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Is Black-Hole Evaporation Predictable?

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If black-hole formation and evaporation can be described by a superscattering operator which is *CPT* invariant, then it can be described by an S matrix which maps pure initial states into pure final states. Thus black holes may be in principle no more unpredictable than other quantum phenomena.

Because gravity provides an attractive force between all forms of matter having positive energy densities, it apparently can lead to a breakdown of physics in which the attraction increases without limit. Such a breakdown, called a singularity, has been shown to occur when matter collapses into a black hole according to the classical theory of general relativity.[1]

It has been thought, according to the Penrose cosmic censorship hypothesis,[2,3] that all singularities that form would be hidden from view inside black holes. Then physics outside would remain predictable, i.e., uniquely determined by the data on some suitable Cauchy hypersurface in the past. However, Hawking found that quantum mechanics allows particles to come out of black holes,[4] so that "even an observer at infinity cannot avoid seeing what happens at a singularity."[5] Furthermore, the emitted particles carry away energy from the black hole and so presumably cause it to shrink and eventually to disappear at a momentarily naked singularity.

Hawking thus argued that the formation and evaporation of black holes would introduce a new level of unpredictability into physics.[5] Part of the information about the system would be lost down the holes. In quantum mechanical terms, a pure initial state would result in a state partially going down the holes and partially remaining outside, so that the final state outside would be a mixed state described by a density matrix. Hawking concluded that one could not describe the process by an S matrix mapping pure states to pure states but must introduce a superscattering operator which maps initial mixed states to final mixed states.

In this Letter I will show that if black-hole formation and evaporation can be described by a superscattering operator which is CPT invariant,[6] then it can be described by an S matrix. Arguments will be made that this is a plausible assumption, though it cannot be proven outside a consistent theory of quantum gravity. Thus black holes may be in principle no more unpredictable than other quantum phenomena. Other possibilities will also be discussed.

Following Hawking's argument in principle if not in detail, let $|X_a\rangle$, $|Y_b\rangle$, $|Z_c\rangle$ be orthonormal bases for Hilbert spaces H_1, H_2, H_3 of incoming states on the initial hypersurface (before the black-hole formation), hidden hypersurfaces (around the black holes that form and disappear), and final hypersurface (after the black-hole evaporation), respectively. Let $|\overline{X}_a\rangle$, $|\overline{Y}_b\rangle$, $|\overline{Z}_c\rangle$ be the corresponding outgoing states obtained by applying the CPT operator.

Assume that an incoming pure state on H_1 evolves into an outgoing pure state on $H_2 \otimes H_3$ according to

$$x_a |X_a\rangle \to S_{abc} x_a |\bar{Y}_b\rangle |\bar{Z}_c\rangle, \tag{1}$$

where x_a is a set of amplitudes defining the state and the summation convention is applied to repeated indices. In order for the inner product between any two initial states on H_1 to give the same inner product between the corresponding two final states on $H_2 \otimes H_3$, one needs

$$S_{abc}\bar{S}_{a'bc} = \delta_{aa'}, \tag{2}$$

where the bar denotes complex conjugation. Applying (1) to an incoming mixed state on H_1 gives an outgoing mixed state on $H_2 \otimes H_3$ which may be reduced to an outgoing mixed state on H_3:

$$|X_a\rangle \rho_{aa'}{}^{(1)} \langle X_{a'}| \to |\bar{Z}_c\rangle \rho_{cc'}{}^{(3)} \langle \bar{Z}_{c'}|. \tag{3}$$

The density matrix on H_3 may be written as

$$\rho_{cc'}{}^{(3)} = \mathcal{S}_{cc'aa'} \rho_{aa'}{}^{(1)}, \tag{4}$$

$$\mathcal{S}_{cc'aa'} = S_{abc}\bar{S}_{a'bc'} \tag{5}$$

being Hawking's superscattering operator.[5]

Now if the evolution is CPT invariant,[6] an incoming pure state on H_3 will evolve backward in time to an outgoing pure state on $H_2 \otimes H_1$ according to

$$z_c |Z_c\rangle \to S_{abc} z_c |\bar{Y}_b\rangle |\bar{X}_a\rangle. \tag{6}$$

To preserve inner products, one needs

$$S_{abc}\bar{S}_{abc'} = \delta_{cc'}. \tag{7}$$

Note that (5) and (7) imply

$$\mathcal{S}_{cc'aa} = \delta_{cc'}, \tag{8}$$

which Hawking found was necessary for thermodynamics.[5]

