Tkachenko Waves in Rotating Superfluid Helium

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The resonant response of a stack of disks driven into torsional oscillation within a container of rotating superfluid helium has been observed. It is shown that the oscillation modes excited are related to Tkachenko waves, that is, vortex displacement waves in the vortex array propagating in a direction transverse to the vortex lines. In particular, the resonances occur at peaks in the vortex-wave density of states.

A container of superfluid helium rotating at an angular velocity Ω contains within it a more or less uniform array of quantized vortices aligned along the rotation axis.[1] The vortices are assumed to be distributed in a triangular array with a mean spacing $b=[h/(3)^{1/2}m\Omega]^{1/2}$ which, on the average, gives rise to a solid-body-rotation velocity field (h is Planck's constant and m is the mass of a helium atom). An isolated vortex line is capable of sustaining waves, which are traveling helical deformations of the line having a dispersion relation[2]

$$\omega(k)=(\hbar k^2/2m)[\ln(1/ka)+0.1159], \tag{1}$$

where a is the effective "radius" of the vortex core and k is the wave vector of the deformation. These waves have recently been observed by Ashton and Glaberson.[3] In a situation where the wavelength is not small compared to the distance between vortices (kb not much greater than 1), collective oscillations can propagate along the rotation axis, as was observed by Hall[4] and Andronikashvili and Tsakadze.[5] In the extreme collective limit ($kb \ll 1$), the dispersion relation in the rotating coordinate system becomes[6]

$$\omega(k)=2\Omega+(\hbar k^2/2m)[\ln(b/a)-1.13]. \tag{2}$$

These longitudinal waves are the superfluid analog of classical inertial waves[7] which, for propagation along the rotation axis, have the simple dispersion relation

$$\omega(k)=2\Omega. \tag{3}$$

It should also be possible to propagate collective waves transverse to the vortex array in the superfluid. These Tkachenko[8] waves are nondispersive, having a frequency

$$\omega(l)=(\hbar\Omega/4m)^{1/2}l, \tag{4}$$

where l is the wave number, and because they depend essentially on the grainy structure of the rotating superfluid, have no classical analog. In this paper we report the observation of Tkachenko waves. The waves observed are mixed longitudinal-transverse standing-wave modes associated with minima in the dispersion relation.

Our experimental cell is shown in Fig. 1. A stack of Macor[9] disks is suspended on a stainless-steel torsion fiber from a relatively massive platform which is itself suspended on a fiber from the cryostat. The disks are 0.005 in. thick, 1.25 in. in diameter and are separated from each other by spacers ranging in thickness from 0.008 in. to 0.030 in. The entire assembly is immersed in helium and mounted in a rotating He^3 refrigerator capable of rotation speeds as high as 10 rad/sec and temperatures as low as 0.3 K. The disks are driven into torsional oscillation magnetically and the response is detected electrostatically. The dual suspension system is responsible for insulating the experimental cell from noise in the cryostat rotation speed. In the most favor-

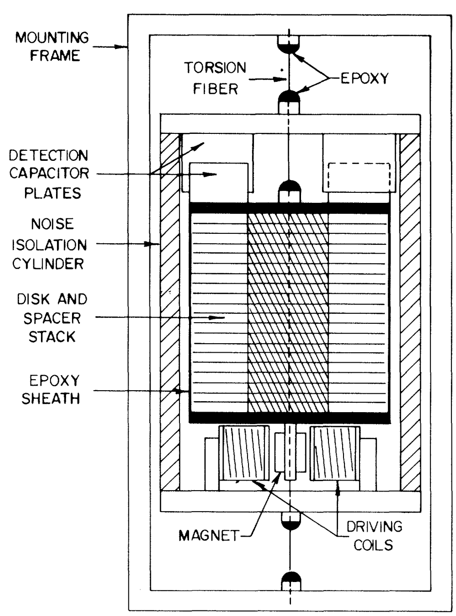

FIG. 1. A schematic drawing of the experimental cell.

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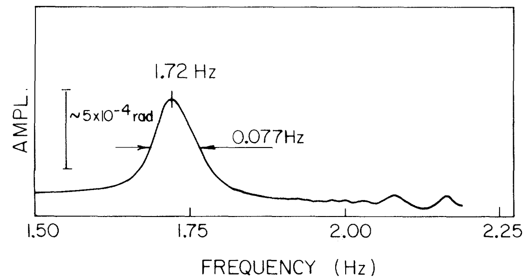

FIG. 2. A plot of the resonant response of the stack of disks as a function of drive frequency. Here, $\Omega = 6.57$ rad/sec, $T = 1.14$ °K, $Q = 22.3$, and the gap spacing is 0.020 in. The oscillation at high frequencies corresponds to noise at a harmonic of the rotation frequency.

