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Triton *s*-Wave Asymptotic Normalization Constants

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The n-d and n-$(np)_{s=0}$ triton s-wave asymptotic normalization constants are calculated from a realistic one-boson-exchange model of the NN interaction. Both parameters are calculated from integral relations which make the analytic continuation to the pole unambiguous. The values obtained are $C_t^2 = 2.60$ and $C_s^2 = -0.69$.

The subject of triton asymptotic normalization constants has received considerable attention in the literature recently.[1] Apart from the accepted usefulness of these quantities in the analysis of direct reactions[2] involving tritions, it has been proposed that these numbers provide a discriminating test of triton wave functions obtained from realistic models of the NN interaction.[3] The purpose of this Letter is to present the values of the $^3\mathrm{H}\to n+d$ and $^3\mathrm{H}\to n+(np)_{s=0}$ s-wave asymptotic normalization constants, designated C_t and C_s, respectively, calculated from a one-boson-exchange (OBE) representation of the NN interaction. To our knowledge, it represents the first calculation of C_s^2 with a realistic NN potential[4] and the first OBE-model calculation of C_t^2.[5] Furthermore, we stress that the C_s^2 defined unambiguously by analytic continuation to the second-sheet pole has not yet been extracted from experiment and a method to do it remains an open problem. Also, we show how to calculate the quantity called $C(^3\mathrm{H}, d^*n)$—the "effective" $^3\mathrm{H}\to n+(np)_{s=0}$ asymptotic normalization—by Plattner, Bornand, and Viollier (PBV)[6] and give its value as predicted from our wave functions.

The OBE model of the NN interaction which we use is one developed by Holinde and Machleidt.[7] This interaction gives an excellent account of the low-energy NN parameters ($a_s = -23.83$ fm, $r_{0s} = 2.703$ fm, $\epsilon_s = 67.6$ keV; $a_t = 5.50$ fm, $r_{0t} = 1.87$ fm, $\epsilon_d = 2.225$ MeV) and the NN phase shifts. Moreover, it leads to a somewhat better description of the triton than does the Reid soft-core potential (RSC).[8] With the OBE interaction effective in the $^1S_0$ and $^3S_1$-$^3D_1$ states, solving the complete set of Faddeev equations, we get $E_3 = 7.38$ MeV for the $^3\mathrm{H}$ binding energy and a triton charge form factor in marginally better agreement with the data. This $^3\mathrm{H}$ wave function, including all its components, we normalize and use in the calculations.

The asymptotic normalization constant C_t is defined from the ^{3}H wave function in the standard manner:

$$\Psi^{[1/2]}(\vec{\rho}, \vec{r}) \xrightarrow[\rho\to\infty]{} C_t(2\alpha_t)^{1/2}\rho^{-1}\exp(-\alpha_t\rho)[[Y^{[0]}(\hat{\rho})\times\chi_{(1)}{}^{[1/2]}]^{[1/2]}\times\Psi_d{}^{[1]}(\vec{r})]^{[1/2]}\eta'(1,23)/\sqrt{2}, \tag{1}$$

where $\vec{\rho}$ and $\vec{r}$ are the coordinate variables conjugate to the Lovelace momenta,[9] $\alpha_t{}^2 = m(E_3 - E_2)$, m is the nucleon mass, E_3 and E_2 are the triton and deuteron binding energies, respectively, $\Psi_d{}^{[1]}(\vec{r})$ is the deuteron wave function, and χ (η') is a spin (isospin) function. In momentum space, this limit can be written as

$$\langle\vec{q}\,\chi_{(1)}{}^{[1/2]}\Psi_d{}^{[1]}|\Psi^{[1/2]}\rangle \xrightarrow[q^2\to-\alpha_t{}^2]{} 2C_t\left(\frac{\alpha_t}{\pi}\right)^{1/2}\frac{1}{q^2+\alpha_t{}^2}\frac{\eta'(1,23)}{\sqrt{2}}, \tag{2}$$

