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Effect of Finite Beta on Drift-Wave Turbulence and Particle Confinement

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This Letter predicts, on the basis of the structure of mode-coupling equations, that if the plasma β exceeds the square of the inverse aspect ratio, $(a/R)^2$, the cross-magnetic-field diffusion is greatly enhanced by the appearance of convective cells even in the presence of magnetic shear.

The purpose of this Letter is to show that under a reasonable scaling for a toroidal plasma with magnetic shear, (1) the electric potential obeys the two-dimensional convective cell equation[1]

$$(\partial/\partial t)\nabla_\perp^2\varphi - (\nabla\varphi\times\hat{z}/B_0)\cdot\nabla(\nabla_\perp^2\varphi) = 0, \qquad (1)$$

if $\beta > (a/R)^2$, while if $\beta \ll (a/R)^2$ it obeys the mode-coupling equation for the electrostatic drift-wave turbulence obtained by Hasegawa and Mima,[2]

$$\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2\varphi}{B_0\omega_{ci}} - \frac{e\varphi}{T_e}\right) - \frac{\nabla\varphi\times\hat{z}}{B_0}\cdot\nabla\left(\frac{\nabla_\perp^2\varphi}{B_0\omega_{ci}} - \ln n_0\right) = 0, \qquad (2)$$

and (2) this difference produces a greatly enhanced diffusion in a plasma as β increases over $(a/R)^2$. Here β is the ratio of plasma to magnetic field pressure, a and R are minor and major radii of the torus, $\hat{z}$ is the unit vector in the direction of the ambient magnetic field, $\nabla_\perp$ is the gradient operator perpendicular to $\hat{z}$, $n_0(x)$ is the plasma density, B_0 is the flux density of the ambient magnetic field, and ω_{ci} $(=eB_0/m_i)$ is the ion cyclotron frequency.

Let us first derive mode-coupling equations for a finite β plasma with magnetic shear. We introduce the following ordering:

$$\frac{1}{\omega_{ci}}\frac{\partial}{\partial t} \approx \kappa_n\rho_s \approx \frac{e\rho_s^2\nabla_\perp^2\varphi}{T_e} \approx \frac{e\rho_s^2\nabla_\perp^2\psi}{T_e} \approx \frac{n_1}{n_0} \approx O(\epsilon), \qquad (3)$$

$$\rho_s\,\partial/\partial z \approx O(\epsilon\delta/q). \qquad (4)$$

Here, κ_n $(=|\nabla \ln n_0|)$ is the measure of the density gradient, ρ_s $[=(T_e/m_i)^{1/2}/\omega_{ci}]$ is the effective ion Larmor radius, T_e is the electron temperature, φ and ψ are the Kadomtsev potentials defined by $E_\perp = -\nabla_\perp\varphi$ and $E_z = -\partial\psi/\partial z$, δ $(=a/R)$ is the inverse aspect ratio, and q is the safety factor. The ϵ ordering is justified by the saturated amplitude of drift-wave turbulence,[3] while δ ordering in Eq. (3) is justified because

$$\rho_s \frac{\partial}{\partial z} = \rho_s k_{\parallel} = \rho_s \frac{k_y x}{L_s} \approx \rho_s \frac{1}{L_s}$$

$$= \frac{\rho_s}{a} \frac{a}{L_s} \approx \epsilon \frac{a}{qR}, \tag{5}$$

where L_s is the shear length and $k_{\parallel}$ is the wave number in the direction of the local magnetic field.

In this Letter, the role of the magnetic shear is identified as such to allow free motion of electrons in the $\hat{z}$ direction. Because of such free electron motions in the $\hat{z}$ direction, electrostatic convective cells are destroyed in the presence of magnetic shear. Hence, we treat electrons to be an inertialess fluid in the equation of motion in the z direction. The usual drift wave ordering allows us to assume ions to move two dimensionally in the x-y plane. These assumptions are equivalent to the assumption

$$k_{\parallel} v_e \gg \omega \gg k_{\parallel} v_i, \tag{6}$$

where v_e and v_i are electron and ion thermal speed and ω is the magnitude of the typical frequency range.

