additions of the two effects for comparison.

We conclude that the variation of P_T^2 with Q^2 from (1) is consistent with the recoil of quarks as they radiate gluons, but that this must be added to terms from the transverse momentum in the fragmentation process. The data are not consistent with a constant P_T^2 as the simple quark model suggests.

The angular distribution is a more sensitive test but its use must await a prescription for combining the two dominant effects according to a full QCD theory.

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Meson Structure Functions from High-Transverse-Momentum Hadron Interactions

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Data on large-transverse-momentum π^0 production by proton, pion, and kaon beams is used to extract the large-x behavior of meson structure functions. Good agreement is found with the proton and pion quark distribution functions obtained from deep inelastic and dimuon production experiments. This study provides the first look at the kaon structure function. A significantly faster falloff is found as x approaches unity than for that of pions.

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Recent experimental results on dilepton production by pion beams has made possible the extraction of the pion structure function.[1–4] If we assume the basic Drell-Yan process of quark-antiquark annihilation, the cross section is proportional to the product of pion antiquark and proton quark structure functions, each evaluated at a unique x value given by the dilepton kinematics. A further indication of the $q\bar{q}$ annihilation comes from the spin alignment exhibited by the dilepton decay angular distribution.[5] In addition, the large-x behavior of this alignment may even provide some information on the scale-breaking effects due to higher twist effects associated with quark binding in the pion.[6] The absolute normalization, however, is not so certain. It has been shown[7] that higher-order quantum chromodynamics (QCD) effects, namely collinear gluon radiation, can produce at least a factor of 2 uncertainty in normalization at present energies. Even so, the overall picture provides a rather satisfying description for large-mass dilepton production in terms of a basic process folded with quark distribution functions.

It has long been recognized[8] that the same type of picture, augmented by quark fragmentation functions available from e^+e^- annihilation into hadrons, could possibly give a correct description of large-transverse-momentum particle production in hadron-hadron interactions. If we are to use the purely hadronic data to extract quark distribution functions, we must face two well-known problems. One is purely technical —the additional kinematic freedom in a basic two-body scattering subprocess followed by fragmentation into the observed hadron. This means that when all external variables are fixed, the cross section still involves a two-dimensional integral over the initial hadrons' distribution functions. We present here a method of extracting the large-x behavior of these functions directly, avoiding the prejudices associated with arbitrary parametrizations. The second difficulty is more fundamental. The momentum-transfer dependence of the cross section predicted by the lowest-order QCD quark-quark scattering is $P_\perp^{-4}$, whereas the data are closer to $P_\perp^{-8}$. It has been shown[9] that a combination of effects, including scale breaking in distribution and fragmentation functions, as well as the quark-gluon coupling, intrinsic parton transverse momenta, and gluons as initial- and final-state partons, can all conspire to bring the theory into agreement with experiment at present energies. In our analysis, we use a ratio of cross sections which minimizes the effect on our results. Future work will include a detailed study of these scale-breaking effects.

The cross section for beam + target → hadron (high $P_\perp$) + anything is written[10]

$$E\frac{d\sigma}{d^3P_H}=\iint dx_1\,dx_2\,f^B(x_1)f^T(x_2)\frac{1}{\pi z}\frac{d\hat{\sigma}}{d\hat{t}}D(z)\,. \qquad (1)$$

The basic process assumes that a parton from the beam with momentum fraction $x_1=2P_1/\sqrt{s}$ scatters from a target parton with x_2. The cross section for momentum transfer $\hat{t}$ in the basic process is $d\hat{\sigma}/d\hat{t}$. A scattered parton with momentum fraction $x_3=2P_3/\sqrt{s}$ then fragments into the observed hadron with $P_H=zP_3$ according to the fragmentation function $D(z)$. The integration region $0<x_1,x_2<1$ is cut off on the low-x region by a boundary curve corresponding to $z=1$. In terms of the scaling variables $x_\parallel=2P_\parallel/\sqrt{s}$ and $x_\perp=2P_\perp=2P_\perp/\sqrt{s}$ for the produced hadron momentum components, the boundary is