where $\vec{q}$ is conjugate to $\vec{p}$. If the projection on the left side of Eq. (2) is formed with $\Psi^{[1/2]}$ replaced by its expression from Faddeev's equations, we get an equation for calculating C_t by matching residues on the left-hand and right-hand sides of Eq. (2):

$$C_t = \tfrac{1}{3}(\pi/3\alpha_t)^{1/2}\int_0^\infty q^2\,dq\int_{-1}^1 dx\,\psi_d(p_1)\Delta(p_2,q)[3\Psi^{(1)}(p_2,q;1) - \Psi^{(1)}(p_2,q;2)], \tag{3}$$

where $\psi_d(p)$ is the s-wave component of the momentum-space deuteron wave function; $\Delta(p,q) = mE_3 + p^2 + q^2$; the ^{3}H wave function components $\Psi^{(1)}(p,q;1)$ and $\Psi^{(1)}(p,q;2)$ correspond to nucleons (23) in spin states $S=0$ and 1, respectively; $\vec{p}$ is conjugate to $\vec{r}$; and $p_1{}^2 = \frac{1}{3}(4q^2 - \alpha_t{}^2 + i4\alpha_t qx)$ while $p_2{}^2 = \frac{1}{3}(q^2 - 4\alpha_t{}^2 + i4\alpha_t qx)$. Integral relations of the form given by Eq. (3) were first derived by Lehman and Gibson[10] (see also Kim, Sander, and Tubis[11]).

From Eq. (3), it is clear that both ψ_d and the $\Psi^{(1)}$'s are required with complex arguments. In the former, the continuation is defined by the homogeneous Lippmann-Schwinger (LS) equation, while in the latter the continuation of the two-nucleon t matrix required in the kernel of the Faddeev equations is defined from the inhomogeneous LS equation. Limited space prevents us from giving further details, but the procedure follows straightforwardly from Eq. (3).

At first glance, the generalization of Eq. (3) to the singlet case is not obvious, because the function which replaces the deuteron wave function is not normalizable,[12] thus leaving the scale indeterminate. However, the unique singlet function with its proper normalization can be found in analogy with an alternative procedure for obtaining the deuteron wave function. The properly normalized deuteron wave function in momentum space can be obtained from the half-shell (s-wave) NN triplet t matrix as follows:

$$\psi_d(p) = \frac{2\pi i\gamma_t{}^{1/2}m}{C_t{}^{(2)}}\frac{\lim_{k\to i\gamma_t}(k - i\gamma_t)\langle p|t^{10}(E_k{}^+)|k\rangle}{p^2+\gamma_t{}^2}, \tag{4}$$

where the deuteron binding energy is $\epsilon_d = \gamma_t{}^2/m$ and the deuteron asymptotic normalization constant $C_t{}^{(2)} \approx (1-\gamma_t r_{0t})^{-1/2}$. Then C_t for the triton can also be obtained from the vertex amplitude for $^3\mathrm{H} - n + (np)_{s=1}$:

$$C_t = -\frac{2\pi i\gamma_t m}{C_t{}^{(2)}}\left(\frac{\pi^2}{3\alpha_t\gamma_t}\right)^{1/2}\lim_{\substack{k\to i\gamma_t\\ q\to i\alpha_t}}(k - i\gamma_t)F(\vec{k},\vec{q};\beta_t), \tag{5}$$

where, in general,

$$F(\vec{k},\vec{q};\beta) = \frac{1}{\sqrt{3}}\sum_{ij}{}_i\langle\psi_{\vec{k}}{}^{(-)},\vec{q};\beta|\bar{\delta}_{ij}V_j|\Psi\rangle. \tag{6}$$

In Eq. (6), $\psi_{\vec{k}}{}^{(-)}$ is a NN scattering state with relative momentum $\vec{k}$, V_j is the NN interaction, and all spin-isospin quantum numbers are subsumed in the index β. Now, the definition of C_s can follow systematically.

