Muon-Induced Fission as a Probe for Fission Dynamics

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(Received 26 November 1979)

Fission of ^{242}Pu induced by radiationless muonic transitions is studied. By solving the time-dependent Schrödinger equation for the muonic wave function in an axially symmetric spatial grid, one finds an observable dependence of the muonic final state—both in the localization of the muon and its excitation probability—on fission dynamics. More refined studies should allow one to extract information about the friction coefficient in fission.

PACS numbers: 25.85.–w, 24.75.+i, 25.30.Ei

Mean lifetimes of muons bound to the fission fragments of actinide nuclei have recently been measured by Schröder *et al.*[1] yielding $\tau_\mu \sim 130$ ns for Np and Pu isotopes. By an analysis of these data in terms of the Goulard-Primakoff formula[2] which describes the muon-capture rates for a wide range of elements, it was found that the muon is predominantly captured by the heavy fragment (with a probability $P_H \sim 0.9$). Since this probability is expected to depend upon the fission dynamics, a comparison of these results with theoretical calculations may shed some light on the time scale and nuclear shapes involved in fission. We will demonstrate in this Letter that experimental values for P_H can be utilized to learn more about the friction parameter.

In an excited muonic atom the $E1$ and $E2$ cascade transitions to the 1s ground state may proceed in radiationless fashion by transferring energy to the nucleus. The maximal transition energies in actinides exceed the neutron separation energy and fission barrier height so that prompt neutron emission or fission becomes possible. In a fission event the muon will either be ionized or remain at one of the fragments from where it decays by weak interaction (muon capture) resulting in delayed-neutron emission. From a measurement of the delayed-neutron production rate in coincidence with the fission fragments, the probability P_H can be deduced.[1]

Since the primary purpose of this Letter is to investigate the presence and order of magnitude of the effect, we decided to use a relatively simple parametrization of the nuclear charge density, not introducing too many parameters while trying to make it sufficiently realistic for this purpose. Therefore, a homogeneous charge distribution and a sharp nuclear surface were utilized. We restricted the shape parameters to the two essential ones in this problem—an elongation parameter and one describing mass asymmetry. The nuclear shapes used consist of two overlapping spheres with a distance of z between their centers. The elongation parameter Q is defined as $Q = z/R_1$ and the mass asymmetry $\alpha = (R_1/R_2)^3$, which in turn are fixed by the requirement of volume conservation. Note that there is no provision for an elongated neck joining the two nascent fragments; for a long neck the wave function should separate into two parts at a later stage, and the effect discussed in this paper should be even more pronounced.

The time development of fission is now approximated by solving the classical equations of motion for $Q(t)$, whereas α is given implicitly as $\alpha(t) = \alpha(Q(t))$, chosen to reproduce the behavior observed in Strutinsky-type calculations: symmetric shapes, $\alpha = 1$, up to the second minimum and then a swift increase to the final value around the second barrier. Figure 1 shows $\alpha(z)$ and also the potential $V(z)$ entering the classical equations of motion for $Q = z/R_1$. This potential was deduced from the experimentally observed barriers and frequencies.[3] The equation of motion

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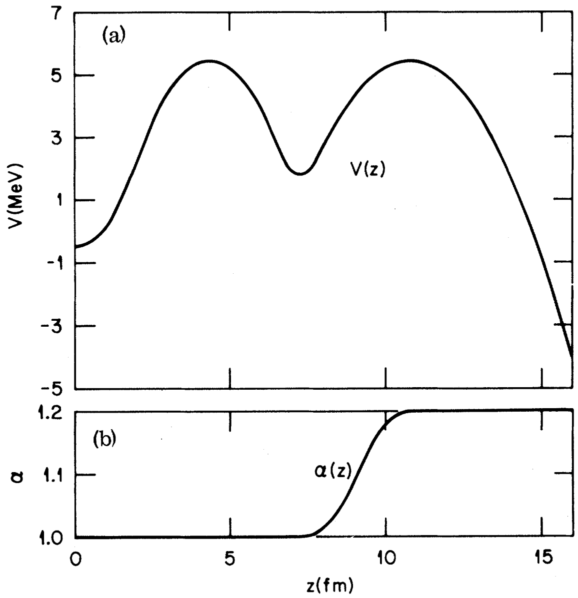

FIG. 1. (a) Empirical fission potential $V(z)$ for ${}^{242}_{94}$Pu. The experimental barrier heights and curvatures are taken from Ref. 3. At large separations $V(z)$ is joined continuously with the Coulomb potential between the fragments. (b) Volume asymmetry parameter $\alpha = (R_1/R_2)^3$. The effect of friction upon the probability P_H was found to be largest for an asymptotic value $\alpha \to A_H/A_L = 1.2$.

for Q reads

$$B_{QQ}\ddot{Q} = -dV/dQ - dE_\mu/dQ - \lambda\dot{Q}. \tag{1}$$

As this is an exploratory calculation, we simply used the reduced mass for evaluating B_{QQ}. This is certainly not a good approximation for small values of Q, but around and beyond the second barrier it does not appear to be too far off[4] and this proved to be the important region. Also, a true B_{QQ} is expected to be larger than the reduced mass, so that our calculation with no friction should give some indication of the upper limit of rapidity in the process. $E_\mu(Q)$ denotes the muonic energy at a given separation Q of the fission fragments. The final term in Eq. (1) is a phenomenological friction term that allows us to study our primary interest: the dependence of the muonic final state on the speed of the fission process.