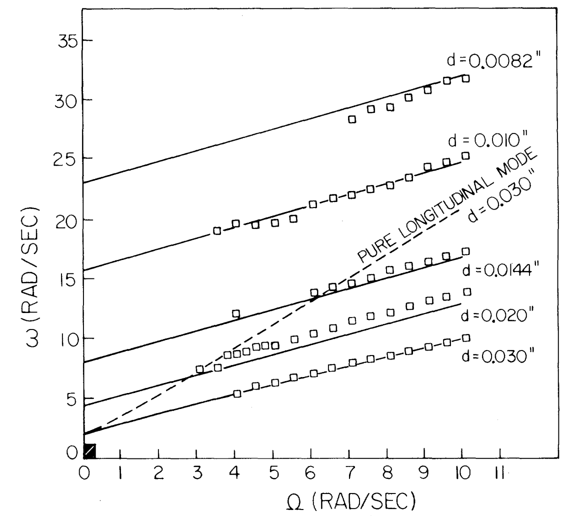

FIG. 3. A plot of the resonance frequency as a function of rotation frequency for various disk spacings (in inches). The dashed curve is the predicted frequency, for the 0.030-in. spacing, and we assume a purely longitudinal oscillation mode (Ref. 2). The solid lines are the theoretical frequencies as discussed in the text. The shaded region at low frequencies represents the range of parameters investigated in Refs. 4 and 5.

able situations the response amplitude sensitivity is $\sim 10^{-7}$ rad. The moment of inertia of the disk assembly is typically ten to fifteen times that of the contained helium.

The resonance frequency of the empty disk assembly is kept substantially below that of the vortex resonances. The response is monitored using phase-sensitive detection with the phase in quadrature with respect to the drive so that the empty disk response is suppressed. As the drive frequency is swept through a vortex resonance, the effective complex moment of inertia of the disk assembly is altered and a response such as shown in Fig. 2 is observed. At relatively high temperatures, the quality factor of the resonance decreases as the temperature is raised, presumably because of mutual friction, but is temperature independent below ~ 1.3 K, where we believe it to be limited there by disk spacing inhomogeneity. The data reported in this paper were obtained at $T = 1.14$ K.

In an attempt to ensure that the vortices are pinned at the disk surfaces, the disks were coated with a layer of 10-μm-diam glass beads. Although this affected the large-amplitude response, no significant effect on the "zero"-amplitude resonance frequencies was observed. We observed hysteretic behavior with respect to drive amplitude changes after a change of rotation speed. The hysteresis disappeared after the disks were driven at sufficiently large amplitude. This suggests that the vortices are indeed strongly pinned at the surfaces, which is consistent with the observations of Yarmchuk and Glaberson[10] but at variance with the presumptions of Hall[4] and Sonin.[11]

In Fig. 3 we show the measured resonance frequencies for various disk spacings as a function of rotation speed. A small dependence of the resonance frequency on drive amplitude was observed, and the resonance frequency plotted is a zero-amplitude extrapolation. We have not included a correction to the resonance frequency for the interaction between the vortex resonance and the empty-disk-assembly resonance, but preliminary calculations indicate that this correction should be small. Overall uncertainties in the vortex resonance frequencies are estimated to be of order 5%. Also shown in Fig. 3, as the dashed line, is the predicted[2] resonance frequency for the lowest purely longitudinal oscillation mode appropriate for the 0.030-in. spacing. This line is based on an analysis which did not assume the extreme collective limit, but is almost the same as that given by Eq. (2) in the experimentally accessible region. The inadequacy of this expression to account for our data is evident.

Consider an infinite array of parallel vortices. A plane-wave vortex displacement of wave vector $\vec{q} = \vec{k} + \vec{l}$, where $\vec{k}$ is the longitudinal component of the wave vector and $\vec{l}$ its transverse component, will have a frequency which, in the long-wavelength limit, is given by[6,11]

$$\omega^2 = \frac{4\Omega^2}{l^2 + k^2}\left[k^2 + \frac{\hbar l^4}{16 m\Omega} + \frac{\hbar C(l^2 + 2k^2)k^2}{2m\Omega}\right], \quad (5)$$

where $C \simeq \frac{1}{2}\ln(b/a) - 0.57$. This expression reduces to Eq. (2) in the limit $l/k \to 0$ and to Eq. (4) in the limit $k/l \to 0$. As pointed out by Williams and Fetter,[6] Eq. (5) has the interesting property that for fixed k the frequency first decreases and then increases as l is increased from zero.[12] This, of course, yields a peak in the vortex-wave density of states for some l which depends on the values of k and Ω. We assert that the vortex-wave resonances we observe are associated with these particular values of l, the wave vector k being determined by the disk spacing d (i.e., $k = \pi/d$). The frequency interval between standing-wave modes corresponding to successive possible values of l in our cell is much smaller than the inhomogeneity-induced spread of the resonance frequency for a particular l, so that we necessarily excite many modes. The observed response is then a convolution of the density of states and the line width associated with disk-spacing inhomogeneity.[13]

Minimizing $\omega(k, l)$ with respect to l at fixed k yields values of l such that $lb \sim 1$, so that the long-wavelength limit, Eq. (5), is not strictly reached in our experiment and a detailed sum over the vortex-line lattice must be performed to obtain the correct dispersion relation. The detailed calculation yields results qualitatively similar to that of the continuum calculation. Furthermore, since $lR \sim 200 \gg 1$, where R is the disk radius, the cylindrical geometry of the experiment has negligible influence on the resonance frequencies. The predicted[14] values of the resonance frequencies, with no adjustable parameters, are shown as the solid lines in Fig. 3. There can be no doubt that we have properly accounted for the data.