The singlet-channel asymptotic normalization constant is defined in analogy with Eq. (5):

$$C_s = \frac{2\pi i\gamma_s m}{C_s{}^{(2)}}\left(\frac{\pi^2}{3\alpha_s\gamma_s}\right)^{1/2}\lim_{\substack{k\to -i\gamma_s\\ q\to i\alpha_s}}(k + i\gamma_s)F(\vec{k},\vec{q};\beta_s), \tag{7}$$

where $-i\gamma_s$ ($\gamma > 0$) is the location of the pole (on the second Riemann sheet in the energy plane) in the

NN singlet half-shell t matrix. Explicitly, we can write

$$C_s = \frac{1}{3}\left(\frac{\pi}{3\alpha_s}\right)^{1/2}\int_0^\infty q^2\,dq\int_{-1}^{1}dx\,\psi_s(p_1)\Delta(p_2,q)[\Psi^{(1)}(p_2,q;1) - 3\Psi^{(1)}(p_2,q;2)], \tag{8}$$

where

$$\psi_s(p) = \frac{2\pi i(-\gamma_s)^{1/2}m}{C_s^{(2)}}\,\frac{\lim_{k\to -i\gamma_s}(k+i\gamma_s)\langle p|t^{01}(E_k^{+})|k\rangle}{p^2+\gamma_s^2}, \tag{9}$$

$C_s^{(2)} \approx (1+\gamma_s r_{0s})^{-1/2}$, and $p_1^2 = \frac{1}{3}(4q^2 - \alpha_s^2 + i4\alpha_s qx)$ while $p_2^2 = \frac{1}{3}(q_2 - 4\alpha_s^2 + i4\alpha_s qx)$. From a computational viewpoint, the key quantity is the residue at the antibound-state pole, $R_s(p)$. Specifically, if we define $R_s(p)$ such that

$$\psi_s(p) = -4\pi i\gamma_s^{3/2}C_s^{(2)}mR_s(p)/(p^2+\gamma_s^2), \tag{10}$$

then $R_s(p)$ satisfies the LS equation

$$R_s(p) = V^{01}(p,i\gamma_s) - \int p'^2\,dp'\,\frac{V^{01}(p,p')R_s(p')}{E_{p'}+\gamma_s^2/m}. \tag{11}$$

This follows from unitarity which gives us the t matrix on the second sheet (II) in terms of the t matrix on the first sheet (I) in the energy plane:

$$\langle p|t^{01}(E_k)|k\rangle_{\mathrm{II}} = \frac{k\cot\delta_s - ik}{k\cot\delta_s + ik}\langle p|t^{01}(E_k)|k\rangle_{\mathrm{I}}. \tag{12}$$

Near the pole, we can use $k\cot\delta_s = -1/\alpha_s + \frac{1}{2}r_{0s}k^2$ which gives

$$\lim_{k\to -i\gamma_5}(k+i\gamma_s)\langle p|t^{01}(E_k^{+})|k\rangle = i2\gamma_s[C_s^{(2)}]^2R_s(p). \tag{13}$$

Two points should be noted: (1) $\psi_s(p)$ is a purely imaginary function which implies that C_s is purely imaginary or $C_s^2<0$ (Ref. 13); (2) the normalization (or scale) of $\psi_s(p)$ is as though the unbounded spatial function were normalized with the integral defined by discarding the infinite contribution from the upper limit.[14]

Using Eqs. (3) and (8), we obtain the values $C_t^2 = 2.60$ and $C_s^2 = -0.69$ for the OBE model. As can be seen in Table I, the OBE value for C_t^2 is considerably smaller than the other realistic potential calculation of Kim and Muslim for the RSC which implies that C_t^2 is model dependent. Unfortunately, no consensus has been reached on the experimental value of C_t^2 (see Table I and Locher and Mizutani[16]). Unanimity for the empirical value is important in order that the model dependence of C_t^2 can be better understood. As far as C_s^2 is concerned, the only theoretical values available are for triton wave functions derived from simple parametrizations of the NN interaction,[4] e.g., separable potential (Y), Bressel-Kerman-Rouben (BKR), Malfliet-Tjon (MT), and Darewich-Green (DG). Our OBE result is comparable to the BKR value, but more important is the fact that C_s^2 is even more model dependent than C_t^2. This emphasizes the significance of developing a method whereby C_s^2 can be extracted from experiment.

