Quark Beta Decay and the Cooling of Neutron Stars

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It is shown that the beta decay of quarks in degenerate quark matter is kinematically allowed. The resulting neutrino emissivity is dramatically larger than that of neutron matter and comparable to that of matter with pion condensate. Thus a star with a quark-matter core would cool at a rate comparable to that for a star with a pion-condensed core, and much faster than an ordinary neutron star.

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In recent years, especially since the launch of the Einstein X-ray Observatory, there has been considerable interest in the cooling of neutron stars. The predominant cooling process early in a neutron star's life is neutrino emission. Since matter with a pion condensate has a considerably greater neutrino luminosity than ordinary neutron-star matter,[1] a number of authors have suggested that the low values for the upper limits on surface temperatures of neutron stars can be understood only if neutron stars have pion-condensed cores.[2-5] In this Letter I will show that an alternative possibility for understanding low surface temperatures is that neutron stars have quark-matter cores.[6,7] One can find the neutrino luminosity of quark matter to be comparable to that of matter with a pion condensate, and conclude that stars with quark-matter cores would cool at rates comparable to those for stars with pion-condensed cores. The essential physical reason for the high neutrino luminosity of quark matter is that, while in ordinary neutron-star matter the simple beta processes $n \rightarrow p + e^- + \bar{\nu}_e$ and $p + e^- \rightarrow n + \nu_e$ cannot occur because of the impossibility of conserving energy and momentum, the analogous processes for quarks can occur.

Let us first discuss the composition of quark matter and the neutrino processes that can occur. We consider three-component (u, d, and s) quark matter in β equilibrium, the condition for which is

$$\mu_d = \mu_s = \mu_u + \mu_e, \tag{1}$$

where μ's are the chemical potentials of the quarks and of the electrons, which are assumed to be the only leptons present.[8] If the masses of these particles and interactions are neglected, Eq. (1) and the electric charge neutrality condition imply for the composition of the matter,[9]

$$n_u = n_d = n_s = n_b,\ n_e = 0; \tag{2}$$

in terms of the number densities n, and each quark component has the same Fermi momentum

$$p_F(q) = 235(\rho/\rho_0)^{1/3}\,\mathrm{MeV}/c, \tag{3}$$

where $\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} is nuclear-matter density and $n_b = (n_u + n_d + n_s)/3$ is the baryon-number density. Because of the finite s-quark mass, and quark-quark interactions the composition is modified from that given by Eq. (2), and one may expect leptons to be present to preserve charge neutrality. Since typical Fermi energies of quarks are high compared with the electron rest mass, a slight difference between μ_d and μ_u (or μ_s and μ_u) implies that the electrons will generally have a relativistic Fermi energy.

The simplest neutrino processes are the direct β-decay reactions

$$d \rightarrow u + e^- + \bar{\nu}_e, \tag{4}$$

$$u + e^- \rightarrow d + \nu_e. \tag{5}$$

For the process to occur, it must conserve momentum. At low temperatures where the momenta of the emitted neutrinos are small and the quarks and electrons which can react all lie close to their respective Fermi surfaces, this requires that one be able to construct a triangle from momenta of magnitudes $p_F(u)$, $p_F(d)$, and $p_F(e)$. Here $p_F(i)$ is the Fermi momentum of the species i. If the quarks are noninteracting and massless, conservation of energy and momenta requires that the u, d, and e momenta be collinear, but if either the finite quark mass or interactions are taken into account the condition is less restrictive.

Let us first consider the effects of interactions. To lowest order in $\alpha_c = g^2/16\pi$ (where g is the quark-gluon coupling constant) one has[10]

$$\mu_i = \left(1 + \frac{8}{3\pi}\alpha_c\right) p_F(i), \quad i = u, d; \tag{6}$$

if one neglects finite-temperature corrections. Note that $\mu_e \simeq p_F(e)$ for the electrons, and therefore from Eqs. (1) and (6) we have $p_F(d) - p_F(u) - p_F(e) \simeq -(8/3\pi)\alpha_c p_F(e) < 0$. This means that one can construct a momentum-conserving triangle from $p_F(d)$, $p_F(u)$, and $p_F(e)$, and thus, Reactions (4) and (5) can proceed.

