Gamow-Teller Matrix Elements from 0°(p,n) Cross Sections

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After simple corrections for distortion effects, 120-MeV, 0° (p,n) cross sections are found to be proportional to the squares of the corresponding Fermi and Gamow-Teller matrix elements extracted from β-decay measurements. It is suggested that this proportionality can be used to extract Gamow-Teller matrix elements for transitions inaccessible to β decay.

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We present new 0° cross-section data for the (p,n) reaction on ^{7}Li, ^{12,13}C, ^{25,26}Mg, ^{27}Al, ^{28}Si, and ^{90}Zr at $E_p = 120$ MeV. The time-of-flight technique used for (p,n) studies permits measurement of cross sections at 0° where the (p,n) reaction preferentially populates states with appreciable Fermi and Gamow-Teller matrix elements. A linear correspondence between the present 0° (p,n) cross sections and the allowed β-decay rates is demonstrated, permitting a nearly model-independent measure of the strengths of the isovector parts of the effective nucleon-nucleon interaction at small momentum transfers. With the interaction strengths determined, the (p,n) reaction at $E_p = 120$ MeV becomes a reliable method for determining Gamow-Teller matrix elements for states that cannot be reached by β-decay.

The present data were taken with a 120-MeV proton beam and the Oak Ridge National Laboratory–Indiana University beam swinger[1] with a 62 m flight path and time compensated detectors.[2,3] The 0° differential cross sections were measured to an estimated absolute accuracy of approximately ± 15%, with uncertainties in target thicknesses and detector efficiencies[4] constituting the larger part of this error. The experimental technique has been described previously.[4-6]

At $E_p = 120$ MeV, strong transitions in the (p,n) reaction are direct. In the distorted-wave approximation the differential cross section is given by[7]

$$\frac{d\sigma}{d\Omega}=\left(\frac{\mu}{2\pi\hbar^2}\right)^2\frac{k_f}{k_i}\frac{1}{(2J_i+1)(2s_p+1)}\sum\left|\int\chi_f^{(-)*}(\bar{r}_p)\langle f|\textstyle\sum_j v_{jp}(1-P_{jp})|i\rangle\chi_i^{(+)}(\bar{r}_p)\,d^3r_p\right|^2, \quad (1)$$

where μ and k denote the reduced energy divided by c^2 and wave number, in the center of mass, respectively, the χ's are the distorted waves, $\langle f|\sum_i v_{ip}(1-P_{ip})|i\rangle$ is the target matrix element, v_{ip} is the effective nucleon-nucleon interaction, P_{ip} is the permutation operator accounting for knockout exchange, and the sum outside the absolute value brackets is over initial- and final-spin projections of the projectile and target. The impulse approximation[8] is also reasonable for $E_p \gtrsim 100$ MeV, so that v_{ip} is expected to be close to the free nucleon-nucleon t matrix.

The (p,n) reaction selects the isovector parts of v_{ip}, and for 0° scattering, only the low-momentum components contribute appreciably to the transition amplitude. The central parts of the isovector effective interaction dominate at low q and the noncentral parts can be neglected.[9-12] With this simplification the 0° cross sections for (p,n) transitions with appreciable Fermi and/or Gamow-Teller strength can be described by[9-12]

$$\frac{d\sigma(0°)}{d\Omega} = \left(\frac{\mu}{\pi\hbar^2}\right)^2 \frac{k_f}{k_i}\{N_\tau{}^D|J_\tau\langle F\rangle|^2 + N_{\sigma\tau}{}^D|J_{\sigma\tau}\mathfrak{M}_{\rm GT}|^2\}, \quad (2)$$

where J_τ and $J_{\sigma\nu}$ are the magnitudes of the volume integrals of $q=0$ components of the effective spin-independent $(\tau_1\cdot\tau_2)$ and effective spin-dependent $(\sigma_1\cdot\sigma_2\tau_1\cdot\tau_2)$ isovector central terms in $v_{ip}(1-P_{ip})$, $N_\tau{}^D$ and $N_{\sigma\tau}{}^D$ are distortion factors that can be calculated, and $\mathfrak{M}_{\rm F}$ and $\mathfrak{M}_{\rm GT}$ are the Fermi and Gamow-Teller matrix elements given by[13]

