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¹V. Enss and B. Simon, "Finite Total Cross Sections in Nonrelativistic Quantum Mechanics" (to be published).

²We intend this to mean that there are only Coulomb potentials but additional short-range potentials [$O(r^{-2-\epsilon})$, $\epsilon > 0$] are allowed without any significant change in our results.

³If the cluster has inversion symmetry, this will be the case unless there is a degeneracy of states of different parity.

⁴By this we mean that if our bound is Cg^γ , γ is correct asymptotically, but not C .

⁵Our constants will be especially bad if the energy is low or the interval of averaging small.

⁶There is, at present, no definitive time-independent approach for multiparticle Coulomb systems.

⁷Except in the region where the Born series converges.

⁸W. O. Amrein and D. B. Pearson, *J. Phys. A* **12**, 1469-1492 (1979).

⁹In addition, A. Martin [CERN Report No. TH 2662 (to be published)] has obtained a bound going for g large as g^4 for central potentials. For general $r^{-2-\epsilon}$ potentials we and Amrein and Pearson, Ref. 8, get a g^2 bound for large g . Also J. M. Combes and A. Tip (private communication from Combes) have informed us that they have a proof of (1) and (2) by different means.

¹⁰W. O. Amrein, D. B. Pearson, and K. B. Sinha, *Nuovo Cimento* **52A**, 115-131 (1979).

¹¹While we have not checked it in detail, we expect that the methods of Refs. 8 and 10 could be extended to handle atom-atom scattering also.

¹²Geometric ideas have been used for some time in rigorous scattering and spectral analysis [see especially, R. Haag, *Phys. Rev.* **112**, 669-673 (1958); G. M. Zhislin, *Mosk. Mat. Obs.* **9**, 81-128 (1960);

P. D. Lax and R. S. Phillips, *Scattering Theory* (Academic, New York, 1967); A. G. Sigalov and I. M. Sigal, *Teor. Mat. Fiz.* **5**, 73-93 (1970) [*Theor. Math. Phys.* **5**, 990-1005 (1970)], but they tended to be overshadowed by the power of time-independent methods. Recently, geometric methods have been shown to be extremely powerful in their own right, see, e.g., V. Enss, *Commun. Math. Phys.* **52**, 233 (1977), and **61**, 285 (1978); B. Simon, *Commun. Math. Phys.* **55**, 259 (1977), and **58**, 205 (1978).

¹³In all steps below the h_R can be carried along and R -independent bounds easily obtained. Moreover, the estimates show that $\lim_{R \rightarrow \infty} \|(S-1)g h_R\|$ exists.

¹⁴In potential scattering, the first inequality is actually an equality as a result of asymptotic completeness. In multiparticle systems, equality should hold, but since we do not know rigorously that asymptotic completeness holds we use the inequality which is always true. Equation (2) is just an interaction-picture formula for $S-1$.

¹⁵L. Hörmander, *Math. Zeit.* **146**, 69-91 (1976); see also M. Reed and B. Simon, in *Methods of Modern Mathematical Physics* (Academic, New York, 1979), Vol. III. The original idea goes back to W. Brenig and R. Haag, *Fortschr. Phys.* **7**, 183-242 (1959).

¹⁶There is an asymmetry (as there should be) between requirements on falloff in the z and x - y directions. Actually, all that is needed is

$$|V(x, y, z)| \leq C(1 + |x|)^{-1/2-\epsilon}(1 + |y|)^{-1/2-\epsilon}(1 + |z|)^{-1-\epsilon}.$$

¹⁷We do not take into account the effect of changing the mass of the charge- Z projectile in our discussion.

¹⁸The possible effects of interference between what has been scattered out and what remains in is accommodated by using $(\int \| \| dt)^2$ rather than $\int \| \| ^2 dt$.

¹⁹Using methods from F. Calogero, *Variable Phase Approach to Potential Scattering* (Academic, New York, 1967), we have proven that for central potentials obeying $(1+r)^{-\alpha} \leq V(r) \leq A(1+r)^{-\alpha}$ there is a lower bound behaving as g^γ .

Turbulent Modification of the $m = 1$ Resistive Tearing Instability

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In the presence of a random spectrum of lower hybrid waves, the $m = 1$ resistive tearing instability becomes an oscillating instability with a significantly enhanced growth rate. For typical tokamak parameters, the growth time can become comparable to plasma disruption times for rather moderate levels of fluctuations.

