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Superconnections in Extended Supergravity

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Closed-form solution for the superspace connections of extended supergravity in terms of supervielbeine are given for arbitrary values of N, and deSitter parameter e. A reduction procedure from the Riemannian geometry is used to deduce the desired solutions.

A superspace formulation of the supergravity geometry hinges crucially on the construction of a so-called "gauge complete" supervielbein and superconnection. By this one means that a supervielbein and a superconnection are constructed in terms of the supergravity multiplet so that the superspace transformation laws for these are consistent with the supergravity transformation laws in the component form. This technique for constructing the superspace "potentials" as well as the first superspace formulation of the supergravity dynamics was developed by the authors[1,2] shortly after the component-form formulation of the $N=1$ supergravity[3] was obtained. However, the "gauge completion" process[1] is intricate since it involves the construction of the superspace quantities order by order in θ.[1,4-6] What one would like are closed-form solutions for the supervielbeine and superconnections so that the iteration procedure in θ and the satisfaction of the integrability conditions to each order in θ are circumvented.

A considerable amount of new progress has occurred in the superspace formulations of supergravity in the last three years.[4-10] Specifically, analyses using the order-by-order integration in θ technique have been extended to values of $N \leq 3$ (Ref. 5) and closed-form solutions for the supervielbein and the superconnection have been given for the case $N=1$.[8] In this Letter we present general closed-form solutions for the superconnection in terms of the supervielbein for extended supergravity for arbitrary N. Details of the analysis and other aspects of the superspace geometry of supergravity are presented elsewhere.[11]

We begin by displaying first the main results obtained in this paper, i.e., the formulas for the superspace connections of supergravity in terms of the supervielbeine. We shall assume for the purpose of this analysis that a "gauge complete" supervielbein $V_\Lambda{}^A(z)$ is known. We denote the superspace connections corresponding to the tangent-space group $O(3,1)\otimes O(N)$ by $h_{mn\Lambda}$ and $h_{ij\Lambda}$. Then the Lorentz tangent-space connection is obtained from $h_{mn\Lambda}=h_{mnA}V_\Lambda{}^A(-1)^{A+\Lambda A}$, where h_{mnA} is given by[12]

$$h_{mns}(z)=\tfrac{1}{2}[\Omega_{mns}(z)+\Omega_{msn}(z)-\Omega_{nsm}(z)], \qquad (1)$$

$$h_{mnai}(z)=\tfrac{1}{2}[\Omega_{main}(z)-\Omega_{naim}(z)], \qquad (2)$$

where Ω_{mns} and Ω_{main} are determined in terms of the vielbein components $V_\Lambda{}^A(z)$ as follows:

$$\Omega_{mAn}(z) = V_m{}^\Lambda[V_\Lambda{}^r{}_{,\Sigma} - (-1)^{\Lambda+\Sigma+\Lambda\Sigma} V_\Sigma{}^r{}_{,\Lambda}](-1)^{A\Sigma}V_A{}^\Sigma \eta_{rn}. \tag{3}$$

The Yang-Mills connections $h_{ij\Lambda}$ are obtained from

$$h_{ij\Lambda}(z) = \tfrac{1}{4}\,\bar{\omega}_{aibj\Lambda}\eta^{ab}, \tag{4}$$

where

$$\bar{\omega}_{aibjm} = \tfrac{1}{2}[\bar{\Omega}_{aimbj} + \bar{\Omega}_{bjmai}], \tag{5}$$

$$\bar{\omega}_{aibjck} = \tfrac{1}{2}[\bar{\Omega}_{aibjck} - \bar{\Omega}_{aickbj} - \bar{\Omega}_{bjckai}]. \tag{6}$$

Here $\bar{\Omega}_{aiCbj}$ is given by

$$\bar{\Omega}_{aiCbj} = (-1)^{C+1}V_{ai}{}^\Lambda[V_\Lambda{}^{dj}{}_{,\Sigma} - (-1)^{\Lambda\Sigma}V_\Sigma{}^{dj}{}_{,\Lambda}](-1)^{C\Sigma}V_C{}^\Sigma\eta_{db}. \tag{7}$$

Equations (1)–(7) represent the closed-form solutions of the superspace connections of extended supergravity for arbitrary N in terms of the supervielbeine. Note that this construction does not calculate the form of the gauge-complete vielbein but only the relation between connection and vielbein. It is the separation of these two problems that simplifies the task of finding the indicated relations.