However, one may deduce more. An incoming mixed state on H_3 will evolve backward into an outgoing mixed state on $H_2 \otimes H_1$ which may be reduced to an outgoing mixed state on H_1. Applying the CPT operator to these states gives a map from outgoing mixed states on H_3 to incoming mixed states on H_1,

$$|\bar{Z}_c\rangle \rho_{cc'}{}^{(3)} \langle \bar{Z}_{c'}| \to |X_A\rangle \rho_{AA'}{}^{(1)} \langle X_{A'}|, \tag{9}$$

where the density matrix on H_1 is

$$\rho_{AA'}{}^{(1)} = \bar{\mathcal{S}}_{cc'AA'} \rho_{cc'}{}^{(3)}. \tag{10}$$

In order for this to be compatible with (4), it is necessary that

$$\bar{\mathcal{S}}_{cc'AA'} \mathcal{S}_{cc'aa'} = \delta_{Aa}\delta_{A'a'}. \tag{11}$$

This in turn implies

$$\rho_{cc'}{}^{(3)} \rho_{c'c}{}^{(3)} = \rho_{aa'}{}^{(1)} \rho_{a'a}{}^{(1)}, \tag{12}$$

and so a pure state on H_1, which is a mixed state with $\rho_{aa'}\rho_{a'a} = 1$, will evolve into a pure state on H_3.[7] Then (1) and (6) imply that the evolution from H_1 to H_3 is given by an S matrix, with H_2 consisting of a single state. Thus if black-hole formation and evaporation can be described by a superscattering operator which is CPT invariant, no information is really lost down the hole.

One might object to this conclusion because the emission from a black hole has been calculated to be completely random and uncorrelated.[5,8] But these calculations have been made in the semiclassical approximation of a fixed background metric, which breaks down long before the final stages of evaporation. For example, in emitting a fraction f of its energy Mc^2, a black hole will in a time $t \sim \hbar^{-1}c^{-4}G^2 f M^3$ create $N \sim \hbar^{-1}c^{-1}GfM^2$ particles, each of rms momentum $\langle p^2\rangle^{1/2} \sim \hbar c^2 G^{-1} M^{-1}$, for a total rms momentum $\Delta P \sim \langle Np^2\rangle^{1/2} \sim \hbar^{1/2}c^{3/2} \times G^{-1/2} f^{1/2}$. If momentum is conserved, the black hole will not remain fixed in its original rest frame as in the semiclassical approximation, but instead its position will recoil in precise correlation with the particles emitted and develop an rms spread $\Delta x \sim \Delta P M^{-1} t \sim \hbar^{-1/2} c^{-5/2} G^{3/2} f^{3/2} M^2$. This will be larger than the hole size $\sim c^{-2}GM$ for $f \gtrsim \hbar^{1/3} c^{1/3} G^{-1/3} M^{-2/3}$ ($\sim 10^{-13}$ for $M \sim 10^{15}$ g, $\sim 10^{-25}$ for $M \sim M_\odot$), so that the semiclassical approximation of a single classical geometry will break down after only a very small fraction of the energy is emitted. What is emitted after the hole has recoiled greatly from its expected position will have significant correlations (in positions as well as momenta) with the particles that caused the recoil. Although these correlations alone do not remove the calculated randomness, their existence suggests other neglected correlations (e.g., those mediated by the back reaction on the shape of the hole) which could give a pure state.

A related objection to a lack of information loss is that the internal states of a black hole should not causally be able to affect the final state outside, so that the information in the initial state that distinguishes between the resulting internal states is really lost when the black hole disappears.[9] One possible answer is that quantum gravity may alter the causal structure so that the information apparently lost down the hole can get back out.[10] A second counter argument is that it may be in principle impossible to separate the Hilbert space of states into internal and external states. Any state that is localized to be completely within the hole would need infinite-momentum components, which would be ruled out by requiring a finite back reaction.

Another objection to the deterministic evolution proposed here is that if the black hole evaporates completely (as it must for CPT invariance), then a description of this by a classical space-time metric will inevitably have a naked singularity.[5,11] But this may be merely a breakdown in the classical description. Since the naked singularity in the classical description could be quite small, quantum effects could conceivably heal it to give a predictable final state. Although one cannot be certain of this outside a proper theory of quantum gravity, it seems reasonable to make the *quantum cosmic censorship hypothesis*: Any initial mixed state composed entirely of regular matter configurations on complete, asymptotically flat hypersurfaces will have a unique evolution under the laws of physics into a final mixed state composed entirely of regular matter configurations on complete, asymptotically flat hypersurfaces. In simpler words, the future is completely predictable from the past.

The argument above showed that if quantum cosmic censorship is realized by a CPT-invariant linear map (the superscattering operator) of the form (5) from initial to final mixed states, then the evolution is given by an S matrix. However, since this assumption cannot be proven yet, it may be illuminating to list a variety of possibilities:

(A) Evolution by an S matrix (discussed above).