Tearing modes are the subject of intense theoretical investigations these days because of their importance in tokamak plasmas.¹ They comprise an important class of ideal magnetohydrodynamic

(MHD) modes of the internal kink type for which the perturbations become resonant at the mode rational surface, where $\vec{k} \cdot \vec{B}_0 = 0$. In this region, it becomes necessary to take into account inertia

and nonideal effects such as resistivity to remove the singularity, and the growth rate is found typically to depend on the resistivity. The evolution of the instability is thus a sensitive function of the dielectric response to the perturbation in the singular region. Much attention is therefore paid to delineating various physical effects that can modify this response.² In this Letter, we examine nonlinear modifications that can arise from the presence of high-frequency microturbulence in the background plasma and its consequences on the evolution of the tearing mode. As a specific example, we consider the evolution of the $m = 1$ resistive tearing mode in the presence of a saturated spectrum of lower hybrid waves. Lower hybrid waves are likely to be present in most tokamak discharges as current-driven microinstabilities. This seems to be borne out by computer simulation studies of anomalous current penetration³ in a plasma and also by some recent measurements on the Microtor⁴ and Alcator tokamaks.⁵ They can also arise from external excitations as in the case of rf heating experiments in the lower hybrid frequency range. Physically, the random spectrum of these fluctuations acts as pump waves which can parametrically drive or suppress the tearing instability through the nonlinear ponderomotive force effect. Mathematically, this nonlinear term modifies both the equation of motion and Ohm's law in the inner layer. Adopting a variational technique, we obtain a modified dispersion relation for the $m = 1$ resistive mode and solve it analytically in convenient limits. The presence of the fluctuations permits a larger number of modes to be excited, including a slowly growing quasimode which is purely driven by the turbulence. However, the most interesting result is that, instead of the purely

growing resistive modes, there now arises a mode with a large real frequency and growing at a greatly enhanced rate. The mode has a broad spatial structure, typically of the order of the tearing layer, and the growth time can become comparable to disruptive time scales (of order 10^{-6} sec) for rather modest levels of fluctuations. Hence this mode may be important from the point of view of understanding major disruption processes.

Our calculations are based on the simple MHD model for the resistive tearing mode carried out in a cylindrical geometry with an equilibrium field, $\vec{B}_0 = (0, B_\theta(r), B_z)$. We assume the existence of an equilibrium electrostatic lower hybrid (LH) field,

$$\varphi_0(\vec{r}, t) = \sum_{\vec{k}} \varphi_{\vec{k}} \exp(i(\vec{k} \cdot \vec{r} - \omega t)) + c.c.$$

The summation is over a spectrum of LH waves so that each (ω, \vec{k}) satisfies the dispersion relation

$$\omega^2 = \omega_{LH}^2 [1 + (k_z^2/k_\perp^2)(m_i/m_e)], \quad (1)$$

where $\omega_{LH} = \omega_{pi}(1 + \omega_{pe}^2/\Omega_e^2)^{-1/2}$ is the lower hybrid frequency and other notations are standard. Assuming the level of fluctuations to be low, so that only terms to order $|\varphi_{\vec{k}}|^2$ need be retained, we consider the interactions among the low-frequency tearing mode (Ω, \vec{q}) and the sideband LH modes at $\Omega \pm \omega, \vec{q} \pm \vec{k}$. Since the tearing mode we are considering has no density perturbations, the dominant nonlinear contribution to the sideband equations arises from the terms $(\vec{V}_T \cdot \nabla) \vec{V}_{LH}$ and $\vec{V}_{LH} \times \vec{B}_T$ in the equations of motion. Here \vec{V}_{LH} is the LH-pump-induced high-frequency velocity fluctuation and \vec{V}_T, \vec{B}_T are the perturbed velocity and magnetic field of the tearing mode. The sideband potentials are then given by

$$\begin{bmatrix} \varphi_+ \\ \varphi_- \end{bmatrix} = k_z^2 \omega_{pe}^2 \left\{ \frac{\vec{k}_\perp \cdot \vec{B}_T}{k_z B_z} + 2 \left(1 + \frac{k_\perp^2 m_e}{k_z^2 m_i} \right) \left(\frac{\vec{k}_\perp \cdot \vec{V}_T}{\omega} \right) \right\} \begin{bmatrix} \varphi_{\vec{k}}^*/\epsilon_+ \\ \varphi_{\vec{k}}^*/\epsilon_- \end{bmatrix}, \quad (2)$$

where $\epsilon_\pm = k_\pm^2 [\omega_\pm^2 - \omega^2(\vec{k}_\pm)]$ with $\omega^2(\vec{k}_\pm)$ satisfying Eq. (1). In deriving relation (2), use is made of the adiabatic approximation $|\vec{q}| \ll |\vec{k}|$ and $|\Omega| \ll |\omega|$.