$$x_2=x_1(x_R-x_\parallel)/(2x_1-x_R-x_\parallel) \qquad (2)$$

with $x_R\equiv(x_\parallel^2+x_\perp^2)^{1/2}$. As $x_R\to 1$, the integration region shrinks to a small wedge in the corner of x_1-x_2 space. We use data on high-$P_\perp$ π^0 production by proton and $\pi^\pm$ beams on protons at lab momenta of 100 and 200 GeV/c.[11] The data cover the ranges $-0.2<x_\parallel<0.8$ and $0.1<x_\perp<0.5$. We consider the beam ratios for protons to pions, denoted by $R(P/\pi)$, in which the overall $P_\perp$ dependence cancels. The trend of this ratio is downward as $x_\parallel$ and/or $x_\perp$ become large, indicating that pions have more large-x constituents than protons. We seek to make this statement quantitative by extracting information directly from these data on the pion structure function, in a manner analogous to the done for dilepton production. The usual method is to parametrize the pion struction function in some way, and adjust parameters to fit the ratio. We will exploit the fact that at large $x_\parallel$ or $x_\perp$ the integration region may be small enough to pull out the integrand and evaluate at some average values $\bar{x}_1$ and $\bar{x}_2$. For π^0 production, the fragmentation function will cancel in the ratio. A similar cancellation is expected for the cross sections,[10] aside from possible interference terms and overall normalization which will be examined later for specific models. The final result is

$$R(P/\pi)=f^P(\bar{x}_1)/f^\pi(\bar{x}_1)\,, \qquad (3)$$

where target proton structure functions at $\bar{x}_2$ have also canceled.

As a first approximation, we neglect the x_1-x_2 variation of cross sections and fragmentation functions, and calculate average x values due to the phase-space boundary curve [Eq. (2)] alone.[12] We obtain

$$\bar{x}_1=\frac{x_R+x_\parallel}{2}+(1-x_R)\left\{(2-x_R+x_\parallel)\left[1-\frac{x_\perp^2}{4(1-x_R)}\ln\left(1+\frac{4(1-x_R)}{x_\perp^2}\right)\right]\right\}^{-1} \qquad (4)$$

with a similar expression for $\bar{x}_2$ with $x_\parallel\to -x_\parallel$. As expected, $\bar{x}_1\to 0.5$ at small x_R. As a check on this method, we have performed integrals with some typical distribution functions, and checked against this extraction procedure. For example, we use $f_p(x)=(1-x)^3$ and $f^\pi(x)=1-x$. Then we should get $R(P/\pi)=(1-\bar{x}_1)^2$ if the method works. Obviously, it shoud be more reliable for $\bar{x}$ near 1. To get a quantitative measure of this effect, we also calculate $\Delta x_1\equiv[\overline{x_1^2}-(\bar{x}_1)^2]^{1/2}$. There is some x_2 dependence, but