The calculation of the neutrino emissivity proceeds in the standard way. Since interaction effects are essential in the processes under consideration, we treat that the quarks in the framework of Landau Fermi-liquid theory. Therefore, for example, the decaying particle in (4) is regarded as a d-quark quasiparticle rather than a bare d quark. The calculations are similar to those for the noninteracting case, except that the quark chemical potentials are given by Eq. (6).

I assume the Weinberg-Salam model of weak interactions extended to semileptonic processes.[11] For the processes with low momentum transfer, the interaction Lagrangian density may be expressed in the form of a current-current interaction, $\mathcal{L}_{Wx} = (G/\sqrt{2})\cos\theta_C \bar{u}\gamma^\mu(1-\gamma_5)d\bar{e}\gamma_\mu(1-\gamma_5)\nu_e$, where the weak-coupling constant is $G \simeq 1.435 \times 10^{-49}$ erg cm^3 and θ_C is the Cabibbo angle ($\cos^2\theta_C \simeq 0.948$).[12] The transition rate for beta decay of the d quark is $W_{fi} = V(2\pi)^4\delta^{(4)}(p_1 - p_2 - p_3 - p_4)|M|^2/\prod_{i=1}^4 2E_iV$, where the four-vectors, $p_i = (E_i, \vec{p}_i)$, are numbered from $i=1$ to 4 to denote those of d, $\bar{\nu}_e$, u, and e, in this order. V is the normalization volume and $|M|^2$ is the squared invariant amplitude averaged over the initial d-quark spin (σ_1) and summed over the final spins of the u quark (σ_3) and the electron (σ_4), $|M|^2 = \frac{1}{2}\sum_{\sigma_1,\sigma_3,\sigma_4}|M_{fi}|^2 = 64G^2\cos^2\theta_C(p_1\cdot p_2)(p_3\cdot p_4)$.

Note that when quark masses and interactions are neglected, the $\vec{p}_i$ must be collinear and $|M|^2$ vanishes. Since the neutrinos are produced thermally, we can neglect the neutrino momentum in the momentum conservation law. Then, to the lowest order in α_c, we have an approximate form for the four-vector products $(p_1\cdot p_2)(p_3\cdot p_4) \simeq E_1E_2 \times E_3E_4[1-(|\vec{p}_1|/E_1)\hat{p}_1\cdot\hat{p}_2](16\alpha_c/3\pi)$, where $\hat{p}_i = \vec{p}_i/|\vec{p}_i|$, and the energy-loss rate due to the neutrino emission process (4) is

$$\epsilon = 6V^{-1}\left(\prod_{i=1}^{4} V\int \frac{d^3p_i}{(2\pi)^3}\right)E_2W_{fi}n(\vec{p}_1)[1-n(\vec{p}_3)][1-(\vec{p}_4)], \tag{7}$$

where n is the Fermi function. The factor 6 accounts for the color (3) and the spin (2) degrees of freedom of the initial d quark. The emissivities due to processes (4) and (5) are equal, and with the help of standard methods for calculating nonequilibrium processes in degenerate Fermi systems we find their sum to be

$$\epsilon_{q\beta} \simeq \tfrac{914}{315}\,\frac{G^2\cos^2\theta_C}{\hbar^{10}c^6}\,\alpha_c p_F(d)p_F(u)p_F(e)(k_BT)^6. \tag{8}$$

The temperature dependence of the emissivity can be understood easily. Each degenerate fermion gives one power of T from the phase-space integral $[d^3p_i \to p_F(i)^2dE_i \propto T]$. Thus we have T^3 from the quarks and the electron. In addition, the phase-space integral for the neutrino gives $d^3p_2 \propto (E_2)^2dE_2 \propto T^3$. One power of T from the emitted neutrino energy, E_2, cancels a factor T^{-1} from the energy-conserving δ function. Since the momenta of the degenerate particles are restricted to lie close to their respective Fermi surfaces, the momentum-conserving δ function is temperature independent. Lastly, the transition probability is also independent of the energy variable. Altogether, we thus have $\epsilon_{q\beta} \propto T^6$.