To describe the tearing mode we use the single-fluid MHD equations and the Maxwell equations. The MHD equations are derived from the two-fluid model with the addition of appropriate nonlinear coupling terms. The predominant nonlinear coupling term is the $(\vec{V}_T \cdot \nabla) \vec{V}_\parallel$ term in the electron equation, where \vec{V}_\perp is the $\vec{E} \times \vec{B}$ drift velocity experienced by the electrons in the LH

field.

We also make the usual incompressibility assumption⁶ ($\nabla \cdot \vec{V}_T = 0$) thereby ignoring the propagation of magnetosonic waves which evolve on an Alfvénic time scale, $\tau_A = a/V_A$ (a is the plasma radius and V_A is the Alfvén speed). We next transform the quantities \vec{V}_T and \vec{B}_T by the familiar substitutions, $\vec{V}_T = \hat{\delta} \times \nabla \varphi$ and $\vec{B}_T = \hat{\delta} \times \nabla \psi$, and seek mode perturbations of the type $\sim g(r) \exp(i(m\theta + q_z z - \Omega t))$. Then, following standard procedures,⁷

we derive a modified set of inner-layer equations:

$$\frac{d^2\bar{\varphi}}{dx^2} + \frac{d^2\bar{\psi}}{dx^2} = \mu \frac{d^3\bar{\varphi}}{dx^3} + \nu \frac{d^3\bar{\psi}}{dx^3}, \quad (3)$$

$$\bar{\varphi} + \frac{\bar{\psi}}{x} = \alpha \left(1 + \frac{S_0}{x} \right) \frac{d^2\bar{\varphi}}{dx^2} + \left\{ \frac{1}{x} (\bar{\eta} + S_0\beta) + \beta \right\} \frac{d^2\bar{\psi}}{dx^2}, \quad (4)$$

where $\bar{\varphi} = \varphi/x_A V_A$, $\bar{\psi} = \psi/B_z x_A$, $\eta = i\eta c^2/4\pi\Omega x_A^2$, $S_0 = r_s/x_A$, $x = (r - r_s)/x_A$, $x_A = \Omega/q_{\parallel}' V_A$, and η is the classical resistivity. Furthermore, $q_{\parallel}(r) = q_z + B_{\theta}(r)/r B_z$ and $q_{\parallel}' = dq_{\parallel}/dr$ at the mode rational surface, r_s . The quantities μ , ν , α , and β are the turbulent contributions and will be discussed a little later. Defining a new variable, $E = x d\bar{\psi}/dx - \bar{\psi}$, we integrate Eq. (3) once and elim-

inate $\bar{\varphi}$ to obtain a single equation in E :

$$\frac{d}{dx} \left[f(x) \frac{dE}{dx} \right] + \left(\mu - \frac{\nu}{x} \right) \frac{dE}{dx} + E(1 - x^{-2}) = C_1, \quad (5)$$

where $f(x) \equiv (1/x^2)(\bar{\eta} + S_0\beta) + (1/x)(\beta - S_0\alpha) - \alpha$ and C_1 is a constant. In the limit $\mu = \nu = \alpha = \beta = 0$, Eq. (5) agrees with the one derived by Hazeltine and Strauss⁷ for the $m = 1$ mode. We note that, for large x , the solution of Eq. (5) is unaffected by the turbulent terms and hence, as shown by Hazeltine and Strauss,⁷ the constant C_1 must be set equal to zero, to ensure proper matching with the outer-layer solutions. Thus, we only need to concern ourselves with the homogeneous part of Eq. (5). We proceed to solve it by employing a variational technique.⁸ For this, we first convert it into a self-adjoint form by writing $E = f^{-1/2} E_1 \exp[\int (\mu - \nu/x) dx/f]$. It can then easily be shown that the functional

$$S = \int_{-\infty}^{+\infty} \left\{ - \left(\frac{dE_1}{dx} \right)^2 + \left[\frac{1}{f} \left(1 - \frac{1}{x^2} \right) - \frac{1}{2} \frac{f''}{f} + \frac{1}{4} \frac{f'^2}{f^2} - \frac{1}{4} \frac{(\mu - \nu/x)^2}{f^2} - \frac{1}{2} \frac{\nu}{x^2 f} \right] E_1^2 \right\} dx \quad (6)$$