Tsakadze[15] has reported observations of the free oscillations of rotating helium-filled cylinders. Following Sonin,[11] he analyzed his data in terms of an empirical slip coefficient, making a rather long extrapolation to the relevant experimental regime, and obtained results consistent with Tkachenko waves. Because the moments of inertia of the cylinders and of the contained helium are comparable, insufficient data are available for detailed analysis in terms of our density-of-states approach.

It is necessary to devote a few words to the question of the coupling of the Tkachenko modes to the disks in our experiment. At first glance it may appear that the coupling should be extremely weak, since $lR \gg 1$ and the vortex tension on the disks parallel to the disks should average almost to zero. The fact is, however, that the environment experienced by vortex lines near the rough disk surface is very different from that in the bulk. It is certainly conceivable that, although the vortex-line displacements far from the surface have the simple distribution of a collective vortex-wave mode, a very different distribution pertains near the surface, particularly in the presence of a finite drive amplitude. It may be that, under these circumstances, all the lines pull on their pins in phase and the force therefore does not average to zero.

In summary, we report the first unambiguous observation of the transverse collective oscillation modes of a vortex array. These modes are intrinsically interesting because of their dependence on the grainy structure of the rotating superfluid, and may be of relevance in understanding the behavior of neutron stars.[16]

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[13]There is also an extremum of $\omega(l)$ at $l=0$. However, $|\partial^2\omega/\partial l^2|$ is relatively large there and, consistent with our observations, we do not expect an observable response peak at the corresponding frequency.

[14]In doing the lattice sums, following Ref. 6, we assumed a triangular array. Because, typically, l is a small fraction of a reciprocal-lattice vector, we do not believe that the results are sensitive to the particular lattice structure. The results depend weakly on the value of the core parameter a and we took it to be 2 Å.

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Observation of Surface Phonons on Ni(111) by Electron Energy-Loss Spectroscopy

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Surface phonons of metals at certain points of the two-dimensional Brillouin zone can be observed in electron energy-loss spectroscopy when suitable coupling is provided by commensurable adsorbate lattices.

Surface phonons on crystalline solids have been a matter of theoretical interest for quite some time.[1] Except for the acoustical surface modes and for the Fuchs-Kliewer surface modes[2] in ionic lattices, little experimental material is available on the subject because of the lack of appropriate spectroscopic tools. Recently we have shown[3] that surface vibrations along the step edges of the stepped platinum (111) surface can be excited by low-energy electrons via the dipole scattering mechanism.[4] The sufficiently high dynamic dipole moment was provided by the isolated position of the step atoms and their particular electronic properties. In this Letter we report on the observation of surface phonons of the flat Ni(111) surface. The appropriate dipole coupling with low-energy electrons specularly reflected from the surface is provided here by submonolayer amounts of oxygen, acetylene, and hydrogen.

The spectrometer was the same as described in previous papers.[5] Clean Ni(111) surfaces were prepared in ultrahigh vacuum by oxidation and reduction cycles and sputtering and annealing. The cleanness of the sample was controlled by Auger spectroscopy using a cylindrical mirror analyzer. A low-energy electron-diffraction (LEED) system to observe the surface structures was also available. In accordance with previous studies we found an ordered (2×2) adsorbate lattice with C_2H_2 (Ref. 6) and with H.[7] With oxygen we found two ordered adsorbate lattices,[8] $p(2\times2)$ and $(\sqrt{3}\times\sqrt{3})R30°$, corresponding to coverages of 0.25 and 0.33 monolayer, respectively. The vibrational spectrum for $p(2\times2)$ is shown in Fig. 1(a). The 580-cm^{-1} (72-meV) loss is the perpendicular vibration of oxygen located in a threefold site.[9]

A temperature of 275 K was chosen for record-

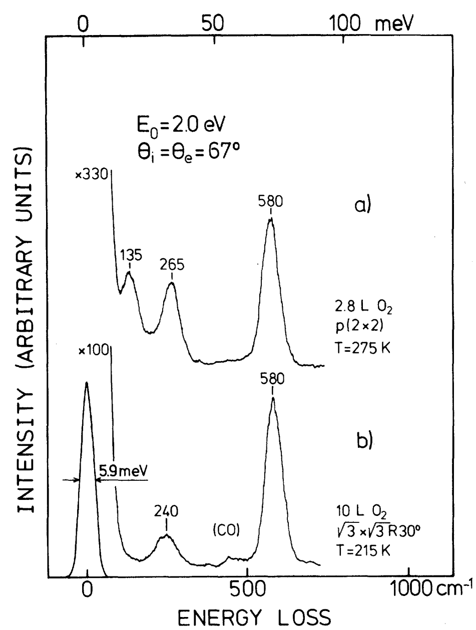

FIG. 1. Electron energy-loss spectra of Ni(111) with a $p(2\times2)$ and $(\sqrt{3}\times\sqrt{3})R30°$ overlayer of oxygen, respectively.

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