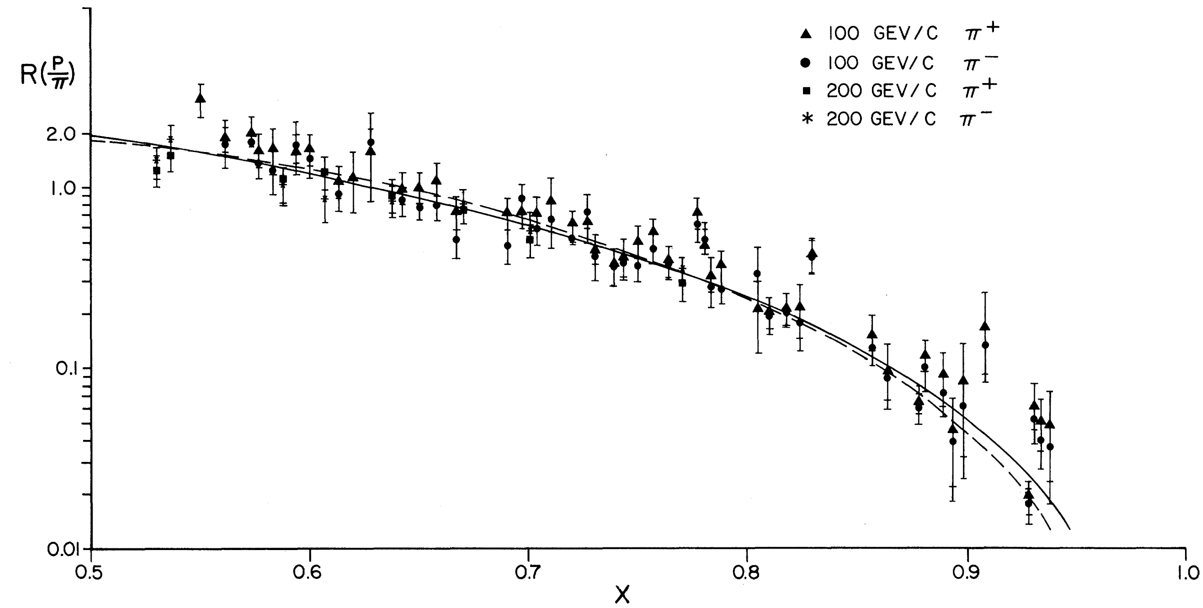

FIG. 1. Ratio of proton to pion structure functions vs x. Data from beam ratio of π^0 production cross sections. The curves are described in the text.

to a fairly good approximation, we can write

$$\Delta x_1 = 0.6(1 - \bar{x}_1). \tag{5}$$

We then fit the ratio of computed integrals to a form $R = A(1 - \bar{x}_1)^N$ with points located at $\bar{x}_1 \pm \Delta x_1$. The parameters so obtained are $A = 1.9 \pm 0.5$ and $N = 2.16 \pm 0.12$. The interpretation is clear: One can fairly accurately extract the large-x behavior of the structure functions, but the magnitude is unreliable. It seems that the large-$\bar{x}_1$ points with relatively small uncertainties Δx_1 are the major influence on the power behavior near $\bar{x}_1 = 1$, whereas the overall magnitude is sensitive to points near $\bar{x}_1 = 0.5$ which sample the falling distribution functions essentially in the entire range $0 < x_1 < 1$. Keeping these points in mind, we proceed to an analysis of the data.

The ratio $R(P/\pi)$ as a function of $\bar{x}_1$ is presented in Fig. 1. Points for both π^+ and π^-, and for both energies, lie approximately along the same curve, indicating approximate scaling and negligible interference terms in the underlying parton cross sections. The fact that all of these points can form a smoothly varying function of $\bar{x}_1$ is somewhat of a triumph for the analysis in itself. Since widely different values of external parameters $x_\parallel$ and $x_\perp$ can give almost the same $\bar{x}_1$ according to (4), this indicates that the average x_1 sampled by each cross section point is the essential factor which determines the effective argument of the distribution functions.

We compare this with the ratio of proton to pion structure functions obtained in electromagnetic probe experiments.[1,3] The result is then multiplied by a constant normalizing factor (1.75) to obtain the best fit to our experimental $R(P/\pi)$. This function is shown by the dashed line in Fig. 1, and is virtually indistinguishable from the best fit to the data points (solid line). We regard this as strong evidence[13] that the distribution functions measured in large-$P_\perp$ hadronic interactions are the same as the quark distribution functions extracted via electromagnetic probes.[14]

We also compare our result to the power-law parametrization for lepton probe experiments. We perform a fit $R = A(1 - x)^B$ and find $A = 9.4 \pm 0.5$, $B = 2.3^{+0.3}_{-0.5}$, with the uncertainty in the power almost entirely due to the uncertainty in $\bar{x}_1$ from (5). This fit is shown by the solid line in Fig. 1. This translates to an effective power of $0.7^{+0.5}_{-0.3}$ for the pion structure function alone, assuming the usual[10,13] $(1 - x)^3$ behavior of u quarks in a proton. The corresponding analysis from the dilepton determination of the pion structure function yields a power 1.01 ± 0.05. We must emphasize again that these power-law fits are not required by our data analysis method. On the contrary, we hope to use this theoretically unbiased extraction to test for consistency with