is variational, in that $\delta S = 0$ yields the differential equation for E_1 . Choosing a trial function, $E_1 = f^{1/2} \times \exp(-\lambda x^2/2)$ with $\text{Re}(\lambda/x_A^2) > 0$, we evaluate S as

$$S(\lambda) = \lambda^{-1/2} \left[1 + \mu^2/2\alpha + \lambda(2 + \nu + \frac{1}{2}\alpha) - (\bar{\eta} + S_0\beta)\lambda^2 \right] + \frac{1}{2} [(\beta - S_0\alpha)^2 + 4\alpha(\bar{\eta} + S_0\beta)]^{-1/2} [(\nu - \mu x_1)^2 Z(x_1\sqrt{\lambda}) - (\nu - \mu x_2)^2 Z(x_2\sqrt{\lambda})], \quad (7)$$

where $x_{1,2}$ are the roots of the quadratic equation $f(x) = 0$, and $Z(x)$ is the plasma dispersion function. A simultaneous solution of $S(\lambda) = dS/d\lambda = 0$ will now give us the dispersion relation. To obtain such a relation, we need to evaluate the nonlinear terms. We write down the expression for one of the nonlinear terms, e.g.,

$$\beta = - \frac{2i|q_z|}{\Omega} \frac{\omega_{pe}^2}{\Omega_e x_A} \sum_{\mathbf{k}} k_z k_{\perp} \frac{c^2 k_{\perp}^2 |\varphi_{\mathbf{k}}|^2}{B_z^2} \left(\frac{1}{\epsilon_+} + \frac{1}{\epsilon_-} \right). \quad (8)$$

Since $|\tilde{\mathbf{q}}| \ll |\tilde{\mathbf{k}}|$ and $|\Omega| \ll |\omega|$, we can expand $\epsilon_{\pm} \equiv \epsilon(\Omega \pm \omega, \tilde{\mathbf{q}} \pm \tilde{\mathbf{k}})$ around $(\omega, \tilde{\mathbf{k}})$, to obtain

$$\frac{1}{\epsilon_+} + \frac{1}{\epsilon_-} = - \frac{[\Omega(\partial/\partial\omega) + \tilde{\mathbf{q}} \cdot (\partial/\partial\tilde{\mathbf{k}})]^2 \epsilon}{(\partial\epsilon/\partial\omega)^2 (\Omega - \tilde{\mathbf{q}} \cdot \tilde{\mathbf{V}}_{gk})^2} \approx \frac{1}{2k_{\perp}^2 (\omega^2 - \omega_{LH}^2)} \text{ for } \Omega \ll \tilde{\mathbf{q}} \cdot \tilde{\mathbf{V}}_{gk}, \quad (9)$$

where $\tilde{\mathbf{V}}_{gk} = -(\partial\epsilon/\partial\tilde{\mathbf{k}})(\partial\epsilon/\partial\omega)^{-1}$. In general, $\Omega \ll \tilde{\mathbf{q}} \cdot \tilde{\mathbf{V}}_{gk}$ unless very special conditions hold, such as the tearing surface coinciding with the local lower hybrid resonance layer ($\omega = \omega_{LH}$). We therefore consider only the nonresonant type of interaction. The nonlinear coefficients then simplify to

$$\beta \approx - \frac{i|q_z|}{\Omega \Omega_e x_A} \sum_{\mathbf{k}} \left(\frac{k_{\perp}}{k_z} \right) \frac{c^2 k_{\perp}^2 |\varphi_{\mathbf{k}}|^2}{B_z^2},$$

$$\alpha \approx - 2i \left(\frac{m_e}{m_i} \right) \frac{|q_z| V_A}{\Omega \Omega_e \omega_{LH} x_A} \sum_{\mathbf{k}} \frac{|k_{\perp}|^3}{k_z^2} \frac{c^2 k_{\perp}^2 |\varphi_{\mathbf{k}}|^2}{B_z^2}, \quad \mu \approx - \frac{\omega_{pi}}{c q_z} \alpha, \quad \nu \approx - \frac{\omega_{pi}}{c |q_z|} \beta. \quad (10)$$