$$\mathfrak{M}_{\rm F} = \left(\frac{1}{2J_i+1}\right)^{1/2}\langle J_i\|T_-\|J_i\rangle\,, \quad (3)$$

$$\mathfrak{M}_{\rm GT} = \left(\frac{1}{2J_i+1}\right)^{1/2}\langle J_f\|\textstyle\sum_k t_-(k)\bar{\sigma}(k)\|J_i\rangle. \quad (4)$$

The reduced matrix elements here are as defined by Bohr and Mottelson[13] and t_- and T_- are isospin-lowering operators. For comparison the transition rate for allowed β decay[14] is

$$6200/ft = \mathfrak{M}_{\rm F}{}^2 + 1.5\mathfrak{M}_{\rm GT}{}^2. \quad (5)$$

Anderson, Wong, and Madsen[15] and Austin[16] have applied similar relationships to the (p,n) reaction at lower energies.

For a pure Fermi or Gamow-Teller transition a plot of

$$K = \frac{d\sigma(0°)}{d\Omega}\left[\left(\frac{\mu}{\pi\hbar^2}\right)^2\frac{k_f}{k_i}N^D\right]^{-1} \quad (6)$$

vs $\mathfrak{M}_{\rm F}{}^2$ or $\mathfrak{M}_{\rm GT}{}^2$ enables one to extract empirically the corresponding interaction strengths, J_τ and $J_{\sigma\tau}$ in Eq. (2). Mixed transitions and Fermi transitions can be grouped on the same plot as pure Gamow-Teller transitions by introducing an *effective* matrix element

$$\mathfrak{M}_{\rm eff}{}^2 = \mathfrak{M}_{\rm GT}{}^2 + \frac{N_\tau{}^D}{N_{\sigma\tau}{}^D}\left(\frac{J_\tau}{J_{\sigma\tau}}\right)^2\mathfrak{M}_{\rm F}{}^2. \quad (7)$$

The distortion factors defined by

$$N^D = \frac{d\sigma({\rm DW})/d\Omega}{d\sigma({\rm PW})/d\Omega}\bigg|_{\theta=0°} \quad (8)$$

have been calculated with the local t-matrix interaction of Love *et al.*[17] for the effective interaction and simple shell-model wave functions to describe the transitions. The plane-wave (PW) cross sections are computed with the proper k_f/k_i weighting but with $Q=0$, so that $\theta=0°$ corresponds to $q=0$. N^D exhibits a smooth exponential A dependence and is only moderately sensitive to the reaction Q values and the model nuclear wave functions. Optical-model parameters were taken from recent elastic-scattering studies.[4,18,19] These parameters give a good description of the available elastic-scattering data and are unique insofar as there is only one set per target at energies near 120 MeV. No attempt was made to adjust these to improve the results of the present calculations.

Some sensitivity of N^D to the interaction components was found with the $N_\tau{}^D$ values being on the average 20% smaller than $N_{\sigma\tau}{}^D$ values in the mass 20–30 region. This can be understood in terms of the shapes of the isovector central terms in the t-matrix interaction of Ref. 17. The $t_{\sigma\tau}$ component has a long-range characteristic of one-pion exchange potential, while t_τ has a much shorter range characteristic of multiple pion exchange.[10,17,20] When t_τ acts, the projectile must penetrate more of the absorbing potential to "see" the interaction than when $t_{\sigma\tau}$ acts. The shape sensitivity may appear to be inconsistent with the "model-independent" tone that we have adopted; however, since the ranges of the t_τ and $t_{\sigma\tau}$ components of the interaction are based on quite general physical expectations[10,17,20] it seems very reasonable to build this effect into the analysis. The eikonal approximation[21] suggests a mass dependence for the distortion factor of the form

$$N^D = C\exp(-xA^{1/3}) \quad (9)$$

with $C=1$ and $x = 2\mu W r_0{}'/\hbar^2 k$, where W is the depth of the imaginary part of the optical poten-

TABLE I. Differential cross sections at 0° for transitions observed in the (p,n) reaction at 120 MeV and the squares of the matrix elements for corresponding β-decay transition.

