$({}_Np_{1/2}^{-1} \otimes {}_\Sigma p_{3/2})_{0^+}$	1/2	2/3	0	1.2
	3/2	4/3	2	0.8
$^1S_0$	1/2	2	0	1.4
	3/2	4	6	0.3
$^3P_0$	1/2	0	0	1.1
	3/2	0	0	1.1

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