Furthermore, for typical tokamak parameters in the collisional regime, $T_e \lesssim 10^3$ eV, $B_z \sim 25$ kG, $n_0 \sim 10^{14}$ cm⁻³, $a/R = \frac{1}{3}$ and moderate levels of LH fluctuations,

$$W_k/n_0 T = k_{\perp}^2 |\varphi_{\mathbf{k}}|^2 / (8\pi n_0 T) < 10^{-4},$$

it can be seen that $\bar{\eta} \ll S_0\beta$. This condition implies that $k_{\perp}^2 |\varphi_{\mathbf{k}}|^2 / B_z^2 > \eta \Omega_e k_z / 4\pi k_{\perp} q_z r_s$. Then the roots of $f(x)$ become $x_1 \approx \beta/\alpha$ and $x_2 = -S_0$. With these simplifications, we can obtain analytic expressions

for the dispersion relation in two limits by expanding the Z function for large or small arguments. We obtain

$$\left(1 - \frac{\beta}{2S_0} \frac{\omega_{pi}^2}{c^2 q_z^2}\right) \left[1 + \bar{\eta} + \beta \left(S_0 - \frac{\omega_{pi}}{c q_z}\right)\right] = 0 \quad \text{for } |S_0 \sqrt{\lambda}| \gg 1, \quad (10a)$$

$$\left\{1 + \frac{\alpha}{2} \frac{\omega_{pi}^2}{c^2 q_z^2}\right\} \left\{1 + \bar{\eta} + \beta \left[S_0 \left(1 - \frac{\omega_{pi}^2}{c^2 q_z^2}\right) - \frac{\omega_{pi}}{c |q_z|}\right]\right\} = 0 \quad \text{for } |S_0 \sqrt{\lambda}| \ll 1. \quad (10b)$$

Physically, these two limits correspond to mode widths $x_w = x_A / \sqrt{\lambda}$ smaller or larger, respectively, than x_A , the tearing layer width. It is now straight forward to solve Eq. (10) to obtain the various roots and we discuss them briefly here:

The first factor in Eq. (10) yields quasimodes which are driven solely by fluctuations. For example, from the factor in Eq. (10b), we get typically

$$\gamma \approx \frac{1}{\sqrt{2}} (q_{\parallel} V_A)^{1/2} \left\{ \frac{m_e}{m_i} \frac{\omega_{LH}}{\Omega_e} \sum_{\mathbf{k}} \frac{|k_{\perp}^3|}{|k_z^2|} \frac{k_{\perp}^2 |\varphi_{\mathbf{k}}|^2}{B_z^2} \right\}^{1/2}$$

which for moderate levels of fluctuations is comparable to classical resistive growth rates. The corresponding quasimode from Eq. (10a) is a damped one. Both these modes also have comparable real frequencies. The second factor yields turbulence-modified versions of the tearing mode [for $\beta = 0$ one can recover the classical result,⁷ $\gamma = (\eta c^2 / 4\pi)^{1/3} (q_{\parallel} V_A)^{2/3}$]; typically again, from Eq. (10b), we get

$$\gamma \approx (q_{\parallel} V_A c)^{2/3} \left[\frac{\eta}{4\pi} - \frac{|q_z| r_s}{\Omega_e} \left(1 - \frac{\omega_{pi}^2}{c^2 q_z^2}\right) \sum_{\mathbf{k}} \frac{|k_{\perp}^3| |\varphi_{\mathbf{k}}|^2}{|k_z| B_z^2} \right]^{1/3}; \quad \Omega_R \approx \frac{q_{\parallel} V_A M}{\omega k_{\perp} m} \gamma. \quad (11)$$

The corresponding root from Eq. (10a) is quite similar except it does not have the factor of $1 - \omega_{pi}^2 / c^2 q_z^2$ multiplying the $q_z r_s$ term. The turbulent terms in γ easily dominate $\eta / 4\pi$ for rather small levels of fluctuations ($W_r / n_0 T \sim 10^{-6}$) and typical tokamak parameters. The condition for instability for mode (11) is thus $\omega_{pi} / c q_z > 1$. Since $\omega_{pi} / (c q_z) \sim 4.4 \times 10^{-8} R \sqrt{n_0}$, where R is the major radius, this condition is easily satisfied for moderate sized machines ($R \sim 30$ cm) and typical tokamak densities of $\sim 10^{14}$ cm⁻³. For $W_r / n_0 T \sim 10^{-6}$, with $q_{\parallel} \sim a^{-2}$ and with use of the parameters quoted earlier, the mode growth rate is of the order of $\sim 10^6$ sec⁻¹ with a correspondingly large real frequency. The growth rate time is thus comparable to disruptive time scales for rather modest turbulence levels.