Equations (1)–(7) may be deduced by the device of starting in the larger Riemannian superspace [with tangent group $\mathcal{K} = \mathrm{OSp}(3,1|4N)$] where the relation between connection and vielbein is trivial to obtain, and taking the limit $k \to 0$. We will see that this limit produces precisely the supergravity connections. We begin, therefore, with a brief summary of some of the relevant formulas of Riemannian superspace. We choose as the basic geometrical quantity the vielbein $V_\Lambda{}^A(z)$ and its inverse $V_A{}^\Lambda(z)$, defined by $V_A{}^\Lambda V_\Lambda{}^B = \delta_A{}^B$. These quantities transform under general coordinate transformations $z^\Lambda = z^{\Lambda'} + \xi^\Lambda(z)$ and local tangent-space transformations generated by $\epsilon_{MN}(z)$. The general tangent-space metric η_{AB} is diagonal with elements η_{mn} and $k\eta_{ab}\delta_{ij}$. Here k is an arbitrary parameter.[6]

The supervielbein connection $\omega_{MN\Lambda}$ may be defined through the covariant derivative of $V_\Lambda{}^A$ so that

$$V_\Lambda{}^A{}_{;\Sigma} = V_\Lambda{}^A{}_{,\Sigma} - (-1)^{\Sigma(A+\Delta)}\Gamma_\Lambda{}^\Delta{}_\Sigma V_\Delta{}^A + \tfrac{1}{2}V_\Lambda{}^B(Z^{MN})_B{}^A\omega_{MN\Sigma}, \tag{8}$$

where $Z^{MN} = -(-1)^{MN}Z^{NM}$ are the $\mathcal{K} = \mathrm{OSp}(3,1|4N)$ generators and $\Gamma_\Lambda{}^\Delta{}_\Sigma$ is the global-space affinity. In a convenient basis, the structure constants of $\mathcal{K}$ (Refs. 11, 13) take the form

$$f_{RS}{}^{MNPQ} = \tfrac{1}{2}\{\eta^{NP}[\delta_R{}^M\delta_S{}^Q - (-1)^{R+S+RS}\delta_S{}^M\delta_R{}^Q](-1)^{N+Q+S} - (-1)^{MN}[M \leftrightarrow N]\} - (-1)^{PQ}(P \leftrightarrow Q). \tag{9}$$

We note the appearance of η^{NP} in Eq. (9) and hence the presence of terms $\sim 1/k$.

So far we have dealt with a general superspace of $4+4N$ dimensions. We now restrict the discussion to a Riemannian space which arises by assuming that the affinity $\Gamma_\Lambda{}^\Delta{}_\Sigma$ is (graded) symmetric[14] and that $V_\Lambda{}^A{}_{;\Sigma} = 0$. Equation (8) then determines both the affinity and the connection in terms of the vielbein in the usual fashion, and we record the latter here:

$$\omega_{ABC} = \tfrac{1}{2}[\Omega_{ABC} + (-1)^{BC}\Omega_{ACB} + (-1)^{A(B+C)}\Omega_{BCA}], \tag{10}$$

where $\omega_{ABC} \equiv \omega_{AB\Lambda}V_C{}^\Lambda(-1)^{C(1+\Lambda)}$ and

$$\Omega_{ABC} = V_A{}^\Lambda(-1)^{C(1+B)}[V_\Lambda{}^D{}_{,\Sigma} - (-1)^{C(\Lambda+\Sigma)+\Lambda+\Sigma+\Lambda\Sigma}V_\Sigma{}^D{}_{,\Lambda}](-1)^{B\Sigma}V_B{}^\Sigma\eta_{DC}. \tag{11}$$

While the Riemannian geometry is based on a tangent-space group $\mathcal{K} = \mathrm{OSp}(3,1|4N)$, supergravity exists in a non-Riemannian space with tangent group $H = \mathrm{O}(3,1)\otimes\mathrm{O}(N)$. Of course H is a subgroup of $\mathcal{K}$ which suggests that it should be possible to extract the superspace connections of supergravity from the above Riemannian ones. We will show now that this is the case, and that the supergravity connections arise from the $k \to 0$ limit of Eqs. (10) and (11). In order to exhibit the supergravity group H, it is necessary to make a change of basis in the Lie algebra of $\mathcal{K}$ and introduce generators $X^{MN} = Z^{RS}C_{RS}{}^{MN}$. The new basis is chosen so that X^{mn} and Y^{ij} generate O(3, 1) and O(N) of H:

$$X^{mn} = Z^{mn} - \tfrac{1}{4}ik(\eta\sigma^{mn})_{cd}Z^{ckdk};$$
$$Y^{ij} = Z^{aibj}k\eta_{ba}. \tag{12}$$

Note that the k factors of Eq. (12) are necessary to cancel the $1/k$ in the structure constants of Eq.