To estimate the magnitude of the emissivity (8) we assume $p_F(d) \simeq p_F(u) \simeq p_F(s)$, as in the noninteracting case. Then $p_F(e) \simeq 3^{1/3}Y_e^{1/3}p_F(q)$ where $Y_e \equiv n_e/n_b$ is the number of electrons per baryon and we have

$$\epsilon_{q\beta} \simeq 8.8\times10^{26}\alpha_c(\rho/\rho_0)Y_e^{1/3}T_9^6 \tag{9}$$

where T_9 is the temperature in units of 10^9 K. We still have two unknown parameters, α_c and Y_e. The value of the strong-coupling constant, which is momentum dependent, is not well determined experimentally. Analysis of charmonium decay suggests the value $\alpha_c \simeq 0.065$ at energy 3.1 GeV,[13] while on the other hand, the Massachusetts Institute of Technology (MIT) bag model of the light-hadron mass spectrum gives $\alpha_c \simeq 0.55$.[14,15] In view of the typical momentum region of interest indicated by Eq. (3), we take the value $\alpha_c = 0.1$ as being a reasonable order of magnitude estimate. We use the value $Y_e = 0.01$, which is typical of what is expected for dense matter. With these values, we have $\epsilon_{q\beta} \sim 1.9\times10^{25}(\rho/\rho_0)T_9^6$ erg cm^{-3} s^{-1}.

Let us now consider finite-quark-mass effects. These may be shown to be unimportant for u and d quarks as long as $m_u, m_d \lesssim 100$ MeV. The presence of s quarks gives rise to neutrino emission via the processes $s \to u + e^- + \bar{\nu}_e$ and $u + e^- \to s + \nu_e$, and the large s-quark mass allows the momenta of the interacting particles to deviate appreciably from collinearity, which tends to increase the matrix element. On the other hand the decay rate is Cabibbo suppressed ($\propto \sin^2\theta_C$), and I estimate that the neutrino energy-loss rate will be at most of the same order-of-magnitude as that from the processes (4) and (5).

One can easily estimate the orders-of-magnitude of the emissivities from other processes, such as the quark-modified URCA process, quark neutrino pair bremsstrahlung,[16] and color plasmon decay in quark matter, which turn out to be negligible compared with (9).

The neutrino emissivity of quark matter is far larger than that of ordinary neutron-star matter. The neutrino emissivity from the nucleon-modified URCA process ($n+n \to n+p+e^- +\bar{\nu}_e$ and $n+p+e^- \to n+n+\nu_e$) calculated by Friman and Maxwell is[17]

$$\epsilon_{\rm URCA} \sim 1.8\times10^{21}(m_n{}^*/m_n)^3(m_p{}^*/m_p)(\rho/\rho_0)^{2/3}T_9{}^8, \quad (10)$$

where $m_n{}^*$ and $m_p{}^*$ are the neutron and proton effective masses. In the nucleon-modified URCA process the number of participating degenerate fermions is two larger than for process (4), which leads to tighter phase-space restrictions and a lower emissivity for the former process. When a neutron star contains a pion condensate (for simplicity, I shall call such a star a pion star) the neutrino emission rate[1] is $\epsilon_{\rm pion} \sim 2.2\times10^{26}(\theta^2/0.1)T_9{}^6$ erg cm^{-3} s^{-1}, where the various parameters are chosen at $\rho \sim 2\rho_0$ and θ^2 (~ 0.1) is the square of the angle of chiral rotation. We see that the neutrino emissivities from quark matter and pion condensates are comparable.

