

## Study of Nuclear Response Tail by High Threshold Reactions in Muon Capture

Nimai C. Mukhopadhyay

*Swiss Institute for Nuclear Research, CH-5234 Villigen, Switzerland, and Department of Physics,  
Technion-Israel Institute of Technology, Haifa, Israel*

and

Jörg Hadermann

*Federal Institute for Reactor Research,  
CH-5303 Würenlingen, Switzerland*

and

Paul Singer

*Department of Physics, Technion-Israel Institute of Technology, Haifa, Israel*  
(Received 29 February 1980)

This Letter demonstrates the importance of two reactions with *high* threshold-fission of nuclei around the lead region and high-multiplicity nucleon emission—in studying the high-energy tail of the nuclear response function in muon capture. Conventional impulse approaches and the recently proposed absorption model of the nonimpulse variety are shown to give significantly different yields for these reactions.

PACS numbers: 23.40.-s

In processes such as muon and radiative pion capture, weak or electromagnetic reactions are used to transfer typically a few tens of MeV of energy to the nucleus. These could be useful to study the poorly-known<sup>1</sup> high-energy tail of the nuclear response function. Already measured inclusive spectra<sup>2,3</sup> are indicative of a large high-energy excitation strength which is difficult to explain in the conventional impulse theories.<sup>4,5</sup> Recently a remarkably common phenomenological basis has been established<sup>6</sup> for the high-energy behavior of response functions in weak, electromagnetic, and strong nuclear processes. The apparent failure of the impulse picture points, on one hand, to the possibly important role of the multinucleon processes<sup>7,8</sup> at high-energy transfer, and, on the other hand, to the difficulty of treating hadronic final-state interactions at these kinematics.

In this paper, we consider muon capture in complex nuclei, and show that the study of specific reaction channels, with high thresholds, can be used to discriminate between currently available models<sup>4-6</sup> of the nuclear excitation function at large energy transfer. Our chosen reactions are the muon-induced fission in nuclei around Pb, and high-multiplicity ( $x \geq 6$ ) nucleon emission. Their observation would demonstrate a large probability for the nucleus to absorb energies above a high threshold. In our calculation, we use two typical impulse models<sup>4,5</sup>, here-

after called *A* and *B*, which give an average nuclear excitation energy of about 15 and 18 MeV, respectively: Model *A* is obtained<sup>4</sup> by using single-particle momentum distributions generated in the Woods-Saxon well. Model *B*<sup>5</sup> is based on a phenomenological nucleon momentum distribution; it does better than *A* in explaining the observed *low-energy* ( $E \leq 30$  MeV) inclusive nucleon spectra. To exemplify the class of nonimpulse models, we take 1% of the total strength at nuclear energy transfer of 80 MeV (model *C*). This figure, predicted<sup>8</sup> for  $E \geq 80$  MeV, relating muon and pion absorptions, is several orders of magnitude higher than what is obtained in the conventional impulse theories.

We first consider muon-induced fission. The threshold here is set by the fission barrier of the compound nucleus produced. The inclusive fission probability  $P_f$  for muon capture in the target nucleus ( $A, Z$ ) is given by

$$P_f = \sum_i \int_{\mathcal{B}_f^i}^{E_0^i} P_f^i(E) I^i(E) dE, \quad (1)$$

where  $I^i(E)$  and  $P_f^i(E)$  are, respectively, the probabilities<sup>4,5,9</sup> for producing a compound nucleus  $i$  at an excitation energy  $E$  and its under-going fission;  $\mathcal{B}_f^i$  and  $E_0^i$  are its fission barrier and the maximum excitation energy available. The summation extends over the different compound nuclei produced. Fission in heavy subactinide nuclei at excitation energies in the range of

30 to 100 MeV is well described<sup>9</sup> by the relation

$$P_f^i(E) \approx \Gamma_f^i / \Gamma_n^i \propto \exp\{2[a_f^i(E^{*i} - B_f^i)]^{1/2} - 2[a_n^i(E^{*i} - B_n^i)]^{1/2}\} \quad (2)$$

derived from the asymptotic nuclear entropy ansatz  $S^2 = 4a(E - \Delta)$ ,  $a$  and  $\Delta$  being the nuclear level density and ground-state mass correction.<sup>9</sup>  $\Gamma_f^i$  and  $\Gamma_n^i$  are the fission and one-neutron emission widths, and  $E^{*i} = E - \Delta^i$ ;  $B_f^i$  and  $B_n^i$  in (2) are liquid-drop thresholds. The potential usefulness of muon-induced fission as an indicator of the nuclear high-energy excitation strength is contained in the exponential rise of its probability above threshold.