On the basis of (6), we treat ions to be cold. Then ion dynamics may be described by the two-dimensional vortex equation[2] of order ϵ^2,

$$\frac{\partial}{\partial t}\left(\frac{\nabla_\perp^2 \varphi}{B_0 \omega_{ci}} - \frac{n_1}{n_0}\right) - \frac{\nabla\varphi \times \hat{z}}{B_0} \cdot \nabla\left(\frac{\nabla_\perp^2\varphi}{B_0\omega_{ci}} - \frac{n_1}{n_0} - \ln n_0\right) = 0. \tag{7}$$

Under the assumption of quasineutrality, the perturbed density n_1 can be described by the electron continuity equation obtainable by taking the zeroth moment of the drift kinetic equation of order ϵ^2:

$$\frac{1}{n_0}\left(\frac{\partial J_z}{\partial z} + \frac{\vec{B}_\perp}{B_0} \cdot \nabla_\perp J_z\right) - e\frac{\partial}{\partial t}\left(\frac{n_1}{n_0}\right) + e\frac{\nabla\varphi \times \hat{z}}{B_0} \cdot \nabla\left(\ln n_0 + \frac{n_1}{n_0}\right) = 0, \tag{8}$$

where $\vec{B}_\perp$ is the perturbed magnetic field given by

$$\partial\vec{B}/\partial t = (\partial/\partial z)\nabla(\psi - \varphi) \times \hat{z}, \tag{9}$$

and the z component of the current density J_z is given by

$$\mu_0 J_z = (\nabla \times \vec{B}_\perp) \cdot \hat{z}. \tag{10}$$

Taking the first moment of the drift kinetic equation, and neglecting the electron inertia, we have

$$\frac{\partial}{\partial z}\left(\frac{n_1}{n_0} - \frac{e\psi}{T_e}\right) + \frac{\vec{B}_\perp}{B_0} \cdot \nabla\left(\ln n_0 + \frac{n_1}{n_0} - \frac{e\varphi}{T_e}\right) = 0. \tag{11}$$

In the derivation of Eq. (11), the nonadiabatic term due to the Landau resonance has been dropped in view of the ordering given by Eq. (6). However, the existence of such a term is implicitly assumed to provide the source of the turbulent energy. Equations (7)–(11) are appropriate nonlinear equations which describe the mode coupling in finite-β drift Alfvén wave turbulence.

We note that these sets of equations contain an additional small parameter, β. If $\beta \ll \epsilon$, $B_\perp = 0$ and Eqs. (9) and (11) give $\varphi = \psi$, $n_1/n_0 = e\psi/T_e = e\varphi/T_e$. Hence Eq. (7) reduces to the Hasegawa-Mima equation (2), the mode-coupling equation for electrostatic drift waves.[4] On the other hand if β is finite, since Eq. (9) gives $\rho_s \nabla \times B_\perp / B_0 = O(\epsilon\delta)$, from Eq. (10) we see that

$$J_z / e n_0 c_s = O(\epsilon\delta/q\beta), \tag{12}$$

where $\beta = c_s^2/v_A^2$, and $v_A^2 = B_0^2/(\mu_0 n_0 m_i)$. If we use Eq. (12) in Eq. (8), we see that the first two terms have order $\epsilon^2\delta^2/(\beta q)$ while the rest have order ϵ^2. Hence if $\beta = \delta^2/q^2$ Eqs. (5)–(9) produce coupled drift-Alfvén wave equations. However, if $\beta > \delta^2$, since $q > 1$ we see that the first two terms become smaller than the rest. Then the ordering requires

$$\frac{\partial}{\partial t}\left(\frac{n_1}{n_0}\right) - \left(\frac{\nabla\varphi \times \hat{z}}{B_0}\right) \cdot \nabla\left(\frac{n_1}{n_0} + \ln n_0\right) = 0. \tag{13}$$

If we use Eq. (13) in Eq. (7), we see that φ satisfies

$$\partial\nabla_\perp^2\varphi/\partial t - (\nabla\varphi \times \hat{z}/B_0) \cdot \nabla(\nabla_\perp^2\varphi) = 0,$$

which is the convective cell equation, Eq. (1). The above finding indicates that in a plasma with $\beta > (a/R)^2$, the line-tying effect by thermal motion of electrons along the field line is ineffective and plasma behaves like a two-dimensional fluid.