We now discuss the implication of the large neutrino emissivity for the cooling of quark stars.[18] Taking the matter to be composed of three component massless quarks, for simplicity, we have from Eq. (3) the specific heat of quark matter, $c_q = 2.5\times10^{20}(\rho/\rho_0)^{2/3}T_9$ erg cm^{-3} K^{-1}, which is to be compared with the specific heat of neutron matter, $c_n = 1.6\times10^{20}(m_n{}^*/m_n)(\rho/\rho_0)^{1/3}T_9$ erg cm^{-3} K^{-1}. Thus, as far as the specific heats are concerned, we expect little difference between neutron matter and quark matter. Let us compare the cooling of pion stars and quark stars: when other cooling mechanisms are neglected both types of stars cool in a similar way, since the temperature dependences of the emissivities and specific heats are the same. Therefore, the ratio of the *internal* temperatures of these stars is time independent, and has the form[19]

$$T_{\rm quark}/T_{\rm pion} \simeq [(c_q/c_n)(\epsilon_{\rm pion}/\epsilon_{q\beta})]^{1/4} = 2.2(\alpha_c/0.1)^{-1/4}(Y_e/0.01)^{-1/12}(\theta^2/0.1)^{1/4}(\rho/\rho_0)^{-1/6},$$

which is close to unity. We therefore expect the *surface* temperatures of these two types of stars to be close to each other, with quark stars being somewhat hotter than pion stars. Estimated surface temperatures of quark and pion stars are both very much lower than those of ordinary neutron stars.[3,5,20,21] Therefore, if surface radiation is observed with a temperature consistent with ordinary neutron stars, one will be able to exclude the possibility of the existence of pion condensates as well as the quark phase. On the other hand an upper limit on the surface temperature which one cannot reconcile with ordinary neutron stars could possibly be an indication of the presence of either a pion condensate or the quark pahse. I stress here that rapid cooling of a star is not a unique signature of a pion star, since quark stars cool almost as fast. Although the dependence of the emissivities on the various parameters is relatively mild, we need more precise knowledge about nuclear physics and quantum chromodynamics (QCD), as well as more detailed cooling calculations, to distinguish quark stars from pion stars.

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Beta Decay in Quark Stars

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It is shown that the energy flux, $\dot{E}_T$, from quark matter in "quark stars" due to the simple URCA process is roughly comparable to that from pion condensates even when quark-quark interaction (QQI) effects are ignored. The form of $\dot{E}_T$ is then independent of the ambiguous electron density and the effective gluon coupling in the "quark star" and accounts for the neutrino in momentum conservation. It is concluded that one need not invoke QQI to explain the recent null x-ray detection of "neutron"-star surface temperatures by the presence of a quark core.

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Evidence that quantum chromodynamics (QCD) is the proper theory of strong interactions and hadrons has been accumulating in recent years. The observation of three-jet events (Berger *et al.*[1]) and large hadron-to-lepton ratios in e^+e^- experiments, the results of deep inelastic scattering of electrons and neutrinos on protons, and the three-jet decay of the Υ (Koller and Krasemann[2]) strongly support the notion of color and the existence of gluons with spin 1. Furthermore, the property of asymptotic freedom in QCD, is comfortably consistent with the naive "parton" models.

Such support for the validity of QCD encourages one to take seriously the detailed calculations, in its perturbative regime, of Baluni[3] and Freedman and McLerran,[4] who find that, at the high baryon densities expected in "neutron" star cores, a phase transition from nuclear matter to quark matter is quite possible. Baluni, in particular, finds that stable quark stars [$n_B(\text{central}) < 2\ \text{fm}^{-3}$] obtain if $0.2 < \alpha_s(3\ \text{GeV}) < 0.3$, where $\alpha_s(3\ \text{GeV}) < 0.3$, where $\alpha_s(3\ \text{GeV})$ is the QCD coupling constant. The new value for $\alpha_s(3\ \text{GeV})$ of 0.23 ± 0.02 derived by Barber *et al.*,[5] as well as the previous estimate of ~ 0.26 by Appelquist and Politzer,[6] are amply within these limits.

The possibility that collapsed stars contain quasifree quarks was suggested in 1970 by Itoh[7] and pursued by Collins and Perrey[8] and Brecher and Caporaso,[9] among others. More recently, Fecher and Joss[10] used the equation of state of Freedman and McLerran[4] and various nuclear equations of state to integrate the Oppenheimer-Volkoff[11] equation of hydrostatic equilibrium. They obtained maximum quark-star masses of from $(1.6-2.3)M_\odot$, maximum moments of from 2×10^{45} to 4.5×10^{45} gm cm^2 and maximum redshifts of from 0.3 to 0.4. None of these results is abnormal or serves to distinguish quark states