For fission, the *optimal choice* of the target is dictated by the following considerations: (1) In order to have equilibration following muon capture, without much dissipation of energy by preequilibrium emission, the muon density must overlap appreciably with the nuclear one. (2) The fission barrier should be large enough so as to distinguish various models of nuclear response function at high-energy transfer, but not too large. This sets  $B_f$  around 15–20 MeV. Nuclei around  $A \approx 200$  fulfill both requirements.

Below we take the example of  $\mu$  capture by <sup>209</sup>Bi. The level-density parameters in (2) are taken from the analysis of pion-induced fission,<sup>9</sup> a reaction characterized by an average compound-nuclear excitation energy of about 60 MeV. The ratio  $a_f/a_n$  is thus taken to be 1:1, with an uncertainty of  $\pm 1\%$ <sup>9</sup>;  $a_n$  is allowed to vary<sup>9</sup> between  $A/7$  and  $A/10$ . The latter is also corroborated by the studies of the high-energy  $\alpha$ -induced reactions.<sup>10</sup>

A crude estimate for the fission probability  $P_f$  may be obtained by neglecting the complexity of the preequilibrium emission chain, and estimating the equilibration probability as simply the nonescape probability  $p_c$  of the primary neutron. Assuming the nucleus to be a sphere, of radius 7 fm in our example, and the neutrons produced isotropically inside it, one obtains  $p_c$  equal to 0.63 and 0.47 for neutron mean free paths of 4 and 7 fm, respectively. Taking  $p_c = 0.5$ , we obtain

$$P_f^A \approx 4 \times 10^{-9}, \quad P_f^B \approx 2 \times 10^{-6}, \quad (3)$$

$$P_f^C \approx 5 \times 10^{-5},$$

where each figure has an uncertainty of a factor of 3 due to the imprecisions of  $a_f$  and  $a_n$ .

We now make estimates for  $P_f$  by a more rig-

orous consideration of the nuclear preequilibrium decay. This is done in the framework of the exciton model.<sup>11</sup> We have computed the equilibrium energy distributions in various generations of compound nuclei, produced at the end of each preequilibrium channel. Results of this calculation, for the three model response functions discussed above, are

$$P_f^A \approx 10^{-10} \approx 0, \quad P_f^B \approx 2 \times 10^{-6}, \quad (4)$$

$$P_f^C \approx 2 \times 10^{-5}$$

with an uncertainty of a factor of 3 in each case. Thus, these figures are not too different from those in (3). The last two numbers are well within the limits of observability. The distinction between  $A$  and  $(B, C)$  is clearly drastic. The actual difference between the values of  $P_f$  in models of the class  $(A, B)$  and that of  $C$  is *bigger* than the figures in (4) would indicate; our neglect of the strength distribution in  $C$  makes  $P_f^C$  only a *lower* bound.

We now turn to the other high threshold reaction—large-multiplicity nucleon emission following muon capture. The threshold here is set by the binding energy of  $x$  nucleons, where  $x$  is the nucleon multiplicity. For the purpose of discriminating the high-energy behavior of various nuclear response functions,  $x$  should be greater than 5; thus, for  $x = 8$ , the threshold is above 60 MeV in heavy nuclei, and its observation is an indication of nuclear excitation process at  $E > 60$  MeV. The optimal choice of target is dictated here mainly by the equilibration criterion. Thus, the nuclei around Pb remain a good choice. It also allows a comparison of the high-energy excitation strength with the fission processes discussed earlier.

The calculation of probabilities for the emission of nucleons of varying multiplicities can be done in two ways. Given the nuclear response function, one can follow the preequilibrium decay chains,<sup>11</sup> and calculate the nuclear evaporation by statistical models.<sup>5,9</sup> Alternatively, one can use the experimental reaction cross-section data<sup>12</sup> [ $(p, xnyp)$  in our case], and fold with the calculated nuclear excitation function. The re-

sults are as follows:

$$P^A(x \geq 6) \simeq 0, \quad (5a)$$

$$P^B(6n) \simeq 10^{-3}, P^B(7n) \simeq 10^{-5}, P^B(8n) \simeq 0, \quad (5b)$$

$$P^C(6n) \simeq 10^{-3}, P^C(7n) \simeq 2 \times 10^{-3}, \quad (5c)$$

$$P^C(8n) \simeq 7 \times 10^{-4},$$

$P^i(x)$  being the probability for the  $(\mu^-, xn)$  reaction for the model  $i$ . The uncertainty in these numbers is about a factor of 2, coming from the imprecise knowledge of the nuclear level density parameters, or the reaction data. Obviously, it is at the *higher* multiplicities that one can expect a more decisive test for the high-energy tails of the various model response functions. The results for  $x \geq 7$  are clearly very different in the three models.