To summarize, we have discussed an important nonlinear effect that can influence the evolution of tearing modes whenever there is a background of microturbulence. For the $m = 1$ resistive tearing mode, we have shown that a modest level of lower hybrid wave turbulence can significantly enhance the growth rate as well as introduce a large real part to the mode frequency. It is important to point out that in these calculations we have highlighted the modification induced by the nonlinear coupling and have neglected effects that might arise from a more complicated formulation of the linear theory of tearing modes. For example, the inclusion of diamagnetic effects in the linear theory would impart a real part to the

mode frequency of order ω_* (the diamagnetic frequency). However, for the collisional regime we have considered this is typically of the order of 10^3 sec and is thus much smaller than the nonlinearly induced real part. Similarly in the limits discussed above, we have assumed that the spatial variations of $x^2 f(x)$ [where $f(x)$ is in some sense an effective resistivity] are mainly determined by the turbulence terms rather than by the spatial dependence of the linear resistivity, η . Such an assumption is valid when $k_{\perp}^2 |\varphi_{\mathbf{k}}|^2 / B_z^2 > \eta \Omega_e k_z / 4\pi k_{\perp} q_z r_s$, a condition easily met for the typical parameters chosen here. The nonlinear modification would also be important for tearing modes in other regimes. There is some evidence of such effects in the weakly collisional regime from computer simulation studies where propagating magnetic islands have been observed along with the presence of lower hybrid turbulence. We are, at present, examining the $m \geq 2$ modes and extending our calculations to the collisionless regime including the effects of diamagnetic drifts and spatial variations in η . These results will be presented elsewhere.

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Effect of Pulse Duration and Polarization on Momentum and Energy Transfer to Laser-Irradiated Targets

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Polished aluminum targets were irradiated with 1.06- μm laser pulses of 60-psec and 2.5-nsec duration and of π and σ polarization. The peak focused intensity was varied over the range 10^{14} – 10^{16} W/cm². Clear evidence of resonant absorption for 60-psec pulses was obtained from target momentum measurements made with a torsion pendulum. No resonant absorption effects dependent on light polarization or angle of incidence were detected for long-pulse irradiations.

A necessary condition for successful laser-induced fusion is the efficient absorption of laser light in the coronal plasma surrounding the target. Equally important, however, is the coupling of this absorbed energy to the dense target material so that inward directed momentum will be imparted to compress and heat the fusion fuel.

Efficient laser-light absorption is believed to occur through the process of resonant absorption.¹ In experiments conducted with short-duration laser pulses ($\tau \leq 100$ psec), increased laser light absorption² and momentum transfer³ have been observed with conditions optimized for the resonant absorption process. Current and planned laser fusion experiments, however, require much longer laser pulses ($\tau = 1$ – 10 nsec) and it is important to determine in this regime whether or not resonant absorption exists and if it will improve the efficiency of momentum transfer to the target.

This Letter deals with the experimental determination of the resonant-absorption contribution to the momentum imparted to targets irradiated with short (60 psec) or long (2.5 nsec) laser pulses. The targets, polished aluminum slabs, were irradiated at oblique incidence with high-

focused-intensity (10^{14} – 10^{16} W/cm²), π and σ polarized, 1.06- μm wavelength laser light. (The target was always placed within the 10- μm -diam diffraction-limited focal spot of the $f/5$ convergent cone of light.⁴) The primary diagnostic was a torsion pendulum which measured the momentum imparted to the target.⁵ Supporting data were provided by charge collectors (Faraday cups).

Figure 1 shows the experimental results. The ratio of momentum to incident energy (P/E) has been plotted as a function of incident intensity. Each point represents an average over 3 shots within an intensity bin extending $\pm 15\%$ from the plotted data point. Absolute calibrations for the momentum and energy measurements are accurate to $\pm 10\%$ and $\pm 5\%$, respectively. The lines are curve fits to the data obtained from equations of the form $P = AE^n$.

The ratio P/E provides a measure of the momentum coupling efficiency to the target. For 60-psec pulses, the coupling efficiency is 35% greater for π than for σ polarization. The coupling efficiencies are independent of focused intensity. For the 2.5-nsec pulses there is no difference between π and σ polarizations and the momentum coupling efficiency decreases with in-