The muonic wave function $\psi(\vec{r},t)$ is determined from the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_{\rm Coul}(\vec{r}, Q(t))\psi. \tag{2}$$

$V_{\rm Coul}(\vec{r},Q)$ is the Coulomb field produced by the nucleus, determined from the charge density as described above. Since the binding energy of the muon is of the order of 10% of its rest mass, the nonrelativistic treatment is not fully justified, but should, on the other hand, not alter the conclusions dramatically. The instantaneous muon energy $E_\mu(Q)$ is obtained by computing the expectation values of the Laplacian and the Coulomb potential operators numerically. The term involving the derivative of $E_\mu(Q)$ in Eq. (1) accounts for the reaction experienced by the nucleus from the muon. It ensures overall energy conservation and also takes into account the change of the fission barrier due to the presence of the muon; this effect was first studied by Leander and Möller.[5]

As the initial conditions we used a spherical nucleus with the muon in the $1s$ ground state and a nuclear excitation energy $E^* = 8$ MeV, which places the fission mode roughly 2.5 MeV above the fission barrier. Even though the nucleus ${}^{242}_{94}$Pu considered has a deformed ground state, we started the calculations with a spherical state, as it was relatively easy to obtain the $1s$-muonic wave function for radial symmetry. This is not a serious approximation, as the wave function develops adiabatically up to the second saddle point and its later behavior does not depend on the precise starting point. We have tested this by utilizing different functions $\alpha(z)$ leading to the same final mass asymmetry A_H/A_L.

The Schrödinger equation (2) was solved numerically in an axially symmetric spatial grid of 100 points along the symmetry axis of the fissioning system and 40 points in the radial direction with a spacing of 1.5 fm between grid points. The algorithm employed is similar to the one used in time-dependent Hartree-Fock calculations in nuclear physics[6] and recently also in atomic physics.[7]

The algorithm was tested very extensively to ensure its accuracy and stability. In the muonic case we do not have to deal with a singular potential as in the atomic problem, where it caused some numerical difficulties.[7] Thus, our main concern was the other difference from the well-tested nuclear problem: the long tails of the wave functions allowed by the infinite range of the Coulomb potential. Tests showed that the size of our mesh as given above was sufficient to reproduce the binding energy of the $1s$ state to within 2%, and we were able to propagate this wave function in time for a nucleus at rest or a uniformly translating system with a change in energy of 3×10^{-4} and in norm of 2×10^{-4} over 450 fm/c, which is about $\frac{1}{5}$ of the duration of the fission process considered.

Figure 2 exhibits the time development of the separation z between the fission fragments for different friction parameters λ and the corresponding probabilities $P_H(z)$ that the muon concentrates at the heavy fragment. Since we require mass symmetry, $\alpha=1$, until the second minimum is reached (see Fig. 1), the probability P_H equals 0.5 in this region. Between the second minimum and the outer barrier, P_H increases slightly to 0.56 and then starts oscillating during the acceleration period from the second saddle to scission. For separations $z>20$ fm, the probability increases smoothly towards its asymptotic value which is reached around $z=50$ fm. In Table I we have listed the measurable quantity $P_H(t\to\infty)$ as a function of λ. Apparently, P_H increases by about 10% if the friction parameter is changed from $\lambda=0$ to 3000 MeV fm/c.

To give an idea of the order of magnitude of the friction in these cases, we have compared the energy dissipated and the time spent in the descent from saddle to scission with the liquid-drop calculations by Davies, Sierk, and Nix.[8] Since the models and the cases studied do not allow for an exact comparison, we can only infer that our friction coefficient $\lambda=3000$ MeV fm/c corresponds to a viscosity of the order of 0.01–0.03 TP in Ref. 8. So the realistic viscosities found in that paper should lie in the range of sensitivity of the muonic effect studied here.