As mentioned above, PBV have defined an effective asymptotic norm, $C^2(^3\mathrm{H},d^*n)$, which they extract from experiment for ^{3}He by a dispersion analysis of p-^{3}He elastic scattering. Their analysis amounts to approximating the cut contribution from the singlet two-nucleon exchange amplitude by a pole. In our notation, the singlet two-nucleon exchange amplitude is

$$f(E) = \frac{\mu}{2\pi\hbar^2}\sum_{\text{spins}}\int d^3k\,\frac{|F(\vec{k},\vec{q};\beta_s)|^2}{E_3+q_2/m+k^2/m}, \tag{14}$$

TABLE I. ^{3}H asymptotic normalization constants.

	C_t^2	C_s^2	Ref.
Theory			
OBE	2.60 ± 0.08	−0.69 ± 0.02	This work[a]
RSC	3.15	?	5
Y	3.81	−0.23	4
BKR	2.48	−0.74	4
MT	3.81	−0.02	4
DG	3.16	−0.01	4
Experiment			
PWDR	3.3 ± 0.1	?	1
FDR	2.6 ± 0.3	?	15

[a] Includes estimated numerical errors.

where $F(\vec{k},\vec{q};\beta_s)$ is defined in Eq. (6) and μ is the p-^{3}H reduced mass. To compare the OBE model with the PBV analysis (which is only qualitative because of Coulomb effects), we note that for low q, $k^2|F|^2$ is strongly peaked at $k=\gamma_s$. Therefore, we write

$$f(E)\approx-\frac{\mu}{2\pi\hbar^2}\frac{mc}{\hbar}\frac{|A(q)|^2}{q^2+\beta_0{}^2}, \tag{15}$$

which gives

$$C^2(^3\mathrm{H},d^*n)=2m^2c^2|A(i\beta_0)|^2/3\sqrt{3}\pi\hbar^2\beta_0,$$

where $\beta_0{}^2=mE_3+\gamma_s{}^2=0.1795\ \mathrm{fm}^{-2}$. The value we obtain is $C^2(^3\mathrm{H},d^*n)=6.50$ which is comparable to the PBV value $C^2(^3\mathrm{He},d^*p)=5.85\pm0.25$, but distinct from it in the location of the pole. PBV obtain an "effective-pole" location shifted by ~ 1.3 MeV, i.e., $\beta_0{}^2=0.2092\ \mathrm{fm}^{-2}$. With $\beta_0{}^2$ equal to the latter value, we obtain $C^2(^3\mathrm{H},d^*n)=14.35$. Clearly, $C_s{}^2$ is the more fundamental quantity since $C^2(^3\mathrm{H},d^*n)$ depends critically on the effective-pole location.[17]

In this Letter, we have given the ^{3}H s-wave asymptotic normalization constants for a OBE NN interaction calculated from integral relations which avoid extrapolation methods. We found that the OBE triplet asymptotic norm is $\simeq 17\%$ smaller than the RSC value. At present, both the OBE and RSC value fall within the extremes of experimental determinations, thus emphasizing the need for a more accurate empirical value so that conclusions can be reached about the theoretical predictions. Also, we emphasize that the singlet asymptotic norm is an even more sensitive quantity than the triplet, but has yet to be extracted from experiment. Our OBE value represents the first calculation of this parameter for a realistic NN interaction. It would be valuable to know the RSC result. Finally, we demonstrated that the "effective" $^3\mathrm{H}\to d^*+n$ asymptotic norm as extracted by PVB for ^{3}He is quite sensitive to the location of the effective pole which decreases the value of this parameter and emphasizes the significance of the true singlet asymptotic norm.

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