Target	J_i^π	J_f^π	E_x [a] (MeV)	$\frac{d\sigma}{d\Omega}(0°)$ [b] (mb/sr)	$\mathfrak{M}_F^2$ [c]	$\mathfrak{M}_{GT}^2$ [d]
^{7}Li	$3/2^-$	$3/2^-$	0.00	31±5	1	1.40 [f]
	$3/2^-$	$1/2^-$	0.43			1.19
^{12}C	0^+	1^+	0.00	6.0±1		.942±.006 [g]
^{13}C	$1/2^-$	$1/2^-$	0.00	4.5±.7	1	.274±.002 [h]
^{25}Mg	$5/2^+$	$5/2^+$	0.00	4.2±.6	1	0.45±.03 [i]
^{26}Mg	0^+	0^+	0.23	2.8±.4	2	
	0^+	1^+	1.06	8.0±1.2	-	1.21±.05 [j]
	0^+	1^+	1.85 [e]	4.1±.6	-	0.64±.04 [j]
	0^+	1^+	2.07			0.14±.03 [j]
^{27}Al	$5/2^+$	$5/2^+$	0.00	3.6±.5	1	0.34±.05 [k]
	0^+	1^+	1.31 [e]	7.0±1.1	-	0.37 [l]
	0^+	1^+	2.10			1.39 [l]
^{90}Zr	0^+	0^+	5.10	6.0±.9	10	-

[a] Excitation energy in final nucleus reached in the (p,n) reaction.
[b] For origin of quoted errors see text.
[c] These values are deduced from the usual isospin assumptions.
[d] Quoted errors are based on uncertainties in measure ft values and the small uncertainties in the parameters in Eq. (5) are not included.
[e] Unresolved in (p,n) measurements.
[f] Ref. 22, p. 64.
[g] Ref. 22, p. 94.
[h] Ref. 22, p. 41.
[i] Ref. 23, p. 131.
[j] Ref. 23, p. 161.
[k] Ref. 23, p. 187.
[l] Deduced from study $M1$ and 62-MeV (p,n) data reported in Ref. 14.