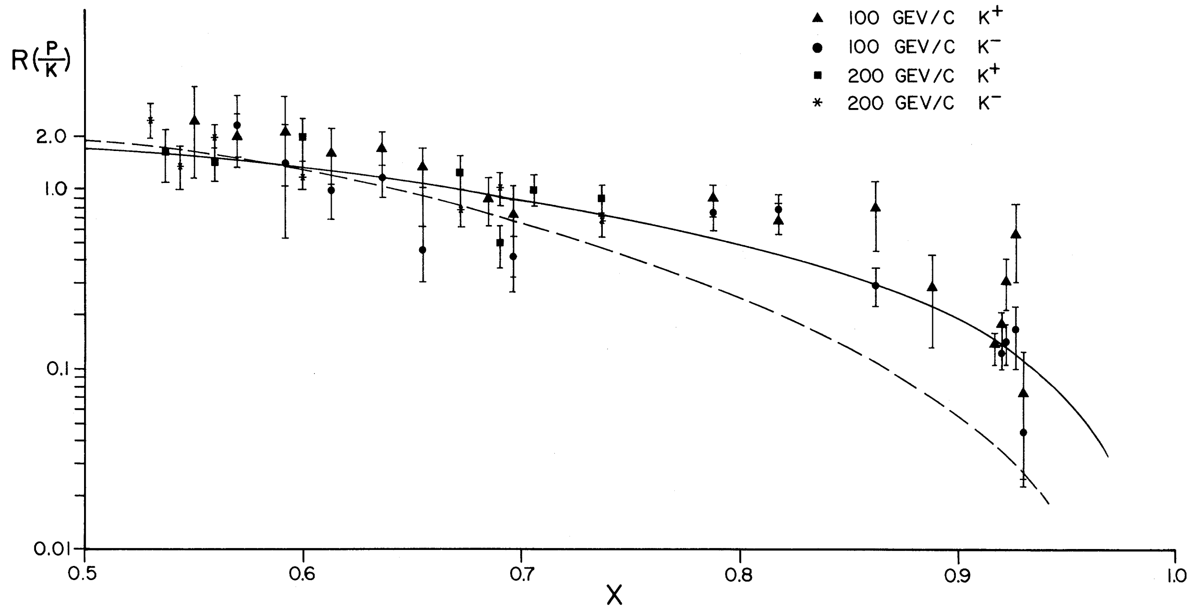

FIG. 2. Ratio of proton to kaon structure functions vs x. Data from beam ratio of π^0 production cross sections. The solid curve is described in the text. The dashed curve is from Fig. 1 for comparison.

more general behavior near $x = 1$.

For example, there exist considerable theoretical arguments[15] that quark structure functions from mesons must involve only even powers of $1-x$ as $x \to 1$. It has been shown[5] that an alternative fit to the pion structure function from dilepton data can be made in the form

$$f^{\pi}(x) \underset{x\to 1}{\longrightarrow} (1-x)^2 + C/Q^2. \tag{6}$$

The second term is associated with binding effects of the quark in a pion, and give rise to a change in the dilepton angular distribution[6] from $1+\cos^2\theta$ to $\sin^2\theta$ as $x \to 1$. This effect has been seen,[5] suggesting that Eq. (6) may be the correct form of the pion structure function. However, the Q^2 variation in Eq. (6) cannot yet be detected in the dilepton data.

Since the hadron interactions probe much larger Q^2 regions than dileptons (Q^2 up to 56 GeV^2 in these data), we should in principle be more sensitive to scale-breaking effects. Efforts are now underway to examine the possibility that the scale-breaking parts of the proton structure function may be masking the effect of those for the pion in our ratio.