Let us now discuss the implication of this result on the particle confinement. Since $\nabla \cdot [\varphi(\nabla\varphi \times \hat{z})] = 0$, the ion flux Γ across the magnetic field in the drift-wave turbulence described by Eq. (2) is given by the nonlinear polarization drift,[2]

$$\Gamma_x = \langle nv \rangle_x$$
$$= -n_0 \langle (\nabla\varphi \times \hat{z}/B_0^2\omega_{ci}) \cdot \nabla\partial\varphi/\partial x \rangle, \tag{14}$$

which is nonvanishing if the spectrum is spatially inhomogeneous and anisotropic. The ion flux in the convective cell turbulence is given by the same term plus an additional term given by $-\langle n_1 \partial\varphi/\partial y/B_0\rangle$.[5,6]

Since the ratio of fluctuation energy of the drift wave turbulence[2] to that of the convective cell turbulence[5] is given by

$$\sum_k (k^2+\rho_s^{-2})|\varphi_k|^2 / \sum_k k^2 |\varphi_k|^2,$$

if $k^2 \ll \rho_s^{-2}$, $|\varphi_k|^2$ of the convective cell becomes much larger than that for the drift-wave turbulence for the same level of fluctuation energy. Now, the mode coupling described by Eqs. (1) (Refs. 5-8) and (2) (Refs. 9 and 10) is found to cascade the energy spectrum into the small wave-number regime (i.e., inverse cascade). However, the rate of cascading at $k_\perp \ll \rho_s^{-1}$ in the electrostatic drift-wave turbulence described by Eq. (2) is much slower than that of Eq. (1) because the nonlinear term of Eq. (2) is smaller than that of Eq. (1) by a factor of $k_\perp^2\rho_s^2$. This further reduces the magnitude of φ in the drift-wave turbulence in the small wave-number regime, resulting in a reduced diffusion. Secondly, the inhomogeneous term, $\nabla \ln n_0$, in Eq. (2) (which gives the drift wave) is shown to produce an anisotropic spectrum at $k_\perp^2\rho_s^2 \ll 1$ which may result in the production of zonal flows in the azimuthal direction.[11] This effect is also considered to reduce the diffusion.

We have shown that at $\beta = 0$ and $\beta > (a/R)^2$, the mode-coupling equations can be reduced to *two-dimensional forms* as given by Eqs. (2) and (1) even in the presence of magnetic shear (where free electron motions are allowed in the $\hat{z}$ direction). In order to confirm the large difference in particle diffusion in these models, numerical simulations have been performed with a two-dimensional particle simulation model in a uniform magnetic field. Although the model is two-dimensional, it suffices to simulate Eqs. (1) and (2). Both ions and electrons are pushed, with use of full dynamics for simulating the convective cell turbulence,[5] Eq. (1), while only the ions are pushed to simulate the drift-wave turbulence, Eq. (2), and the electrons are assumed to be an adiabatic fluid (Boltzmann distribution).[12] In addition, the background density of electrons is continuously adjusted to match with that of ions in the latter case so that no ambipolar potential develops.

Figure 1 shows the particle positions for the drift-wave turbulence (top) and the convective-cell turbulence (bottom) at $t = 6000\omega_{pi}^{-1}$. The initial macroscopic density profile was taken to be $n(x) \sim \exp(-x^2/\Delta^2)$ for both cases where Δ was