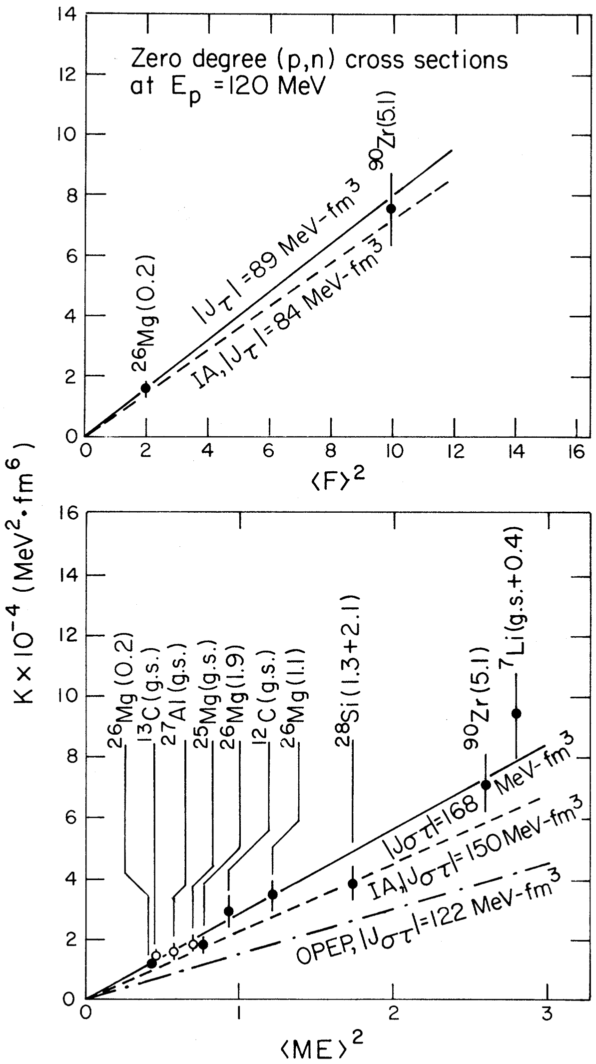

FIG. 1. Graphs of K, defined in Eq. (6) and deduced from the measured 0° (p,n) cross sections vs $\mathfrak{M}_F{}^2$ and $\mathfrak{M}_{eff}{}^2$. The top graph contains only the pure Fermi transitions while the lower graph contains the complete data set. Error bars on K reflect uncertainties in measured cross sections only. The transitions are labeled by specifying the target nucleus with the excitation energy in the final nucleus given in parentheses. The solid curves represent the fits to ^{26}Mg data, the dashed curves are the impulse approximation strengths, and the dot-dashed curve the one-pion-exchange-potential strength.

tial and r_0' is its radius. With average values of Wr_0' from the optical potentials used in this work, $x=0.237$. The calculated values of N^D are well described with $C=1.74$, $x=0.541$. The difference in the mass dependence of the calculated N^D and the mass dependence predicted by the eikonal formula indicates that refraction is still significant for 120-MeV nucleons.

The 0° (p,n) cross sections and the corresponding β-decay matrix elements for the transitions studied are summarized in Table I. The results of the analysis based on Eq. (2) and the associated discussion are displayed in Fig. 1. The upper graph shows K vs $\mathfrak{M}_F{}^2$ for the pure Fermi transitions and the lower graph is a plot of K versus the effective matrix $\mathfrak{M}_{eff}{}^2$ which contains the complete data set. The values $J=89$ MeV fm^3 and $J=168$ MeV fm^3 are the fits to the ^{26}Mg data. The values $J=84$ MeV fm^3 and $J=150$ MeV fm^3 are deduced from the impulse approximation[10,17] and are shown as dashed lines on Fig. 1. The dot-dashed line in the lower part of Fig. 1 represents the strength of the one-pion-exchange potential contribution to $J_{\sigma\tau}$. This accounts for a little more than half of the 0° spin-flip (p,n) cross section, a result which differs from the recent finding of Moake *et al.*[24] who have made a similar but more limited study with an eikonal approximation to estimate the effects of distortion.

A striking feature of the present results is the value of

$$\epsilon = (N_{\sigma\tau}{}^{D}/N_{\tau}{}^{D})(J_{\sigma\nu}/J_{\tau})^2 \qquad (10)$$

which measures the effectiveness of the (p,n) reaction in producing spin-flip transitions relative to analog transitions. For the present 120-MeV data, an average value of ϵ is about 4.5. At lower energies ϵ is found to be roughly unity.[16,17] This energy dependence, which is discussed in more detail elsewhere[5,6,9-11] makes the (p,n) reaction at medium energy a powerful probe of GT strength in nuclei. Application of this technique to identify giant GT resonances in heavier nuclei is in progress and extensions of the ideas to study isovector transition strengths for higher multipoles is being considered.

In conclusion, a definite correlation between medium-energy 0° (p,n) cross sections and allowed β-decay rates has been denomstrated. We believe that we can extract GT matrix elements from (p,n) reaction data to an accuracy only slightly worse than that of the cross-section measurement, or about 20%. It has also been shown that the strength factor for spin-flip transitions relative to analog transitions is about 4.5 at $E_p = 120$ MeV in sharp contrast to the situation at lower energies where this factor is only about 1. This makes the (p,n) reaction at medium energy a particularly powerful probe of GT matrix-element strength in nuclei. Application of this technique to identify giant GT resonances in heavier nuclei are in progress and extensions of the ideas to study isovector transition strengths for higher multipoles are also being considered.

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