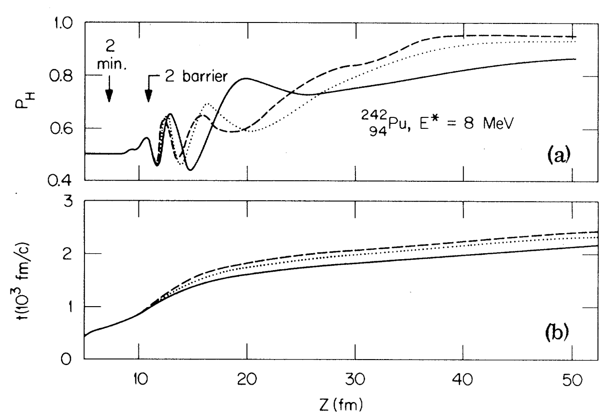

FIG. 2. (a) Probability P_H to find the muon bound to the heavy fission fragment as function of the separation z between the fragment centers. (b) Time development of the separation coordinate z. The results are displayed for different friction parameters λ (MeV fm/c): $\lambda=0$ (solid line), $\lambda=2300$ (dotted line), and $\lambda=3000$ (dashed line).

We have decomposed that part of the muonic wave function localized at one fragment into different angular momenta. Table I also shows the probabilities $W_H(l=1)$ and $W_L(l=1)$ for the muon to be in any p state, given that it is around the heavy or light fragment, respectively. Since the numerical method employed requires a complicated expansion in the radial direction, these results are estimated to be reliable only within a factor of 2. The trend, however, can be established with confidence. There is very little excitation at the heavier fragment, but the excitation of the muon at the light fragment can be quite considerable and depends strongly on the fission dynamics.

From the above results, it is clear that there is a definite observable dependence of the muonic final state on fission dynamics, both in the localization of the muon and in its excitation probability. For the future we plan the use of a more refined and realistic model for the behavior of the nuclear charge distribution during fission and a more accurate determination of the final-state excitation probabilities.

During preparation of this article we have obtained a preprint by Olanders, Nilsson, and Möller[9] yielding similar results for P_H. However, in that paper the fissioning system is approximated by point charges and the muonic wave function is made up of just two basis states. Since our theoretical treatment does not rely on these assumptions, it is felt to be more accurate.

The authors would like to thank Dr. W. W. Wilcke for drawing their attention to this problem. One of us (J.A.M.) also wishes to express his gratitude for the hospitality extended to him

TABLE I. Final probability P_H that the muon is bound to the heavy fission fragment, as a function of the friction coefficient λ. The mass asymmetry is fixed at $A_H/A_L=1.2$, where the influence of friction is largest. $W_H(l=1)$ and $W_L(l=1)$ denote the probabilities for the muon to be in any p state (angular momentum $l=1$), given that it is around the heavy or light fragment, respectively.

λ (MeV fm/c)	P_H(final)	$W_H(l=1)$	$W_L(l=1)$
0	86%	2%	3%
1500	88%	2%	6%
2300	93%	2%	7%
3000	97%	2%	10%

during his stay at Vanderbilt University and Oak Ridge National Laboratory. This research was sponsored in part by the Division of Basic Energy Sciences, U. S. Department of Energy, under Contract No. W-7405-eng-26 with the Union Carbide Corporation.

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Do Quasifree Reaction Mechanisms Explain Reaction Cross Sections in Intermediate-Energy Proton-Nucleus Scattering?

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The 800-MeV (p,p') inclusive proton spectra at forward angles are considered. A plane-wave impulse approximation is used to calculate quasifree nucleon knockout and quasifree isobar production. When the quasifree peaks are normalized to data, it is found that the integrated cross section for the two processes can account for the total reaction cross section.

PACS numbers: 25.40.Ep, 24.10.Fr, 24.50.+g, 25.40.Rb

When intermediate-energy protons are scattered from light nuclei, the inclusive proton spectrum shows strong quasifree knockout peaks at forward angles. Tandy, Redish, and Bollé[1] pointed out that for 160-MeV protons scattering on ^{12}C, the angle and energy integral of the quasifree knockout peaks[2] accounts for nearly the entire reaction cross section. This implies (i) that quasifree processes are the dominant reactions, and (ii) that the loss of flux from the elastic channel should be well approximated by the single-scattering term of the optical potential.[1,3]

Although the (p,p') inclusive spectra show a strong signature of quasifree scattering in the form of a broad peak that moves with the same kinematical relation as free scattering, it is not likely that most of the reactions corresponding to the protons in this peak are actually quasifree knockout, i.e., (p,pN) reactions. Indeed, $(p,2p)$ cross sections suggest that only a small fraction of the struck protons will actually emerge from the nucleus carrying all of the energy decrement. Rather, we conceive of the quasifree scattering process as forming a doorway state in a sense similar to those found in low-energy processes. The projectile strikes a target nucleon as if it were free. The struck system then evolves in a complex way. The validity of the quasifree doorway model only means that as far as the projectile is concerned, the reaction can be treated as quasifree.

It was pointed out in Ref. 1 that for 1-GeV proton data,[4] integration of the quasifree nucleon peaks accounts for one-third of the observed total

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