We now make some observations concerning the experimental possibilities and presently available information. Since low-energy muon beams have pion contamination, a serious problem is the resultant background, since pions stopping in heavy nuclei<sup>9</sup> also give rise to high nucleon multiplicities, and relatively large fission probabilities ( $\sim 10^{-2}$  and  $7 \times 10^{-3}$  in Bi and Hg, respectively). Clearly, the success of the muon experiment requires a very high pion rejection in the muon beam. Fortunately, a pion rejection rate better than 1 in  $10^4$  is easily achievable.

While there are so far no muon-induced fission experiments to bear on our conjectures here, careful radiochemical studies on the high multiplicity muon reaction have begun. Results for  $\mu$  capture in  $^{209}\text{Bi}$  are as follows<sup>13</sup>:

$$\begin{aligned} P(6n) &= (15 \pm 1.5) \times 10^{-3}, \\ P(7n) &= (1.4 \pm 0.2) \times 10^{-3}, \\ P(8n) &= (2.8 \pm 0.2) \times 10^{-3}. \end{aligned} \quad (6)$$

This has two immediate consequences. First, a comparison of (6) with (5a)–(5c) shows that (6) disagrees with  $A$  and  $B$  for the high multiplicities, and favors  $C$ , even as  $C$  is inaccurate in detail. This experiment definitely indicates a substantially high nuclear excitation probability at  $E \sim 70$ – $80$  MeV, far above the impulse predictions. Second, it suggests nuclear equilibration at reasonably large energy, and, thus, implies a measurable fission probability for Pb, at least at the level  $P_f^C$  in (4). Checking the latter experimentally will constitute a *consistency test* for the nonimpulse models.

We should stress that our use of the 1% strength

concentrated at 80 MeV in type- $C$  models is mainly to illustrate the dramatic threshold effects implicit in the high-energy tail of the nuclear response function. More accurate tests of these models will require a detailed picture of the strength distribution as a function of energy, as yet unavailable for the class- $C$  models.<sup>14</sup> Should the 1% nuclear excitation strength be distributed over the energy interval 80–100 MeV, this would give fission probabilities larger than  $P_f^C$  in Eq. (4), and would relatively enhance higher multiplicities, due to opening of new channels with larger thresholds.

In summary, we have proposed here two reactions with high thresholds, muon-induced fission and high multiplicity nucleon emission, for nuclei in the Pb region, which can throw light on the nuclear weak response function at high-energy transfer. Conventional impulse theories and the nonimpulse approaches imply different probabilities for such processes. Between the two processes considered, high-multiplicity neutron emission seems to be more efficient in distinguishing between the tails of nuclear response functions obtained in the impulse and nonimpulse theories. Preliminary experimental results on the fission process indicate a preference for the nonimpulse tail. More experiments on the reactions discussed here should test this better, and thus help formulate a more satisfactory theory of the nuclear weak response function valid over the entire regime of capture kinematics. Further work on the apparent inadequacy of the impulse theories is in progress, and will be reported elsewhere.<sup>15</sup>

We are grateful to A. Zglinski, T. Kozlowski, and M. Lifshitz for their valuable collaborations in the exciton-model calculations. We thank them, and T. Fujita, K. Junker, and K. Dubodera, for their critical remarks; H. S. Pruyss and A. Wytenbach, for performing high-multiplicity experiments suggested by one of us (N.C.M.), and keeping us informed of their results; B. Hahn and S. Polikanov, for their valuable advice on the feasibility of the fission experiment. N.C.M. has the pleasure of thanking Paul Singer for arranging a memorable sojourn at the institution where parts of this work were done, under the partial support of the Israeli Commission for Basic Research.

<sup>14</sup>N. C. Mukhopadhyay, Phys. Rep. **30C**, 1 (1977), and in Proceedings of the Eighth International Conference

on High Energy Physics and Nuclear Structure, Vancouver, 1979, to be published, and Nucl. Phys. **A335**, 111 (1980).

<sup>2</sup>T. Kozłowski *et al.*, in Proceedings of the Eighth International Conference on High Energy Physics and Nuclear Structure, Vancouver, 1979 (to be published).