(9) for the Z^{MN} basis so that X^{mn} and Y^{ij} correctly generate $O(3,1)\otimes O(N)$ (whose structure constants are *independent* of k). In the new basis, Riemannian connections $\tilde{\omega}_{MN\Lambda}$ are given by $X^{MN}\tilde{\omega}_{MN\Lambda}=Z^{MN}\omega_{MN\Lambda}$ and the $O(3,1)\otimes O(N)$ pieces are then

$$\tilde{\omega}_{mnC}=\omega_{mnC};\qquad \tilde{\omega}_{ijC}=\tfrac{1}{4}\omega_{aibjC}=\tfrac{1}{4}\omega_{aibjC}\frac{1}{k}\eta^{ab}. \tag{13}$$

We now examine the $k\to 0$ limit of Eq. (13) and show that the supergravity connections $h_{mn\Lambda}$ and $h_{ij\Lambda}$ are

$$h_{mnC}=\omega_{mnC}|_{k\to 0};\qquad h_{ijC}=\left[\tfrac{1}{4}\omega_{aibjC}\frac{1}{k}\eta^{ab}\right]_{k\to 0}. \tag{14}$$

From Eq. (11) one sees that the k dependence of ω_{ABC} comes from the final η_{DC} factor when C is a Fermi index. Thus for h_{mnC}, the limit $k\to 0$ in Eq. (14) produces precisely the results stated in Eqs. (1) and (2). To verify that these formulas correctly represent the Lorentz connections of supergravity, recall that in the gauge-completing arguments of Brink *et al.*[5] $h_{mn\Lambda}$ is constructed order by order in $\theta^{\alpha i}$ by integrating the gauge transformation equations

$$\delta h_{mn\Lambda}(z)=\epsilon_{mn,\Lambda}(z)+(-1)^{\Lambda+\Sigma\Lambda}\xi^{\Sigma}{}_{,\Lambda}h_{mn\Sigma}+h_{mn,\Sigma}\xi^{\Sigma} \tag{15}$$

subject to the boundary condition $h_{mn\mu}(x,\theta^{\alpha i}=0)=\omega_{mn\mu}(x)$ where $\omega_{mn\mu}(x)$ is the total connection (Einstein plus torsion) of the component form of supergravity. It is obvious that the $h_{mn\Lambda}$ constructed from Eq. (14) correctly obeys Eq. (15) (since ω_{mnC} transforms like a connection). In order to verify the $\theta^{\alpha i}=0$ boundary condition, one must substitute the gauge-complete vielbein into Eqs. (1) and (2) and evaluate at $\theta^{\alpha i}=0$. Such vielbeine have been constructed in Ref. 5 for $N\leqslant 3$ and we find by explicit calculation that the boundary condition $h_{mn\mu}(x;\theta^{\alpha}=0)=\omega_{mn\mu}(x)$ indeed holds. Thus Eqs. (1) and (2) are correct expressions for the supergravity Lorentz connections. The gauge-complete vielbein for $N\geqslant 4$ has not yet been constructed. It is easy to verify, however, for this case that gauge completion of the vielbein must yield from Eqs. (1) and (2) the boundary condition

$$h_{mn\mu}(x,\theta^{\alpha i}=0)=\omega_{mn\mu}(x)+\Delta_{mn\mu}(x), \tag{16}$$

where $\Delta_{mn\mu}$ is independent of derivatives. This is an acceptable boundary condition to choose and so Eqs. (1) and (2) are valid expressions for the Lorentz connections for arbitrary N.[15]

For the Yang-Mills connection $h_{ij\Lambda}(z)$, the boundary condition chosen by Brink *et al.*[5] is $h_{ij\mu}(x,\theta^{\alpha i}=0)=eA_{\mu}{}^{ij}(x)$ where $A_{\mu}{}^{ij}$ are the Yang-Mills fields of $O(N)$ and e is the deSitter parameter. Here the limit $k\to 0$ of Eq. (14) is more subtle because of the factor $1/k$ there. However, using the gauge-complete vielbeine of Ref. 5 for $N\leqslant 3$ one finds by explicit calculation that at $\theta^{\alpha i}=0$ the possible $1/k$ singular term actually cancels out [because of the Majorana trace in Eq. (14)] and that the whole $\Omega_{aibjm}(z)$ is at most a "constant of integration" and can be omitted. For $N\leqslant 3$, the vielbeine of Ref. 5 then yield by direct calculation the correct boundary condition for $h_{ij\mu}(x,\theta=0)$. For $N\geqslant 4$, one may easily extend the results of Ref. 5 on gauge completing the vielbein to show that a regular boundary condition of the form

$$h_{ij\mu}(x,\theta^{\alpha i}=0)=eA_{\mu}{}^{ij}(x)+\Delta_{ij\mu}(x) \tag{17}$$

always holds where $\Delta_{ij\mu}(x)$ is independent of derivatives, which represent valid boundary conditions for gauge completing the connection $h_{ij\mu}(z)$.