Finally, we examine the corresponding cross sections for $K^{\pm}$ beams. The ratio data[16] are shown in Fig. 2. Since the uncertainties in data are considerably larger than for pion beams, we see somewhat more scatter in the points. A fit to a power-law behavior gives $B = 1.4 \pm 0.3$ and is shown by the solid line. The corresponding pion ratio function is shown by the dashed curve. We see that the kaon and pion structure functions must have similar behavior at small x, but as $x \to 1$ the kaon function must fall more rapidly than the pion function. The effective power-law exponent is $n = 1.6 \pm 0.3$ (compared with 0.7 for the pion). Actually, the kaon function is an "effective" value, a combination of strange and nonstrange quark distributions. The weighting of these is of course unequal, since the s-quark fragmentation into a π^0 is much smaller than for u or d quarks.[17] We are presently analyzing η^0 production data in the same way. We hope to be able to extract the difference between strange- and nonstrange-quark distribution functions for K mesons.

In conclusion, we have been able to extract the $x \to 1$ behavior of meson structure functions from purely hadronic interactions. The agreement with those extracted from deep inelastic and dilepton data for pions is suggestive that large-$P_{\perp}$ hadrons may also be produced by hard scattering of quarks. The corresponding distributions for kaons are different. The possibility that the large Q^2 variation available in hadronic interactions can be used to explore scaling violations is being pursued.

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[13]There remains the possibility that some other constituents accidentally have the same ratios as for lepton-probed quarks and dominate the P_t^{-8} behavior of strong processes in this P_t and energy range.
[14]M. Dris *et al.*, Phys. Rev. D 19, 1362 (1979), and M. D. Corcoran *et al.*, Phys. Rev. Lett. 44, 514 (1980), have extracted an analogous quantity from two-jet production cross-section beam ratios. In this case, the kinematics again gives unique x values, and agreement with the shape of the pion distribution function of Ref. 3 is shown for $x < 0.6$.
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Comment on Chiral and Conformal Anomalies

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It is shown that the trace anomaly is also identified with the Jacobian factor for the functional measure under the conformal transformation in the path-integral formalism. The path-integral formulation of anomalous Ward-Takahashi identities is then translated into a simple algebraic characterization of chiral and conformal anomalies. This exemplifies some of the common features shared by the topological and nontopological anomalies.

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The anomalous chiral Ward-Takahashi (W-T) identities[1] (Adler-Bell-Jackiw anomaly) can be formulated in a simple manner with the aid of the index theorem[2] in the path-integral formalism.[3] It was also recognized that the chiral anomaly is related to the fact that the covariant "energy" operator $\not{D}$ and γ_5 cannot be simultaneously diagonalized.[4] In the present note I show that the trace anomaly[5] is also formulated as the Jacobian factor arising from the conformal transformation in the functional measure.[6] I then translate the path-integral formulation of anomalous W-T identities into a simple algebraic characterization of chiral and conformal anomalies.

I start with the simplest conformal-invariant theory of a "free" scalar field in the background gravitation field (R is the scalar curvature):

$$S \equiv \tfrac{1}{2}\int [\, g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \tfrac{1}{6}R\varphi(x)\varphi(x)](\sqrt{-g})d^4x \quad (1)$$

suitably continued to the Euclidean space. The Wick rotation is performed in the *local Lorentz frame*[7] as $h_0{}^\mu \to +ih_4{}^\mu$ and $\sqrt{-g} \equiv \det h_\mu{}^a \to -i\sqrt{g} \equiv -i\det h_\mu{}^a$, where $h_a{}^\mu(x)$ is the tetrad satisfying $h_a{}^\mu(x)h^{\nu a}(x) \equiv g^{\mu\nu}(x)$.

As is well known, (1) is invariant under the local conformal transformation

$$g^{\mu\nu}(x) \to e^{2\alpha(x)}g^{\mu\nu}(x) \quad [g_{\mu\nu}(x) \to e^{-2\alpha(x)}g_{\mu\nu}(x)],$$
$$\varphi(x) \to e^{\alpha(x)}\varphi(x), \quad (2)$$

without the use of the equation of motion. This