<sup>3</sup>M. Balandin *et al.*, Yad. Fiz. **28**, 582 (1978) [Sov. J. Nucl. Phys. **28**, 297 (1978)]; K. S. Krane *et al.*, Phys. Rev. C **20**, 1873 (1979).

<sup>4</sup>J. Hadermann and K. Junker, Nucl. Phys. **A271**, 401 (1976).

<sup>5</sup>M. Lifshitz and P. Singer, Phys. Rev. Lett. **41**, 18 (1978), and Phys. Rev. C (to be published).

<sup>6</sup>P. Singer, N. C. Mukhopadhyay, and R. D. Amado, Phys. Rev. Lett. **42**, 162 (1979).

<sup>7</sup>P. Singer, Phys. Rev. **124**, 1602 (1961).

<sup>8</sup>J. Bernabéu, T. E. O. Ericson, and C. Jarlskog, Phys. Lett. **69B**, 161 (1977).

<sup>9</sup>N. C. Mukhopadhyay, J. Hadermann, and K. Junker,

Nucl. Phys. **A319**, 448 (1979).

<sup>10</sup>G. Chenevert *et al.*, Nucl. Phys. **A122**, 481 (1968).

<sup>11</sup>T. Kozłowski and A. Zglinski, Nucl. Phys. **A305**, 368 (1978) and references therein. Numbers quoted in (4) and (5) are not drastically altered by changing the initial exciton number from two to four.

<sup>12</sup>*Landolt-Börnstein: Numerical Data and Functional Relationships in Science and Technology*, edited by K. H. Mellwege (Springer, Berlin, 1973), Group 1, Vol. 5b.

<sup>13</sup>H. S. Pruyss and A. Wytttenbach, private communications.

<sup>14</sup>At minimum, this requires a knowledge of the *s*- and *p*-wave pion-nuclear scattering length and volume for the nuclei considered here (as yet unavailable) and an extrapolation of the nuclear excitation strength away from the kinematic end point of the muon-capture reaction.

<sup>15</sup>T. Fujita and N. C. Mukhopadhyay, to be published.

## Evidence Regarding Precritical Phenomena in $^{12}\text{C}$

J. R. Comfort

*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260*

and

W. G. Love

*Department of Physics, University of Georgia, Athens, Georgia 30602*

(Received 19 February 1980; revised manuscript received 2 June 1980)

Data and calculations for  $^{12}\text{C}(p, p')^{12}\text{C}$  reactions at 122 MeV are discussed with respect to predictions and implications of precritical phenomena near pion-condensation thresholds. The evidence is largely unresponsive of the expectations of precritical behavior.

PACS numbers: 21.65.+f, 24.50.+g, 25.40.Ep

The topic of a pion condensate phase of dense nuclear matter, perhaps dynamically stimulated in nuclei, has been of considerable interest for several years.<sup>1,2</sup> Experimental interest has been sharpened recently by suggestions that precritical phenomena related to the proximity of the critical point might be observable.<sup>3,4</sup> The phenomenon that seems to be most amenable to observation is a modification of differential cross sections at large momentum transfers for inelastic nucleon-nucleus scattering into channels that carry pion quantum numbers ( $\Delta S = \Delta T = 1$ ).

Toki and Weise<sup>4</sup> consider  $(p, p')$  reactions to  $1^+$  states in  $^{208}\text{Pb}$ . The one-pion-exchange (OPE) interaction is modified with a Landau parameter  $g'$  to account for additional short-range effects and is divided by a momentum-dependent polarization denominator constructed from considerations of virtual intermediate nucleon-hole and isobar-

hole excitations. Greatly enhanced cross sections<sup>4</sup> for momentum transfers  $q \sim 250\text{--}500$  MeV/ $c$  are expected for  $g'$  above but near a critical value of about 0.33. The authors<sup>4</sup> understand this to arise from collective particle-hole contributions from a large number (e.g., 30) of oscillator shells, an effect that would not be reproducible in conventional calculations.

Since  $N \neq Z$  in  $^{208}\text{Pb}$ , there is an ambiguity regarding the separation of isoscalar and isovector excitation modes, a problem made worse by a general lack of understanding of the isoscalar spin-flip mode. On the other hand, the symmetric nucleus  $^{12}\text{C}$  has several states where the terms in the effective interaction can be separated better and tested individually. Of particular interest is the excitation of the  $1^+$ ,  $T = 1$  state at 15.11 MeV and the  $2^-$ ,  $T = 1$  state at 16.58 MeV. Calculations of the polarization effect for the