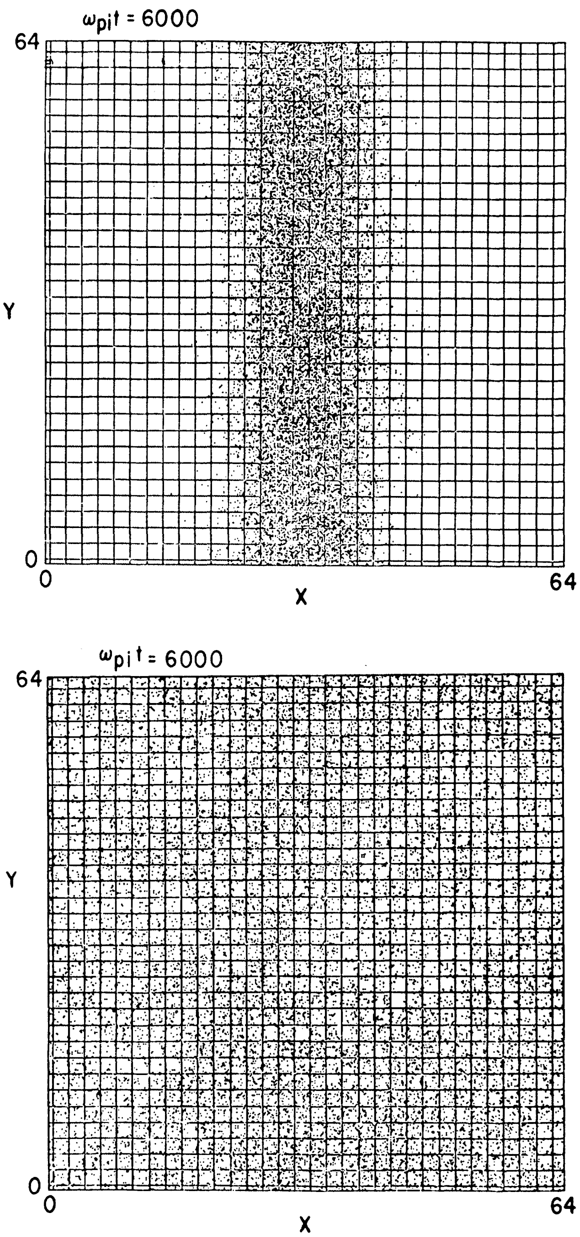

FIG. 1. Plot of the particle positions at $t = 6000\omega_{pi}^{-1}$ for drift-wave turbulence (top) and convective-cell turbulence (bottom), starting from the same condition. 64×64 mesh, 128×128 particles, and $\omega_{pi}/\omega_{ci} = 0.5$ are used for both cases.

typically a few ion gyroradii. The initial fluctuation energy was the same for both cases. The large difference in diffusion is apparent. The density profile for the drift-wave turbulence remains almost unchanged while it is completely spread out for the convective-cell turbulence. We note that in the limit of zero ion temperature, the mode-coupling equations for these models can be shown to reduce to Eqs. (1) and (2), respectively.

In conclusion, we have shown that convective cells can be excited when β exceeds the square of the inverse aspect ratio even in the presence of a magnetic shear. In this regime of β, the particle confinement is expected to deteriorate drastically.

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Study of Current-Driven Magnetohydrodynamic Instability in the Heliotron-D Device

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Ohmic-current-driven magnetohydrodynamic instabilities in the Heliotron-D device, which has large external rotational transform ($\iota_F > 1$) and strong shear ($\theta > 0.5$), have been studied experimentally. Since those instabilities satisfy the regular relations between the external and the ohmic-current transforms (ι_{OH}), kink and resistive tearing-mode instabilities are supposed to exist. The strong shear plays a major role to suppress the magnetohydrodynamic instabilities, and the Ohmic current exceeds the $\iota_{OH} = 1$ limit stably.

In many stellarators such as Wendelstein VII A (W VII A),[1] CLEO,[2] L-2,[3] and TORSO,[4] Ohmic current is used to produce and heat the plasma. In these devices, studies have been carried out to analyze the effect of the stellarator field on current-driven magnetohydrodynamic (MHD) instabilities. The observed MHD activity is considered to be due to the kink and resistive tearing modes.[5,6] In addition, the current disruption, which is familiar in tokamaks, is observed in these devices. In W VII A which has an external rotational transform with low shear, the internal disruption has been observed when the resonant surface of $\iota = 1$ is present in the plasma column, where ι is the sum of the rotational transforms due to the Ohmic current (ι_{OH}) and the external field (ι_F). This experiment also shows that the $m = 2$ (poloidal mode number) oscillation seems to play the dominant role in major disruptions. In the case of L-2 and CLEO, which have fairly large shear, the rather peaked plasma current has a tendency to make the ι profile flat. Once the $\iota = 1$ condition is satisfied within the plasma, the major disruption occurs. Thus, in these stellarators, the value of ι cannot stably exceed unity, just as in tokamaks.

On the other hand, the external rotational transform of the Heliotron D is larger than unity at the boundary of the plasma column as shown in Fig. 1(a). The magnetic surface with $\iota = 1$ exists in the middle of the plasma; or it can be removed with a small Ohmic current, corresponding to $\iota_{OH}(a)$ from 0.2 to 0.4, where the argument a refers to the value at the boundary. In this Letter