(B) Evolution by (1)–(5) but not (6)–(12) (resulting in a CPT-noninvariant superscattering matrix).[12] CPT invariance is a theorem in nongravitational physics,[13] but Penrose has argued it might break down with gravity as Poincaré invariance goes badly wrong in highly curved space-times.[3] However, without (6)–(7) one loses the quantum mechanical justification for (8), which is necessary to exclude the possibility of perpetual-motion machines.[5]

(C) Evolution backward in time by (5)–(10) but not (1)–(4), (11), and (12) (the time reverse of B). Then the past would be predictable from the future, but the future would not be predictable from the past. This possibility would suit historians better than physicists.

(D) Evolution (either forward or backward in time) by a superscattering matrix which is not of the form (5). However, if it is a linear map from the convex set of unit-trace, Hermitian, positive semidefinite density matrices to itself that is onto (as required for CPT invariance), then it can be shown to arise from a unitary or antiunitary S matrix.[9]

(E) Evolution of density matrices deterministically but nonlinearly (i.e., not by a superscattering matrix). This would be a breakdown of ordinary quantum mechanics.

(F) Evolution in which black holes or naked singularities form but do not disappear. This would violate CPT invariance.[5] If the black holes or singularities have a finite but nonzero number of final states, it would also violate unitarity and lead to a piling up of states as different initial states (with and without holes or singularities) evolve into the same final state (with holes or singularities).

(G) Evolution in which the disappearance of black holes results in mixed states that are unpredictable (breakdown of quantum cosmic censorship), i.e., God would not use the laws of physics completely in creating the future from the past as He usually does but would create it in a way we could not predict. (To us it might seem as if He were throwing dice. Nevertheless, "the lot is cast into the lap, but its every decision is from the Lord."[14])

(H) Replacement of density matrices by something more fundamental (overthrow of quantum mechanics).

At present none of these possibilities seems to be ruled out experimentally. In view of the historical developments in the concept of nature, one might say that the most radical, (H), is the most realistic. However, in the absence of further information, it would seem most productive to pursue the most conservative possibility (A). Hawking suggested that "God not only plays dice, He sometimes throws the dice where they cannot be seen."[5] But it may be that "If God throws dice where they cannot be seen, they cannot affect us."

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[3]R. Penrose, in *General Relativity: An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge, England, 1979).

[4]S. W. Hawking, Nature 248, 30 (1974), and Commun. Math. Phys. 43, 199 (1975).

[5]S. W. Hawking, Phys. Rev. D 14, 2460 (1976).

[6]I mean that the operator giving the evolution in terms of the initial or final state is *CPT* invariant, not that the state and its evolution is itself *CPT* invariant. The gravitational field is regarded as part of the quantum state, not as a fixed classical background metric. I thank E. Kazes for pointing out the need for this clarification.

[7]R. M. Wald and R. Sorkin and also S. W. Hawking and C. N. Pope have verified this conclusion by an independent argument (private communication).

[8]R. M. Wald, Commun. Math. Phys. 45, 9 (1975); L. Parker, Phys. Rev. D 12, 1519 (1975).

[9]R. M. Wald, private communication.

[10]R. P. Geroch, private communication.

[11]H. Dodama, "Inevitability of a Naked Singularity Associated with the Black Hole Evaporation," Kyoto University Report No. KUNS 500 (to be published).

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Extraction of Gluon Momentum, Spin, Parity, and Coupling from $\gamma^* N \to \psi N$ Data

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The unique ability of heavy-quark leptoproduction to probe constituent gluons is exploited. From recent $\gamma^* N \to \psi N$ data, the gluon distribution $\eta^{-1}(1-\eta)^p$, $p = 5.6^{+0.8}_{-1.2}$, is extracted and α_s/π is inferred. It is shown that measurement of σ_L/σ_T as a function of Q^2 for charm production can determine the gluon spin and parity.

As emphasized by Shifmar *et al.*[1] and by Leveille and Weiler,[2] leptoproduction of heavy-quark states offers a unique possibility for extracting information on the gluon constituents of the nucleon. This is because unlike any other process, leptoproduction's lowest-order contribution, viz., $\gamma^* G \to Q\bar{Q}$, is directly proportional to the gluon component of the nucleon wave function and independent of the quark component. In this Letter, recent ψ muoproduction data[3] along with ψ photoproduction data[4] are used to extract information on the fractional momentum distribution, spin, parity, and coupling strength of nucleon glue.

The gluon momentum distribution is inferred from the ψ excitation curve. One may prcceed either by extracting and inverting moments, or by fitting the gluon model directly to data. The starting point for