This completes the verification that Eqs. (1)–(7) correctly represent the relation between connections and vielbeine in supergravity.

Concluding remarks.—It is perhaps not surprising that the limit $k\to 0$ of the Riemannian connections leads to the supergravity connections. Thus, as shown in Ref. 1, the full dynamics of $N=1$ supergravity arises in this limit from the Riemannian gauge supersymmetry theory. What is remarkable is that the device of starting from the Riemannian geometry and contracting down leads to the relatively simple formulas Eqs. (1)–(7) for the supergravity connections. In Ref. 10 a full discussion is given as to how all the supergravity geometrical objects are embedded in the larger Riemannian geometry. Further dynamical relations of gauge supersymmetry also show that under spontaneous breaking, the Riemannian geometry of gauge supersymmetry reduces, in the limit $k\to 0$, to a vacuum state whose global space invariance becomes $OSp(N|4)_{R\to\infty}$ and tangent-space invariance is reduced from $\mathcal{H}=OSp(3,1|4N)$ to $H=O(3,1)\otimes O(N)$. Thus under spontaneous breaking, gauge supersymmetry in the limit $k\to 0$ has the same vacuum-state symmetry and tangent-space invariance as supergravity does.

A serious drawback of supergravity lies in the fact it is *not* a unified gauge theory. This is because its tangent group is a product group $H = O(3,1) \otimes O(N)$. In analogy to the framework of grand unified theories, supergravity resembles the nonunified theory of $SU(3)^c \otimes SU(2) \otimes U(1)$ rather than unified theory such as SU(5) or O(10). In usual gauge theories, a product invariance group is the symptom that a spontaneous breaking has already occurred. As mentioned above, the vacuum state and tangent symmetry of supergravity can indeed be thought of as arising from the spontaneous breaking of the truly unified Riemannian space with tangent group $\mathcal{K} = OSp(3,1|4N)$.

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[12] Global indices are denoted by Greek letters, $\Lambda \equiv (\mu, \alpha i)$, $\alpha = 1, \ldots, 4$, $i = 1, \ldots, N$, and local indices by Latin letters, $A \equiv (m, ai)$, $a = 1, \ldots, 4$. Middle lowercase letters are vectorial, early letter are (Majorana) spinorial, and $i, j, \ldots$ denote internal symmetry labels. η_{mn} is the Lorentz metric (with signature +2) and $\eta_{ab} = -(C^{-1})_{ab}$ is the Fermi metric (C = charge conjugation matrix). Comma, e.g., $\varphi_{,\Lambda}$ denotes right derivative.

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[15] We conjecture that actually $\Delta_{mn\mu}(x)$ [and the $\Delta_{ij\mu}$ of Eq. (17)] vanishes for $N \geq 4$ also, since there are no local supersymmetry tensors independent of derivatives of the fields.

Nonspectator Quark Interactions and the Λ_c^+ Lifetime

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It is found that nonspectator interactions between c and d quarks in the Λ_c^+ give a substantial enhancement of its decay rate. The lifetime relation $\tau(\Lambda_c^+)/\tau(D^+) = [1 + \tau(D^+)/(3\times10^{-13}\text{ s})]^{-1}$ is derived, based on the assumption that the D^+ decays purely via charm quark decay. The semileptonic branching fractions are related by $B_l(\Lambda_c^+) = B_l(D^+) \times \tau(\Lambda_c^+)/\tau(D^+)$.

Inclusive weak decays of a hadron with a new flavor are usually assumed to proceed through the heavy quark, with the light quarks acting as spectators.[1,2] The lifetimes of charm states D^+ ($c\bar{d}$), F^+ ($c\bar{s}$), D^0 ($c\bar{u}$), and Λ_c^+ (cud) are thereby expected to be equal, and of order a few times 10^{-13} s. Recent lifetime measurements[3-6] give D^+ and F^+ values in qualitative accord with this, but may indicate D^0 and Λ_c^+ lifetimes that are significantly shorter.[3,4] The discrepancy suggests that interactions involving light quarks in the decaying hadron may play a role.[7] The subject of this Letter is the Λ_c^+ lifetime. We find that the nonspectator transition $cd \to su$ of Fig. 1(a) is at least comparable to the spectator process of Fig. 1(b), leading to a Λ_c^+ lifetime that is

